Some Methodological Suggestions

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Methodological suggestions are made for two separate issues. First, I show how a consistent estimate of the *level* of the expected inflation can be gleaned from inflation swap rates. Second, I indicate how the dynamic general equilibrium model in question can be modified to generate the observed persistence in commodity price movements.

JEL Codes: E52, G13, Q43.

1. Introduction

Rather than providing a big picture about monetary policy, I appoint myself as a discussant of the two papers presented in this session, one on inflationary expectations by Galati, Poelhekke, and Zhou, and the other on the recent commodity boom by Erceg, Guerrieri, and Kamin. For each of the two papers, which I take to be the best industry practice, I will suggest an alternative methodology. My commentary is almost self-contained; the reader should be able to get the main messages without reference to the two underlying papers.

2. How to Estimate the *Level* of the Expected Inflation Rate

The Galati, Poelhekke, and Zhou paper examines the inflation expectations that can be inferred from financial markets. There are two sources: inflation-indexed bonds and inflation swaps. The methodological challenge is to disentangle the risk premium from expectations. I will focus on inflation swaps rather than on indexed bonds because my methodological suggestion can be stated more cleanly for the former.

A zero-coupon inflation swap, which is a promise to exchange the value proportional to the level of the CPI (consumer price index) at maturity for an amount agreed upon today and which involves no exchange of cash flows until maturity, is a futures contract. Since swaps for different maturities are traded, one can construct, much as we do routinely for bonds, the constant-maturity "yield curve" for inflationary expectations. For example, say today's date is September 16, 2010, and say two different kinds of contracts are traded: those for the end of August 2020 delivery and those for the end of September 2020 delivery. One can construct a measure of the expected inflation rate over the next ten years from an average of the August 2020 swap rate and the September 2020 rate. Therefