Liquidity, Moral Hazard, and Interbank Market Collapse*

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This paper proposes a framework to analyze the functioning of the interbank liquidity market and the occurrence of liquidity crises. The model relies on three key assumptions: (i) ex ante investment in liquid assets is not verifiable—it cannot be contracted upon, (ii) banks face moral hazard when confronted with liquidity shocks—unobservable effort can help overcome the shock, and (iii) liquidity shocks are private information—they cannot be diversified away. Under these assumptions, the aggregate volume of capital invested in liquid assets is shown to exert a positive externality on individual decisions to hoard liquid assets. Due to this property, the collapse of the interbank market for liquidity is an equilibrium. Moreover, such an equilibrium is more likely when the individual probability of the liquidity shock is lower. Banks may therefore provision too few liquid assets compared with the social optimum.

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1. Introduction

The financial market turmoil that has been under way since the summer of 2007 hit the core of the global financial system, the interbank market for liquidity. While this paper does not endeavor to account for all the features of the recent crisis, be it hard evidence or casual stories about the motivations of market players, it argues that a proper modeling of the collapse in the market for liquidity involves a close look at incentives to provision/hoard liquidity and moral hazard mechanisms in the interbank market. In addition, it makes sense to do so in a framework where banks can actually fail and default on their borrowing. Both of these assumptions are strongly vindicated by salient features of the recent crisis. Many observers have argued that securitization may have provided the wrong incentives regarding the monitoring of underlying asset quality, in a clear-cut case of moral hazard. In addition, recent developments have shown that bank failure scenarios are only too realistic.

We investigate the possible role of insufficient ex ante liquidity provision in paving the way to an interbank market collapse. We thus highlight the benefits of situations where banks set aside large amounts of liquid assets in order to better deal with shocks affecting their illiquid investments. By liquidity provisions, we mean specifically holdings of assets that can be used to safely transfer wealth over a short period of time. This may be seen as a form of "balance-sheet liquidity." In practice, such liquid holdings could be remunerated reserves held at the central bank, or short-term Treasury securities.[1] Indeed, the secular decline in the share of liquid assets on banks' balance sheets is a striking stylized fact that has been underscored by Goodhart (2008) as a troubling feature of risk management. A situation where market and funding liquidity appeared to be high may thus have hidden vulnerabilities stemming from limited holdings of liquid assets.

Against such a background, this paper shows that across equilibria, the risk-adjusted return on liquid assets can be increasing with

[1]We do not model a risk-free asset market as such; however, we will simply assume that a technology providing a risk-free rate of return is available as an alternative to illiquid investments on the one hand, and to interbank lending on the other hand.

the aggregate volume of such assets in the economy. When a bank faces a liquidity shock, it needs to reinvest in its impaired assets. Moreover, success in reinvestment depends on the effort the bank undertakes. When it has provisioned a large volume of liquidity ex ante, reinvestment is mostly financed through internal funds. Hence, the distressed bank pays particular attention to improving the probability that reinvestment succeeds. Consequently, the moral hazard problem is mitigated and the distressed bank benefits from a large capacity to borrow liquidity on the interbank market. This tends to raise the demand for liquidity and hence the price of liquidity, which in turn raises incentives to provision liquid assets ex ante. As a result, both the risk-adjusted return on liquidity provisioning and the total volume of liquidity in the economy are large.

By contrast, with low ex ante liquidity provision, the argument is reversed: the moral hazard problem is amplified through the aforementioned channel—reinvestment is mostly financed through external funds. Intact lending banks then impose a tight constraint on the volume of liquidity distressed banks can borrow on the interbank market so as to restore their incentives to deliver effort. This, however, reduces the demand for liquidity and drives down the price of liquidity, which in turn depresses banks' incentives to provision liquidity ex ante. Consequently, the risk-adjusted return on liquidity provisioning and the total volume of liquidity in the economy are low. The two polar cases of high and low liquidity provisions can therefore both be equilibrium outcomes.

Turning to comparative statics, the credit-rationing equilibrium happens to be more likely when the liquidity shock is less likely. We call this property *the curse of good times*. When the probability of facing the liquidity shock is low, banks reduce their liquidity holdings because they are less likely to need these liquid assets for reinvestment. This tightens the moral-hazard-induced liquidity constraint, reducing the demand for liquid assets and thereby the return on liquid assets on the interbank market, which in turn reduces incentives to provision liquidity ex ante. Conversely, the equilibrium with large liquidity provision and high risk-adjusted return is more likely when the liquidity shock is more likely, a property we call *the virtue of bad times*. When the probability of facing the liquidity shock is high, banks raise their liquidity holdings because they are more likely to need these provisions for reinvestment. This relaxes the moral-hazard-induced liquidity constraint, raising the demand for liquidity and thereby the price of liquidity on the interbank market, which in turn raises incentives to provision liquidity ex ante. Hence, when the probability of the liquidity shock is intermediate, multiple equilibria emerge: large (resp. low) aggregate investment in liquid assets tends to raise (resp. reduce) the return on liquid assets and thereby raise (resp. reduce) individual incentives to invest in liquid assets.

Finally, the paper investigates how policy can prevent or dampen a collapse of the market for liquidity. The main result is that policies aimed at tackling the collapse of the interbank market ex post—i.e., after the collapse has happened—are unlikely to reach their goal. In particular, liquidity injections as well as interest rate cuts cannot help distressed banks overcome their liquidity shocks. By contrast, ex ante policies, especially those that modify the relative return of liquid assets compared with illiquid assets, can be successful in preventing a collapse of the interbank market. In other words, monetary policy, by setting short-term interest rates which provide incentives to invest in liquid assets, can be helpful in reducing the occurrence of liquidity crises. Regulatory policies requiring liquidity provision can also be useful.

The model in this paper builds on the standard literature on moral hazard and liquidity crisis. The demand for liquidity is modeled in a basic, standard fashion, similar to that of Holmström and Tirole (1998). Agents (in our case, banks) with long-term assets face stochastic liquidity shocks which trigger a reinvestment need and a moral hazard problem: success in reinvestment depends on unobservable effort by banks.[2] We, however, depart from this seminal paper in an important way, by assuming that idiosyncratic liquidity shocks cannot be diversified away: this opens the door to an interbank market where liquidity can be reallocated interim. Because of this feature, our framework is closely related to the

[2]The main alternative modeling of liquidity is based on the Diamond and Dybvig (1983) approach—enriched by Diamond and Rajan (2001)—in which banks with illiquid assets supply liquidity to consumers through liquid deposits (funding liquidity). While this approach can account for bank runs that have taken place during the current financial crisis, the Holmström and Tirole (1998) approach, focused on market liquidity, seems more relevant given the particular initial circumstances of the crisis.

model of liquidity demand developed by Caballero and Krishnamurthy in a series of papers (in particular, Caballero and Krishnamurthy 2004) dealing with access to international financing. Our model shares their features that (i) idiosyncratic shocks cannot be written into insurance contracts, generating the need for domestic financial transactions, and (ii) borrowers cannot transfer the full surplus generated by reinvestment resources. Likewise, we therefore have situations where private decisions are biased against hoarding liquidity.

Our paper is connected to the literature on interbank markets, as a mechanism for managing, and potentially eliminating, risks stemming from idiosyncratic liquidity shocks. Bhattacharya and Gale (1987) in particular studied the case where neither banks' investments in the illiquid technology nor liquidity shocks are observable. In their framework, banks have an incentive to underprovision liquidity ex ante and free-ride the common pool of liquidity. Rochet and Tirole (1996) adapted the Holmström-Tirole framework to the interbank market in order to study systemic risk and "too-big-to-fail" policy. The existence of interbank market imperfections has been established empirically by Kashyap and Stein (2000), which showed the role of liquidity positions, the so-called "liquidity effect." Building on such evidence, Freixas and Jorge (2008) analyzed the functioning of the interbank market in order to show the consequences of its imperfections for monetary policy. In particular, they established the relevance of heterogeneity in banks' liquid asset holdings for policy transmission.

Our work is also related to recent work on liquidity crises. A recent strand of literature has explored the propagation of crises through banks' balance sheets, while treating the level of liquidity held by banks as endogenous. This approach builds on Allen and Gale's (1998) analysis of distressed liquidation of risky assets, to explore the mechanism whereby anticipation of fire-sale pricing of such assets determines banks' ex ante portfolio allocation. Allen and Gale (2004) as well as Acharya, Shin, and Yorulmazer (2007, 2009) have concentrated on this interaction between equilibrium liquidity and endogenously determined fire sales. In particular, Acharya, Shin, and Yorulmazer (2007) showed that banks' holdings of liquidity may be too low or too high compared with the social optimum, depending on the pledgeability of their assets and the possibility to take advantage of fire sales. Interestingly, in their model, liquidity holdings are decreasing in the health of the economy, a result similar to our *curse of good times* property.[3]

In related work, Acharya, Gromb, and Yorulmazer (2008) studied the consequences of imperfect competition in the interbank market for liquidity. In a model where there are frictions in the money and asset markets, if banks that provide liquidity have market power, they may strategically underprovide liquidity and thus precipitate fire sales.

Finally, Caballero and Krishnamurthy (2008) provided a model of crises that features liquidity hoarding and provides a motivation for lender-of-last-resort intervention. Their approach is primarily based on Knightian uncertainty that leads each agent to hedge against the worst-case scenario.

A common feature of this literature is that bank holdings of liquidity are not necessarily optimal. The public provision of liquidity, such as liquidity injections, can therefore often improve on the allocation of liquidity resulting from the decentralized outcome. Our work shares these features. It also rejoins the result of Acharya, Shin, and Yorulmazer (2007, 2009) by which banks or outside arbitrageurs hold too little liquidity in good times.

Our paper, however, departs from this literature in two key aspects. First, the motivation for banks' ex ante provisioning of liquidity is not to have the possibility to purchase low-priced distressed assets, but rather to have the resources to reinvest in its own distressed projects, or to lend on the interbank market. Second, these papers do not feature interbank liquidity crises in the sense of a breakdown in the money market, simply because they typically do not consider interbank lending. While in the "fire-sales" literature the source of inefficiency is liquidation to outsiders, the focus of this paper is the interbank market collapse. Namely, we provide conditions under which the market for liquidity itself (as opposed to the distressed asset market) may cease to function. In addition, we show that the equilibrium where liquidity-affected banks face credit

[3]Acharya, Shin, and Yorulmazer (2009) also feature the result that arbitrage capital is lower in good times, leading to bigger fire-sale discounts.

rationing remains when allowing for liquidation of risky assets once the liquidity shock hits.[4]

In sum, this paper's contribution consists in combining standard features of the moral hazard literature in order to account for a collapse in interbank lending. To the best of our knowledge, the feedback loop between aggregate investment in liquid assets and the return to liquid assets as well as implications in terms of insufficient aggregate liquidity provision and multiple equilibria have not been studied previously.

The paper is organized as follows. The following section lays down the main assumptions of the model. The first-best allocation is derived in section 3. The problem of intact and distressed banks in a second-best environment is analyzed in section 4. Section 5 details the decentralized equilibrium, characterizing the full-reinvestment and credit-rationing equilibria. Section 5 also discusses the nature of the externality at the source of the multiple equilibria property. Section 6 looks at its robustness by relaxing some of the model's assumptions. Section 7 derives some policy implications. Section 8 concludes.

2. Timing and Technology Assumptions

We consider an economy with a unit mass continuum of banks. Banks are risk neutral and maximize expected profits. The economy lasts for three dates: 0, 1, and 2. At date 0, each bank has a unit capital endowment and two investment possibilities. The first is to invest in a liquid asset: a unit of capital invested in the liquid technology at date $t \in \{0; 1\}$ yields one unit of capital at date $t + 1$. The volume of capital that a bank invests at date 0 in the liquid technology is denoted l. Alternatively, each bank can invest in an illiquid project. The volume of capital a bank can invest in an illiquid project at date 0 is hence equal to $1 - l$. The volume of capital invested in each technology is observable but not

[4]This result holds assuming that liquidity-affected banks can borrow on the interbank market against the product of liquidation. If this is not possible—if liquidation takes time, for instance—then interbank market total collapse is still an equilibrium even if liquidity-affected banks can liquidate their risky assets.

verifiable. Contingent contracts on ex ante liquidity provisioning are thus precluded.[5]

Illiquid projects are invested in at date 0. At date 1, they may face a liquidity shock. With probability $1-q$, the liquidity shock is avoided and the bank that has financed the project is said to be "intact." The illiquid project yields R units of capital at date 2 per unit of date 0 investment. With probability q, the liquidity shock occurs and the bank that has financed the project is said to be "distressed." Following Holmström and Tirole (1998), a liquidity shock at date 1 triggers (i) a reinvestment need and (ii) a shirking possibility: a distressed bank which reinvests ck units of capital ($0 < c \leq 1$ and $k \leq 1-l$) and delivers an effort e at date 1 reaps $R(e)k$ units of capital at date 2 with a probability e. With probability $1-e$, it gets nothing. Importantly, effort e is private information and hence a source of moral hazard. Similarly, the liquidity shock is private information and hence cannot be diversified away across banks.[6]

To simplify, and without any implications for further analysis, effort e can be either high, $e = e_h$, or low, $e = e_l$ ($e_h > e_l$), with $R(e_h) = R$ and $R(e_l) = \mu R$ with $\mu > 1$. High effort e_h is efficient and low effort is dominated: $e_l \mu R < 1 < e_h R$.[7] Finally we add the following parameter restrictions: (i) parameter c is normalized to 1, (ii) the illiquid project is more profitable on average than the liquid technology $(1-q)R > 1$, and (iii) moral hazard—scaled by the μ parameter—is sufficiently large, i.e., $\frac{e_h - e_l}{e_h - \mu e_l} > R$.[8]

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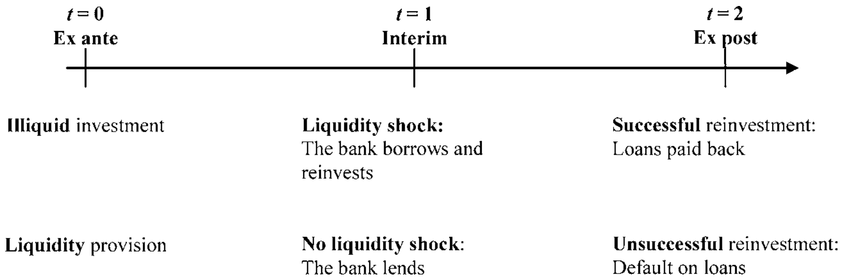
[5]The assumption that ex ante liquidity provisioning is neither observable nor verifiable is a sufficient condition (although not necessary), under which the results of the model hold. In reality, the volume of liquid assets a bank holds at a given point in time may be observable. However, the funding source for these assets—capital or short-term deposits, for instance—is much more difficult to assess for an outside agent in real time. Hence, even observability can be an issue in practice.

[6]The alternative arrangement under which banks would sign ex ante insurance contracts against liquidity shock is not possible here. If banks receive a payment when they declare to be distressed, then "intact" banks would always report untruthfully their situation as "distressed" since (i) liquidity shocks are unobservable and (ii) banks can invest the payment from the insurance contract in the liquid technology from date 1 to date 2 and finally consume the output at date 2.

[7]The parameter μ incorporates private benefits stemming from delivering low effort e_l.

[8]This last parameter restriction ensures that the moral hazard problem does not disappear when the interest rate on the interbank market is sufficiently low.

Figure 1. Timing of the Model



Timing, shown in figure 1, is as follows. At date 0, banks decide on capital allocation between liquid and illiquid assets. At date 1, a fraction q of banks face the liquidity shock. The interbank market then opens, and intact banks can lend to distressed banks. Distressed banks reinvest their own liquidity plus borrowed funds in their illiquid project and deliver some effort. Banks, both intact and distressed, can also invest in the risk-free liquid technology at date 1 if they prefer to do so. Finally, at date 2, distressed banks learn if reinvestment has been successful. If so, they pay back their liabilities.

3. The First-Best Allocation

To derive the first-best allocation, we remove two assumptions regarding market imperfections. First, date 0 allocation between liquid and illiquid assets is now verifiable. Second, both the liquidity shock at date 1 and the effort e delivered by distressed banks are now public information.

Let $(l; k; e)$ be a generic contract where l is date 0 investment in the liquid technology, k is date 1 reinvestment in a project that faces a liquidity shock, and e is effort undertaken in case of reinvestment. The first-best allocation solves

$$\begin{aligned} \max_{l;k;e}\ & (1-q)(1-l)R + qkeR(e) + (l-qk) \\ & \text{s.t. } qk \leq \min\{l; q(1-l)\}. \end{aligned} \tag{1}$$

Each unit of capital endowment is divided between l units of capital invested in the liquid asset and $1-l$ units of capital invested in the illiquid asset. The illiquid asset is intact with probability $1-q$. In this case, it returns $(1-l)R$ at date 2. With probability q, the illiquid asset is distressed. If k units of capital are reinvested in each distressed project, total date 1 reinvestment is equal to qk. Since there are l units of capital available at date 1 for reinvestment, and given that reinvestment k in each distressed project cannot be larger than $(1-l)$, total reinvestment qk cannot be larger than l and $q(1-l)$. Moreover, each distressed project in which k is reinvested yields an expected return $keR(e)$. Finally, when total capital available at date 1 is larger than aggregate reinvestment, $l > qk$, the remaining available capital $l - qk$ is invested in the liquid technology with a unit marginal return. We can then derive the following result.

PROPOSITION 1. *The first-best capital allocation is such that each bank invests l^* units of capital in the liquid technology at date 0 with*

$$l^* = \frac{q}{1+q}\mathbf{1}[e_h > 1-q].$$

Proof. Optimality requires that $e = e_h$ since $e_h R(e_h) > e_l R(e_l)$ and $qk = \min\{l; q(1-l)\}$ since $e_h R > 1$. The problem therefore simplifies as

$$\max_l \ (1-q)(1-l)R + \min\{l; q(1-l)\}e_h R + (l - \min\{l; q(1-l)\}).$$

This problem is piecewise linear in l. So one extreme value of l must be optimal. When $l \leq q(1-l)$, the optimal capital allocation writes as

$$l^* = \frac{q}{1+q}\mathbf{1}[e_h \geq 1-q],$$

where $\mathbf{1}[x]$ is equal to 1 if x is true and zero otherwise. On the contrary, when $l \geq q(1-l)$, then given that $(1-q)R > 1$ and $e_h R > 1$, optimal capital allocation writes as $l^* = \frac{q}{1+q}$.

The first-best optimal ex ante liquidity provision is $l^* = \frac{q}{1+q}$ when $e_h \geq 1-q$ and $l^* = 0$ when $e_h < 1-q$. Typically, when the probability q of the liquidity shock is sufficiently low—i.e., $q < 1-e_h$—then it is not worth provisioning liquidity, because there will be very few illiquid projects hit by the liquidity shock. Put differently, the expected return to illiquid investments without any ex ante liquidity provision $(1-q)R$ is very large. The social planner then prefers to maximize illiquid investments. In what follows, we will assume that the parameter restriction $e_h > 1-q$ always holds so that first-best ex ante liquidity provision is always $l^* = \frac{q}{1+q}$.

4. Intact and Distressed Banks

We now turn to the resolution of the model described in section 2, which can be done by backward induction. We first solve the problem of intact and distressed banks at date 1. Then we solve the date 0 problem of optimal ex ante liquidity provision.

4.1 Distressed Banks' Optimal Demand for Liquidity

Consider bank i which, at date 0, invested l_i units of capital in the liquid technology and $1-l_i$ in an illiquid project. If bank i is distressed at date 1, it can either reinvest in its illiquid project or give up this project and lend its liquid assets on the interbank market. In case a distressed bank reinvests in its illiquid project, d_i denotes the volume of capital it borrows at date 1 and e_i the effort it undertakes. Its date 2 expected profit then writes as

$$\pi_b = e_i[(l_i + d_i)R(e_i) - rd_i]. \tag{2}$$

At date 1, a distressed bank uses the proceeds of its date 0 liquid investments l_i and borrows d_i to reinvest in the illiquid project initiated at date 0. Hence, reinvestment is equal to $l_i + d_i$. Conditional on success, date 2 output net of nonpecuniary cost of delivering effort is $(l_i + d_i)R(e_i)$, the face value of liabilities is rd_i, and e_i is the probability of successful reinvestment. Note that the interest rate r is independent of bank i decisions and in particular of its effort e_i, because effort is unobservable. The problem at date 1 of a distressed bank which reinvests in its illiquid project consists in choosing the effort level e_i and the volume of borrowing d_i which solve the problem

$$\begin{aligned} \max_{d_i;e_i} \pi_b &= e_i[(l_i + d_i)R(e_i) - rd_i] \\ \text{s.t. } l_i + d_i &\leq 1 - l_i. \end{aligned} \tag{3}$$

The constraint that total reinvestment $(l_i + d_i)$ cannot be larger than the reinvestment need $(1 - l_i)$ imposes a limit on the volume d_i that can be borrowed on the interbank market. We can then derive the following proposition.

PROPOSITION 2. *Denoting* $\psi = \frac{e_h - e_l \mu}{e_h - e_l}$, *if the interest rate on the interbank market verifies* $r \leq R$, *a distressed bank's demand for liquidity* d_i *is such that* $l_i + d_i = 1 - l_i$. *It delivers effort* e_i *such that*

$$e_i = \begin{cases} e_h \text{ if } (r - \psi R) d_i \leq \psi R l_i \\ e_l \text{ if } (r - \psi R) d_i > \psi R l_i. \end{cases} \tag{4}$$

Proof. If bank i is distressed and reinvests in its illiquid project, then optimal borrowing d_i^* writes as

$$d_i^* = (1 - l_i - l_i)\mathbf{1}[R(e_i^*) \geq r]. \tag{5}$$

Consequently, as long as $r < R$, $d_i^* = (1 - l_i - l_i)$ and optimal effort e_i^* is given by

$$e_i^* = \begin{cases} e_h \text{ if } r d_i^* \leq \psi R\big(l_i + d_i^*\big) \\ e_l \text{ if } r d_i^* > \psi R\big(l_i + d_i^*\big). \end{cases} \tag{6}$$

A distressed bank is more likely to deliver high effort e_h when reinvestment is proportionally more financed through internal funds—i.e., when ex ante liquidity provisioning l_i is larger and/or borrowing d_i is lower.

Having determined optimal borrowing and effort conditional on reinvestment, we can now examine whether distressed banks prefer to reinvest in their illiquid assets or to give up their illiquid project and lend their liquid holdings on the interbank market. The following lemma derives this choice.

LEMMA 1. *If the interest rate on the interbank liquidity market verifies* $r \leq R$, *then distressed banks always prefer to reinvest in their illiquid project rather than lend their liquid assets on the interbank market.*

Proof. Denoting d_i^* the volume of capital a distressed bank borrows, when the interest rate on the interbank market verifies $r \leq R$, its expected profits from reinvestment π_b then write as

$$\pi_b = e_i \left[R(e_i)\big(l_i + d_i^*\big) - rd_i^* \right],$$

with e_i being the distressed bank's optimal effort. Expected profits π_b' from lending liquid assets on the interbank market are simply $\pi_b' = e_i r l_i$ because the repayment probability of distressed banks is e_i. Given the assumption $R(e_i) \geq r$, d_i^* is always positive and profits from reinvestment π_b are always larger than profits from lending liquid assets on the interbank market.

4.2 Intact Banks' Optimal Supply of Liquidity

We now turn to the case where bank j is intact at date 1. Recall that at date 0 it invested l_j units of capital in the liquid technology and $1 - l_j$ in an illiquid project. It hence reaps $(1 - l_j)R$ at date 2. Moreover, it can lend its liquid assets to distressed banks at date 1. When the interest rate on the interbank market is r, and distressed banks deliver effort e, intact bank j enjoys date 2 expected profits:

$$\pi_g(l_j) = (1 - l_j)R + l_j \max\{er; 1\}. \tag{7}$$

An intact bank can always invest its liquid assets l_j at date 1 in the liquid technology. Hence, intact banks supply their liquid holdings on the interbank market if and only if $er \geq 1$. A distressed bank delivers high effort e_h if and only if its ex ante liquidity provision l_i and its interbank market borrowing d_i verify

$$(l_i + d_i)\psi R \geq rd_i. \tag{8}$$

Given that it borrows at most $(1 - l_i - l_i)$ on the interbank market, there can be two different situations:

(i) If (8) holds for $d_i = (1 - l_i - l_i)$, then the distressed bank always delivers high effort e_h. Intact banks then supply their liquid holdings on the interbank market as long as the interest rate r verifies $e_h r \geq 1$.

(ii) If (8) does not hold for $d_i = (1 - l_i - l_i)$, then the distressed bank delivers low effort e_l and intact banks' participation constraint $er \geq 1$ cannot be met.

When a distressed bank delivers low effort e_l, the interest rate r it is charged cannot be larger than μR—otherwise, the distressed bank would not borrow—and by assumption we have $e_l \mu R < 1$. To make sure that the distressed bank delivers high effort e_h, intact lending banks impose a liquidity constraint. The volume of liquidity the distressed bank can then borrow verifies the incentive constraint:

$$e_h((l_i + d_i)R - d_i r) \geq e_l((l_i + d_i)\mu R - d_i r).$$

Denoting $[x]^+ = \max(x; 0)$, this condition simplifies as a borrowing constraint:

$$d_i \leq \overline{d}(l_i) \equiv \frac{\psi R}{[r - \psi R]^+} l_i. \tag{9}$$

In this case, a distressed bank's total borrowing from the interbank market is a positive function of its ex ante liquidity provision.[9]

5. The Decentralized Equilibrium

In the previous section, we derived the optimal date 1 decision rules for intact and distressed banks in terms of lending, borrowing, and effort. Based on these results, we now turn to the optimal date 0 liquidity provision policy in order to characterize the different equilibria of the economy.

Definition 1. *An equilibrium is an ex ante liquidity provision policy l and an interest rate r on the interbank market such that banks' date 0 expected profits are maximized:*

$$\max_l (1 - q)[(1 - l)R + l \max\{e_h r; 1\}] + q e_h[(l + d)R - rd]$$
$$s.t.\ d = \mathbf{1}(R \geq r) \min\left\{\tfrac{\psi R}{[r - \psi R]^+} l; 1 - l - l\right\}$$

[9]Recall that ex ante liquidity provisions are observable, so that the size of illiquid projects as well as reinvestment needs, assuming a shock has occurred, are also observable. However, the implementation of a borrowing constraint by intact banks on distressed banks requires the additional (implicit yet standard) assumption that total interbank borrowing is observable by lenders. Without such an assumption, no borrowing constraint can ever be enforced.

and the interest rate r balances the supply and the demand of liquidity at date 1; i.e., $L_s = L_d$, with

$$L_s = (1-q)l \text{ and } L_d = q \min\left\{\frac{\psi R}{[r-\psi R]^+} l; 1-l-l\right\}.$$

Aggregate liquidity supply L_s is the sum of intact banks' available liquid assets $(1-q)l$. Aggregate demand of liquidity L_d is the minimum of distressed banks' liquidity constraint and the maximal amount of liquidity these banks need to borrow. The following two subsections are devoted to laying down the conditions under which each of these two situations can be an equilibrium.

5.1 The Full-Reinvestment Equilibrium

5.1.1 Optimal Ex Ante Liquidity Provision with Full Reinvestment

Let us focus first on the case where distressed banks are able to reinvest fully in their illiquid project. Assuming the interest rate on the interbank market verifies $R > r$, the problem of bank i at date 0 then writes as

$$\begin{aligned} \max_{l_i} E\pi_i &= (1-q)[(1-l_i)R + l_i e_h r] + q e_h[(l_i + d_i)R - r d_i] \\ &\text{s.t. } d_i = 1 - l_i - l_i \text{ and } d_i \leq \overline{d}(l_i). \end{aligned} \tag{10}$$

Proposition 3. *Denoting $r_1 = \frac{1-q+qe_h}{1+q}\frac{R}{e_h}$, optimal individual ex ante liquidity provision for a bank that reinvests fully in its illiquid project when distressed is given by*

$$l_i^* = \begin{cases} \dfrac{\frac{r-\psi R}{r}}{1+\frac{r-\psi R}{r}} & \text{if } r \leq r_1 \\ 1 & \text{if } r \geq r_1. \end{cases} \tag{11}$$

Proof. Expected profits are decreasing in ex ante liquidity provision for $r \leq r_1$, since

$$\frac{\partial E\pi_i}{\partial l_i} = (1+q)e_h\left[r - \frac{1-q+qe_h}{1+q}\frac{R}{e_h}\right] \leq 0.$$

Banks then choose to provision as little liquidity as they can. Optimal ex ante liquidity provision then verifies $l_i + \overline{d}(l_i) = 1 - l_i$. On the contrary, expected profits are increasing in ex ante liquidity provision for $r \geq r_1$. Banks then choose to provision as much liquidity as they can; i.e., $l_i^* = 1$. In between—i.e., for $r = r_1$—they are indifferent to ex ante liquidity provisioning.

5.1.2 Equilibrium Interbank Interest Rate with Full Reinvestment

The equilibrium with distressed banks achieving full reinvestment exists if and only if two conditions are met: First, ex ante liquidity provision l_i^* maximizes expected profits; i.e., there should be no profitable deviation ex ante for banks. Second, the aggregate supply of liquidity must balance the aggregate demand for liquidity:

$$(1-q)\int_{[0;1]} l_i^* di = q \int_{[0;1]} (1 - l_i^* - l_i^*) di. \qquad (12)$$

Moreover, the cost of liquidity in the interbank market r must be such that distressed banks are willing to borrow and intact banks are willing to lend their liquid assets on the interbank market:

$$1 \leq e_h r \leq e_h R. \qquad (13)$$

Let us denote $r_2 = \frac{\psi R}{1-q}$ and $r^* = \min\{r_1; r_2\}$. We can then derive the following proposition.

PROPOSITION 4. *The first-best allocation—where banks provision liquidity $l_i = l^*$ and fully reinvest in their project when distressed—is an equilibrium if and only if*

$$(1-q)R \leq e_h r^* \leq e_h R. \qquad (14)$$

Proof. See the appendix.

Conditions (14) are more likely to be verified when the individual probability q of the liquidity shock is high. In other words, the equilibrium with full reinvestment is more likely to hold in deteriorated environments. More precisely, when the equilibrium interest rate is $r^* = r_1$, the individual rationality constraint for intact banks, $e_h r_1 \geq (1-q)R$, is always verified. Similarly, the individual rationality constraint for distressed banks, $r^* \leq R$, always holds since by assumption $e_h \geq 1-q$. Alternatively, when the equilibrium interest rate is $r^* = r_2$, the individual rationality constraint for distressed banks, $r^* \leq R$, is necessarily verified since $r^* = r_2$ implies $r_2 \leq r_1$ and we always have $r_1 \leq R$. Finally, the individual rationality constraint for intact banks, $e_h r_2 \geq (1-q)R$, is more likely to be verified when the probability q to face the liquidity shock is relatively large, since r_2 increases with the probability q.

When the probability q to face the liquidity shock is high, there are on the one hand more distressed banks, but on the other hand, banks raise their liquidity holdings because they are more likely to need these ex ante provisions for reinvestment. At the aggregate level, the former effect dominates and the demand of liquidity from distressed banks on the interbank market is large. This drives up the interbank market interest rate, which provides incentives for banks to provision liquidity ex ante. The full-reinvestment equilibrium is therefore more likely when the liquidity shock is more likely, a property we refer to as *the virtue of bad times*. Note finally that the equilibrium where distressed banks achieve full reinvestment is efficient in the sense that it replicates the first-best capital allocation between liquid and illiquid assets.

5.2 The Credit-Rationing Equilibrium

In the equilibrium described in the previous subsection, distressed banks are able to carry out full reinvestment thanks to their relatively large ex ante liquidity provision. This subsection examines what happens when the volume of liquidity that banks provision ex ante is not sufficiently large to ensure both full reinvestment and high effort.

5.2.1 Optimal Ex Ante Liquidity Provision under Credit Rationing

When the constraint $d_i \leq \overline{d}(l_i)$ on the volume of liquidity that can be borrowed from the interbank market is binding, each distressed bank borrows $\overline{d}(l_i)$ from intact banks. Assuming the cost of borrowing liquidity is lower than the return on reinvestment—i.e., $r < R$—the problem of an individual bank i at date 0 therefore consists in choosing the volume of ex ante liquidity provision l_i which solves

$$\begin{aligned} \max_{l_i} E\pi_i &= (1-q)[(1-l_i)R + e_h r l_i] + q e_h [(l_i + d_i)R - r d_i] \\ &\text{s.t. } d_i = \overline{d}(l_i) \text{ and } d_i \leq 1 - l_i - l_i. \end{aligned} \tag{15}$$

PROPOSITION 5. *Optimal individual ex ante liquidity provision for a bank whose liquidity constraint binds is given by*

$$l_i^* = \begin{cases} 0 \text{ if } \dfrac{\partial E\pi_i}{\partial l_i} \leq 0 \\ \dfrac{\frac{r-\psi R}{r}}{1+\frac{r-\psi R}{r}} \text{ if } \dfrac{\partial E\pi_i}{\partial l_i} \geq 0. \end{cases} \tag{16}$$

Proof. When expected profits are decreasing in ex ante liquidity provision, then banks choose to provision as little liquidity as they can; i.e., $l_i^* = 0$. On the contrary, when expected profits are increasing in ex ante liquidity provision, then banks choose to provision as much liquidity as they can. This level of ex ante liquidity provisioning solves $l_i + \overline{d}(l_i) = 1 - l_i$.

The function $\frac{\partial E\pi_i}{\partial l_i}$ is potentially nonmonotonic in the interest rate on the interbank market. On the one hand, a high interbank market interest rate r raises the return to liquidity for intact banks. On the other hand, however, it raises the cost of borrowing liquidity for distressed banks, and it reduces the volume of liquidity they can borrow on the interbank market. Banks therefore choose low ex ante liquidity provisioning when the interest rate on the interbank market is either very low or very high.

5.2.2 Equilibrium Collapse of the Interbank Market

Given optimal date 0 ex ante liquidity provisioning (16), the aggregate demand of liquidity L_d at date 1 is

$$L_d = q\frac{\psi R}{r-\psi R}\int_{[0;1]} l_i^* di$$

and the aggregate supply of liquidity L_s at date 1 is

$$L_s = (1-q)\int_{[0;1]} l_i^* di.$$

We define a collapse of the interbank market as a situation where banks do not provision liquidity ex ante, and intact banks do not lend to distressed banks. We can then derive the following proposition.

PROPOSITION 6. *The collapse of the interbank market is the unique equilibrium of the credit-rationing regime. It exists if and only if*

$$1 + q\frac{e_h R - 1}{1 - \psi e_h R} < (1-q)R. \tag{17}$$

In this equilibrium, the interest rate verifies $e_h r = 1$.

Proof. See the appendix.

Condition (17)—under which the interbank market collapse equilibrium exists—is more likely to be satisfied when the probability q to face the liquidity shock is relatively low. When the liquidity shock is less likely, banks provision less liquidity ex ante and invest more in illiquid assets. Distressed banks are then more likely to deliver low effort when they reinvest in their illiquid project, as reinvested funds will be mostly borrowed. Intact lending banks then impose credit rationing to ensure that distressed banks deliver high effort. However, credit rationing reduces the demand for liquidity and thereby depresses the return on ex ante liquidity provision for intact banks. This in turn reduces ex ante incentives to provision liquidity, especially when the probability to remain intact is large. The credit-rationing equilibrium is therefore more likely when the liquidity shock is less likely, a property we refer to as *the curse of good times:* an environment with good fundamentals is conducive to credit rationing and interbank market collapse.

5.3 Multiple Equilibria and the General Equilibrium Externality

5.3.1 Multiple Equilibria

When ex ante liquidity provisioning is low, then both liquidity supply and liquidity demand are relatively low. Supply is low because intact banks have relatively few provisions. Demand is also low because the liquidity constraint stemming from moral hazard introduces a positive relationship between aggregate liquidity provisioning and the aggregate demand for liquidity. Hence, with little provisioning, the demand for liquidity is also low. In this case it turns out that the equilibrium interest rate on interbank liquidity is relatively low. This has two opposite consequences: On the one hand, this reduces the return to ex ante liquidity provisioning for intact (lending) banks. On the other hand, it raises the return to ex ante liquidity provisioning for distressed (borrowing) banks because (i) borrowing liquidity is not expensive and (ii) the volume of liquidity that can be borrowed on the interbank market increases with ex ante liquidity provisioning. When the probability q of facing the liquidity shock is relatively low, then the former effect (for intact banks) dominates the latter (for distressed banks), which gives rise to a negative feedback loop: a low expected return on ex ante liquidity provisioning reduces bank incentives to provision liquidity, and low ex ante liquidity provisioning generates a low demand for liquidity, which depresses the expected return on such provisioning. An equilibrium of low ex ante provisioning and low expected return on provisions therefore emerges. As a matter of fact, the necessary and sufficient condition (17) under which the interbank-market-collapse equilibrium exists can be simplified as an upper bound on the probability q of liquidity shocks:

$$q < \overline{q} \equiv \frac{R-1}{R + \frac{e_h R - 1}{1 - \psi e_h R}}.$$

Conversely, when ex ante liquidity provisioning is large, then both liquidity supply and liquidity demand are relatively high. Supply is high because intact banks hold a large volume of liquid assets. Demand is also high because with large ex ante liquidity provisioning, the liquidity constraint is not binding and distressed banks can therefore achieve full reinvestment. When the probability q of facing the liquidity shock is high, the interest rate at date 1 is relatively high because a larger number of banks are distressed, which raises the relative demand for liquidity. The expected return on ex ante liquidity provisioning is then high. This gives rise to a positive feedback loop: a large expected return on ex ante liquidity provisioning raises bank incentives to provision liquidity, while large ex ante liquidity provisions translate into a large expected return on liquid assets. As a result, an equilibrium with high ex ante liquidity provisioning and high expected return on provisions emerges. As a matter of fact, the necessary and sufficient condition (14) under which the full-reinvestment equilibrium appears can be simplified as a lower bound on the probability q of liquidity shocks:

$$q \geq \underline{q} \equiv 1 - \sqrt{e_h \min(e_h; \psi)}.$$

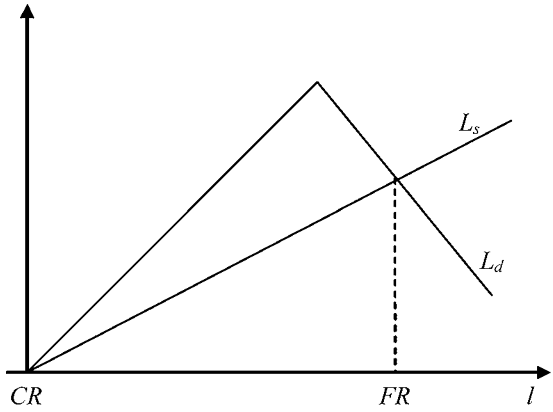
The economy is therefore subject to multiple equilibria when the probability q to face the liquidity shock verifies $\underline{q} \leq q < \overline{q}$. In this region, there is a probability p that agents coordinate on the interbank-market-collapse equilibrium and a probability $1 - p$ that agents coordinate on the full-reinvestment equilibrium. Outside this region the equilibrium is unique. When the probability q is sufficiently high, the full-reinvestment equilibrium occurs with probability one while when the probability q is sufficiently low, the interbank-market-collapse equilibrium occurs with probability one.[10]

5.3.2 Aggregate Supply of and Aggregate Demand for Liquidity

The multiple equilibria property can be examined in a diagram (figure 2) representing aggregate liquidity supply L_s and aggregate

[10]The interbank-market-collapse equilibrium could be eliminated if banks could sign contracts contingent on the volume of date 0 liquidity provisioning. For instance, banks could agree at date 0 to make the cost of borrowing liquidity at date 1 contingent on individual ex ante liquidity provisioning. If the interest rate r charged to distressed bank i writes as $r(l_i) = r^* + (R - r^*)\mathbf{1}[l_i < l^*]$, then bank i's ex ante liquidity provision l_i would always verify $l_i \geq l^*$ and the credit-rationing equilibrium would be ruled out. The assumption that ex ante liquidity provisioning is not verifiable is therefore required to obtain the credit-rationing equilibrium.

Figure 2. Aggregate Supply of and Aggregate Demand for Liquidity



demand L_d as a function of the aggregate ex ante liquidity provision l. Due to the existence of moral hazard, the aggregate demand for liquidity L_d is decreasing in the volume of aggregate ex ante liquidity provisioning l if and only if l is sufficiently large. When provisioning is low, the moral hazard problem binds and the demand for liquidity increases with aggregate ex ante liquidity provisioning.

Liquidity supply L_s is increasing in the volume of aggregate ex ante liquidity provisioning l. As a consequence, there are two equilibria. The credit-rationing equilibrium is situated at point CR, where banks provision no liquidity. The moral-hazard-induced liquidity constraint then binds for distressed banks which cannot borrow liquidity, and intact banks have no liquidity to offer at date 1. If intact banks had liquidity—e.g., assuming intact illiquid projects did generate some output at date 1—they would be compelled to store it in the liquid technology. The full-reinvestment equilibrium is situated at point FR. In this case, the date 1 market for liquidity clears and banks' capital allocation between liquid and illiquid assets is identical to the first-best allocation.

As can be noted from the above discussion, the risk-adjusted return to ex ante liquidity provisioning and the aggregate volume of ex ante liquidity provisioning are higher under the full-reinvestment equilibrium. Hence, across equilibria, the expected return on liquid assets increases with the volume of liquid assets that banks provision ex ante.

6. Extensions

In this section we investigate the robustness of our main result, i.e., the existence of multiple equilibria including the possibility of a collapse in the market for liquidity. To do so, we consider the consequences of relaxing two assumptions made so far.

6.1 Aggregate Shocks

While this model shows that the fragility of the market for liquidity does not necessarily stem from the presence of aggregate shocks, it can easily be extended to allow for such shocks. Suppose, for instance, that the individual probability q to face a liquidity shock can take different values, F denoting the cumulative distribution function for q. Then when $q < \underline{q}$, which happens with probability $F(\underline{q})$, the interbank market collapse is the unique equilibrium and therefore happens with probability one. When $\underline{q} < q < \overline{q}$, which happens with probability $F(\overline{q}) - F(\underline{q})$, there are multiple equilibria and the interbank-market-collapse equilibrium happens with probability p. Finally, when $q > \overline{q}$, which occurs with probability $1 - F(\overline{q})$, the interbank market never collapses. Hence, the unconditional probability θ of an interbank market collapse is given by

$$\theta = F(\underline{q}) + p[F(\overline{q}) - F(\underline{q})].$$

PROPOSITION 7. *An increase in the return to illiquid investment R reduces the unconditional probability θ of a market collapse if and only if*

$$R > \sqrt{\frac{1}{\psi e_h}}.$$

Proof. Deriving the expression for θ with regard to R yields

$$\frac{\partial \theta}{\partial R} = \frac{e_h(1-\psi)}{1-\psi e_h R} \frac{R - \frac{R-1}{1-\psi e_h R}}{\left(R + \frac{e_h R - 1}{1-\psi e_h R}\right)^2} pf\left(\frac{R-1}{R + \frac{e_h R-1}{1-\psi e_h R}}\right),$$

where $f(.)$ is the distribution function for q. This expression is positive if and only if

$$R > \frac{R-1}{1-\psi e_h R},$$

which simplifies as $e_h \psi R^2 > 1$.

An increase in the return to illiquid investment R has two opposite effects. On the one hand, it raises the return to illiquid investments and hence raises banks' incentives to invest in illiquid assets. On the other hand, it raises the return to liquid investment in the credit-rationing regime because a larger return R raises the borrowing capacity on the interbank market and thereby raises incentives to invest in the liquid technology. When the return to illiquid investment is low, the former effect dominates the latter: an increase in R then raises incentives to invest in illiquid assets. As a consequence, the probability of liquidity shocks $\overline{q}$ below which the market-collapse equilibrium is possible tends to increase. On the contrary, when the return to illiquid investment is large, an increase in R reduces incentives to invest in illiquid assets. As a consequence, the probability $\overline{q}$ below which the market-collapse equilibrium exists tends to decrease.

6.2 Interim Liquidation of Illiquid Assets

We have assumed so far the liquidation value of distressed illiquid projects to be zero. Let us assume instead that distressed banks can liquidate (part of) their illiquid projects with a strictly positive liquidation value. Specifically, a distressed bank can liquidate a fraction α of its illiquid project ($0 < \alpha < 1$). It then gets ρ units of capital for each unit of capital liquidated ($0 < \rho < 1$).

Denoting l_i the amount of capital bank i has invested in the liquid technology at date 0, and v_i the part of the illiquid project liquidated at date 1, the date 1 problem of bank i when distressed now writes as

$$\begin{gathered}\max_{d_i;v_i} \pi_b = e_h[(l_i + \rho v_i + d_i)R - rd_i] \\ \text{s.t. } l_i + \rho v_i + d_i \leq 1 - l_i - v_i \text{ and } v_i \leq \alpha(1 - l_i) \\ d_i \leq \frac{\psi R}{r - \psi R}(l_i + \rho v_i).\end{gathered} \tag{18}$$

The distressed bank reinvests $(l_i + \rho v_i + d_i)$, with d_i being what the distressed bank borrows on the interbank market. Hence, its profit conditional on reinvestment being successful is $(l_i + \rho v_i + d_i)R - rd_i$, while e_h is both the effort the distressed bank undertakes and the probability that reinvestment is successful. Finally, the distressed bank i faces the following three constraints. First, reinvestment $(l_i + \rho v_i + d_i)$ cannot be larger than the illiquid project's size $(1 - l_i - v_i)$. Second, the distressed bank cannot liquidate more than a fraction α of its illiquid project. Third, the distressed bank faces an incentive constraint stemming from the moral hazard problem: what a distressed bank can borrow on the interbank market is at most a fraction $\frac{\psi R}{r - \psi R}$ of the distressed bank's own available capital $(l_i + \rho v_i)$ at date 1.[11]

Expected profits of an intact bank are modified as follows: if bank i has invested ex ante l_i units of capital in the liquid technology and $1 - l_i$ units of capital in the illiquid technology, then it reaps $(1 - l_i)\beta + l_i$ at date 1, with β being the interim marginal return to an intact illiquid project. Intact bank expected profits therefore write as

$$\pi_g = (1 - l_i)R + [(1 - l_i)\beta + l_i] \max\{e_h r; 1\}. \tag{19}$$

PROPOSITION 8. *When distressed banks can liquidate interim a fraction α of their illiquid assets with a marginal return ρ, and intact banks enjoy an interim marginal return β on their illiquid assets, then a credit-rationing equilibrium where*

(i) banks make no liquidity provision at date 0,

(ii) distressed banks are unable to achieve full reinvestment, and

[11] This incentive constraint is based on the implicit assumption that ex ante liquidity provision l_i and interim liquidation v_i are both observable when the interbank market opens. In reality, interim liquidation v_i is likely to be more difficult to observe than ex ante liquidity provision when the interbank market opens because liquidating assets takes time. Put differently, there are very few assets that banks can liquidate over night. Moreover, interim liquidation could well happen at the same time or even *after* distressed banks borrow on the interbank market. In this case—where liquidation would happen after borrowing on the interbank market takes place—then allowing for interim liquidation does not change any of the properties of the model.

(iii) intact banks store part of their liquid assets in the liquid technology at date 1, exists if and only if the parameters α, β, and ρ verify

$$1 + q\frac{e_h R - 1}{1 - e_h \psi R} \leq (1-q)(R+\beta) + q\alpha\rho\frac{e_h R[1-\psi]}{1 - e_h \psi R}$$
$$q\frac{e_h \psi R}{1 - e_h \psi R}\rho\alpha < (1-q)\beta$$
$$\rho\frac{\alpha}{1-\alpha} \leq 1 - e_h \psi R.$$

Proof. See the appendix.

Here the possibility for banks to borrow in the interbank market based on liquid assets l and liquidated distressed projects v prevents a total collapse of the interbank market. However, when the share α of illiquid assets that distressed banks can liquidate is sufficiently low, there is still a credit-rationing equilibrium in which some liquidity is traded on the interbank market as opposed to the previous credit-rationing equilibrium where a total collapse of the interbank market takes place. However, distressed banks still face credit rationing and are still unable to achieve full reinvestment.

7. Policy Implications

In this section we investigate whether and how policy can avoid a collapse of the interbank market. To do so, we focus on two types of public interventions. First we look at ex post interventions, i.e., policies that take place after the interbank market has collapsed. Then we focus on ex ante interventions, i.e., interventions aiming at preventing the collapse of the interbank market.

7.1 Ex Post Interventions

There are basically two types of interventions that can take place after the interbank market has collapsed: liquidity injections and changes in interest rates which modify the return on the liquid technology. Typically, a central bank can lend liquidity to distressed banks when the interbank market does not function. It can also influence the cost of liquidity by modifying short-term interest rates. In our case, both these policies are unlikely to be successful in helping distressed banks to achieve reinvestment. Given that banks do not make any ex ante liquidity provision in the equilibrium where the interbank market collapses, any loan from the central bank or from any intact bank violates the incentive constraint stemming from moral hazard. This implies that liquidity injections from the central bank toward distressed banks—assuming the central bank can distinguish between intact and distressed banks—would end up financing negative net-present-value projects, as distressed banks would deliver low effort given that reinvestment is fully financed with external funds. In other words, unless the central bank has access to a monitoring technology that market participants do not have access to, liquidity injections are doomed to fail.

Similarly, cutting interest rates to dampen the effects of a market collapse is unlikely to work. In theory, a reduction in interest rates relaxes the moral hazard problem and raises distressed banks' incentives to deliver high effort. As a consequence, the incentive-compatible level of interbank borrowing is larger with a lower interest rate. However, this effect depends on banks' ex ante liquidity provisions. Given that banks make no ex ante liquidity provision in the equilibrium with a market collapse, the reduction in interest rates does not modify distressed banks' borrowing capacity, which remains at zero. The positive impact of an interest rate cut on distressed banks' borrowing capacity depends positively on banks' ex ante liquidity provision. Hence, interest rate cuts are most effective when banks have made relatively large ex ante liquidity provisions. In a nutshell, interest rate cuts are most effective when not needed.

7.2 Ex Ante Interventions

A regulator can affect the banks' date 0 allocation of capital by imposing a liquidity ratio, requiring that banks invest at least some fraction of their portfolio in liquid assets. Imposing this type of regulation eliminates the equilibrium characterized by a collapse in the interbank market. However, one of the important assumptions of the model is that liquidity is not contractible; i.e., it is not possible to write contracts contingent on the share of assets invested in the liquid technology. Yet imposing a liquidity ratio is equivalent to writing such a contingent contract between the regulator and banks, stating that the bank would be shut down if the share of liquid assets was lower than a given threshold. Imposing such a regulation in this type of model therefore ends up giving discretion to the regulator, which can be costly for reasons outside the scope of this paper (e.g., in terms of capture of the regulator by the regulated agents).

A central bank can, however, affect the return to liquid assets through its policy rates. In particular, the central bank can raise the return to liquid assets between date 0 and date 1 to raise banks' incentives to invest in liquid assets and thereby prevent the collapse of the interbank market at date 1. Assume that the central bank can (at no cost) modify the return r_0 on liquid assets between date 0 and date 1. We can then derive the following result.

PROPOSITION 9. *The central bank can always prevent the collapse of the interbank market by imposing an interest rate r_0 such that*

$$r_0 > \frac{R}{1 + \frac{q}{1-q}\frac{e_h(1-\psi)R}{1-e_h\psi R}}.$$

Proof. Let us consider the credit-rationing regime where distressed banks' borrowing constraint binds. Denoting r_0 the return to liquid assets between date 0 and date 1, bank i date 0 expected profits write as

$$\pi_i = (1-q)R(1-l_i) + e_h r\left[1 - q + q\frac{(1-\psi)R}{r-\psi R}\right] r_0 l_i$$

and bank i optimal liquidity provision l_i^* writes as follows:

$$l_i^* = \begin{cases} 0 \text{ if } (1-q)R \geq e_h r\left[1-q+q\dfrac{(1-\psi)R}{r-\psi R}\right] r_0 \\ \dfrac{\frac{r-\psi R}{r}}{r_0 + \frac{r-\psi R}{r}} \text{ if } (1-q)R \leq e_h r\left[1-q+q\dfrac{(1-\psi)R}{r-\psi R}\right] r_0. \end{cases}$$

Hence, any return r_0 verifying

$$r_0 > \frac{(1-q)R}{e_h r\left[1-q+q\frac{(1-\psi)R}{r-\psi R}\right]}$$

will preclude the collapse of the interbank market since then banks will make ex ante liquidity provision $l_i^* = \frac{\frac{r-\psi R}{r}}{r_0 + \frac{r-\psi R}{r}}$ and thus distressed banks will be able to carry out full reinvestment.

Given that the right-hand side of the above inequality is decreasing in the interest rate r, the above inequality always holds if it holds for the lowest possible interest rate, i.e., when $e_h r = 1$. In this case, the inequality simplifies as

$$r_0 > \frac{(1-q)R}{1-q+q\frac{(1-\psi)e_h R}{1-e_h \psi R}}.$$

The bottom line is therefore that a sufficiently high interest rate ex ante, by raising banks' incentives to invest in liquid assets, can help avoid the collapse of the interbank market.

8. Conclusion

The model we analyzed in this paper provides a framework for analyzing the occurrence of liquidity crises and discussing policy responses to situations of interbank market collapse. To the extent that such a collapse may be explained by the ingredients we focus on (in particular, moral hazard and nonverifiability of ex ante liquidity provisions), this model provides some insights on the scope for ex ante policies to prevent this outcome. In addition, this framework presumably lends itself well to the analysis of the role of international liquidity and its impact on domestic liquidity provision in an open-economy setting. These are possible research avenues for future work.

Appendix

Proof of Proposition 4: The Full-Reinvestment Equilibrium

When distressed banks achieve full reinvestment, the equilibrium interest rate cannot verify $r > r_1$, since banks would then invest their capital in liquid assets and the interbank market would be in excess supply at date 1. The equilibrium interest rate therefore always verifies $r \leq r_1$. When $r < r_1$, then each bank makes ex ante liquidity provisions $l(r) \equiv \frac{\frac{r-\psi R}{r}}{1+\frac{r-\psi R}{r}}$. The equilibrium interest rate is $r = r_2$, which yields an equilibrium ex ante liquidity provision $l = l(r_2) = l^*$. When $r = r_1$, then the equilibrium volume of liquidity each bank provisions ex ante is $l = l^*$. When banks achieve full reinvestment, they always provision the first-best volume of liquidity, and the equilibrium interest rate on the interbank market is $r^* = \min\{r_1; r_2\}$. To determine whether this case is an equilibrium, let us examine if there are profitable deviations. A bank can deviate by provisioning a lower level of liquidity. Assuming the interest rate on the interbank market verifies $r \leq R$, then the profit of a deviating bank is

$$\pi_d = (1-q)(1-l_i)R + e_h r\left(1 - q + q\frac{[1-\psi]R}{r - \psi R}\right) l_i.$$

Denoting $\frac{\partial E\pi}{\partial l_i} = e_h r(1 - q + q\frac{(1-\psi)R}{r-\psi R}) - (1-q)R$, the optimal ex ante liquidity provision policy of the deviating bank l_d is given by

$$l_d = \begin{cases} 0 \text{ if } \dfrac{\partial E\pi}{\partial l_i} \leq 0 \\ l(r) \text{ if } \dfrac{\partial E\pi}{\partial l_i} \geq 0 \end{cases},$$

where r is the equilibrium interest rate when banks achieve full reinvestment; $r = r^*$. If the interest rate r^* is such that $\frac{\partial E\pi}{\partial l_i} \geq 0$, then the deviating bank provisions $l_d = l(r^*)$. In this case, deviation is not strictly profitable, since we have $\pi_d = \pi_h$. On the contrary, if the interest rate on the interbank market r^* is such that $\frac{\partial E\pi}{\partial l_i} \leq 0$, then the deviating bank chooses to make no ex ante liquidity provision $l_d = 0$. Deviation is then profitable if and only if

$$(1-q)R > e_h r^*.$$

When $r^* = r_2$, this inequality simplifies as $e_h < 1 - q$. By assumption, this inequality never holds, since we have $e_h \geq 1-q$. When the interest rate is $r^* = r_1$, deviation is profitable if and only if

$$1 - q < \frac{1 - q + e_h\phi}{1-q}.$$

However, since by assumption we have $e_h \geq 1 - q$, this condition cannot be satisfied. As a consequence, there are no profitable deviations, and the situation where banks achieve full reinvestment is an equilibrium.

Proof of Proposition 6: The Credit-Rationing Equilibrium

This proof is divided into two parts. The first part establishes that (17) is a necessary and sufficient condition for the existence of a market-collapse equilibrium. The second part shows that the market-collapse equilibrium is the unique equilibrium in the credit-rationing regime.

To establish that (17) is a necessary and sufficient condition for the existence of a market-collapse equilibrium, we proceed in two steps.

First Step. Assume that the liquidity constraint $d_i \leq \overline{d}(l_i)$ binds. Then distressed banks borrow $d_i = \overline{d}(l_i)$ from the interbank market and the first-order condition to the problem of an individual bank implies that zero ex ante liquidity provision is optimal if and only if $\frac{\partial E\pi}{\partial l_i} < 0$; i.e.,

$$e_h r \left[1 - q + q\frac{(1-\psi)R}{r - \psi R}\right] < (1-q)R.$$

When optimal ex ante liquidity provision l_i^* is zero, the demand for liquidity is $L_d = 0$ and the supply of liquidity is $L_s = 0$. Hence, any interest rate r verifying $r > \psi R$ and

$$e_h r \left[1 - q + q\frac{(1-\psi)R}{r - \psi R}\right] < (1-q)R$$

is an equilibrium interest rate of the interbank market. In particular, $r = e_h^{-1}$ is such an equilibrium interest rate if and only if $e_h \psi R < 1$—which by assumption always holds—and

$$1 + q\frac{e_h R - 1}{1 - e_h \psi R} < (1-q)R.$$

When this last condition is verified, the situation where banks do not provision liquidity ex ante is possibly an equilibrium and the liquidity constraint $d_i \leq \overline{d}(l_i)$ is indeed binding.

Second Step. Let us now show that the liquidity constraint $d_i \leq \overline{d}(l_i)$ is always binding when (17) holds and $e_h r = 1$. To do so, consider a bank that decides to provision ex ante a volume of liquidity such that the liquidity constraint $d_i \leq \overline{d}(l_i)$ does not bind. Given that the interest rate on the interbank market verifies $e_h r = 1$, the bank's expected profits π_d writes as

$$\pi_d = (1 - q)[(1 - l_i)R + l_i] + q[(1 - l_i)(e_h R - 1) + l_i].$$

Moreover, the liquidity constraint does not bind if and only if the bank's ex ante liquidity provision l_i verifies

$$l_i \geq l(e_h^{-1}) = \frac{1 - e_h \psi R}{1 + 1 - e_h \psi R}.$$

The bank can then achieve full reinvestment. Expected profits π_d are strictly decreasing in ex ante liquidity provisioning l_i because

$$\frac{\partial \pi_d}{\partial l_i} = -[(1 - q)R - 1 + q(e_h R - 1)]$$

and, by assumption, we have $(1 - q)R > 1$ and $e_h R > 1$. As a consequence, the optimal ex ante liquidity provision l_d of a bank seeking to maximize π_d is $l_d = l(e_h^{-1})$. Its optimal expected profits π_d can hence be written as

$$\pi_d(l_d) = (1 - q)[(1 - l_d)R + l_d] + q e_h \frac{(1 - \psi)R}{1 - e_h \psi R} l_d.$$

However, when a bank does not provision liquidity, expected profits are $(1 - q)R$. The zero ex ante liquidity provision policy is therefore optimal if and only if $(1 - q)R > \pi_d(l_d)$. This inequality simplifies as (17), which by assumption is supposed to hold. As a consequence, when (17) holds, it is never optimal for a bank to provision ex ante a volume of liquidity such that the liquidity constraint $d_i \leq \overline{d}(l_i)$ does not bind. The situation where banks do not provision liquidity is hence an equilibrium if and only if (17) holds, in which case the interest rate on the interbank market verifies $e_h r = 1$.

We now turn to establishing the unicity of the market-collapse equilibrium in the credit-rationing regime. Suppose banks make strictly positive ex ante liquidity provision while being credit constrained. There may be two types of such equilibria.

First, we examine whether the case where banks' optimal ex ante liquidity provision is $l(r)$ and

$$\left[1 + q\frac{R - r}{r - \psi R}\right] e_h r \geq (1 - q)\, R \tag{20}$$

can indeed be an equilibrium of the economy. When banks' ex ante optimal liquidity provision is $l(r)$, the equilibrium interbank market interest rate is necessarily $r = r_2$. Otherwise, the interbank market would not be balanced. Banks' expected profits then write as $e_h r_1$. Let us now show that the strategy which consists in provisioning a larger volume of liquidity is more profitable. When $r_2 < r_1$, then a bank that wants to achieve full reinvestment chooses to provision the same volume of liquid assets $l(r)$, and expected profits are identical. On the contrary, if $r_2 > r_1$, then a bank that wants to achieve full reinvestment chooses to invest all its capital in liquid assets, $l_d = 1$, and its expected profit is $e_h r_2$, which by assumption is larger than $e_h r_1$. As a consequence, the situation where banks provision a volume of liquidity $l(r)$ and (20) holds cannot be an equilibrium.

Second, we examine the case where banks are indifferent to provisioning ex ante any volume of liquid assets in $[0; l(r)]$ and

$$\left[1 + q\frac{R - r}{r - \psi R}\right] e_h r = (1 - q)R. \tag{21}$$

In this case, banks' expected profits write as $(1-q)R$. If the interest rate r which solves (21) is such that $r < r_1$, then a bank that wants to achieve full reinvestment would choose to provision a volume of liquidity $l(r)$. In this case, expected profits are identical. On the contrary, if the interest rate r which solves (21) is such that $r > r_1$, then a bank that wants to achieve full reinvestment would invest all its capital in liquid assets $l = 1$, and its expected profit would be $e_h r$. This situation is an equilibrium if and only if the interest rate r which solves (21) verifies the condition

$$e_h r < (1 - q)R. \tag{22}$$

Given that we consider the case where $r > r_1$, a necessary condition for this situation to be an equilibrium is that (22) must hold for $r = r_1$. This necessary condition simplifies as

$$e_h < 1 - q,$$

which by assumption does not hold. Consequently, the situation where the interbank market interest rate verifies (21) and banks are indifferent to provisioning any amount l_i of liquid asset such that $0 \leq l_i \leq l(r)$ cannot be an equilibrium. The equilibrium with zero ex ante liquidity provision and interbank market collapse is therefore the only equilibrium, when it exists, in the credit-rationing regime.

Proof of Proposition 8: Credit-Rationing Equilibrium with Interim Liquidation

Given its program (18), a distressed bank's optimal choices are as follows: optimal liquidation is such that $v = \alpha(1-l)$ and optimal borrowing d on the interbank market writes as

$$d = \min\left\{1 - 2l - \alpha(1+\rho)(1-l); \frac{\psi R}{r - \psi R}(l + \rho\alpha(1-l))\right\},$$

with l being the bank's optimal ex ante liquidity provision. Assuming the distressed bank's liquidity constraint is binding and assuming the interest rate r on the interbank market satisfies participation constraints—i.e., $1 < e_h r < e_h R$—the equilibrium at date 1 on the interbank market writes as

$$(1-q)[(1-l)\beta + l] = q\frac{\psi R}{r - \psi R}(l + \rho\alpha(1-l)).$$

Having determined banks' date 1 decisions, we can turn to banks' date 0 problem, which writes as

$$\max_l (1-q)[(1-l)R + [(1-l)\beta + l]\max\{e_h r; 1\}] + q e_h r\frac{R[1-\psi]}{r - \psi R}(l + \rho\alpha(1-l)).$$

The solution is given by

$$l = \begin{cases} \lambda(r) \text{ if } \left[(1-q)(1-\beta) + q\frac{R[1-\psi]}{r-\psi R}(1-\rho\alpha)\right] e_h r \geq (1-q)R \\ 0 \text{ if } \left[(1-q)(1-\beta) + q\frac{R[1-\psi]}{r-\psi R}(1-\rho\alpha)\right] e_h r \leq (1-q)R, \end{cases}$$

where $\lambda(r)$ is such that

$$1 - 2\lambda(r) - \alpha(1+\rho)(1-\lambda(r)) = \frac{\psi R}{r - \psi R}(\lambda(r) + \rho\alpha(1-\lambda(r))).$$

When banks choose $l = 0$, the equilibrium interest rate on the interbank market would write as $r = \psi R(1 + \frac{\rho\alpha}{\beta(1-q)})$. However, for α and/or ρ sufficiently low compared with β, i.e.,

$$q\frac{e_h\psi R}{1 - e_h\psi R}\rho\alpha < (1-q)\beta,$$

intact banks' participation constraint is violated. As a consequence, the "equilibrium" interest rate is such that $e_h r = 1$, distressed banks face rationing in their demand for liquid assets, and there is an excess supply on the interbank market.

Hence, banks make no liquidity provision ex ante and intact banks store part of their liquid assets in the liquid technology at date 1 instead of lending on the interbank market if and only if

$$q\frac{e_h\psi R}{1 - e_h\psi R}\rho\alpha < (1-q)\beta$$

$$1 + q\frac{e_h R - 1}{1 - e_h\psi R} \leq (1-q)(R+\beta) + q\alpha\rho\frac{e_h R[1-\psi]}{1 - e_h\psi R}.$$

Finally, distressed banks are unable to achieve full reinvestment if and only if

$$\rho\alpha + \frac{\psi R}{r - \psi R}\rho\alpha < 1 - \alpha,$$

where the interest rate r verifies $e_h r = 1$. This inequality can be simplified as

$$\rho\frac{\alpha}{1-\alpha} \leq 1 - e_h\psi R.$$

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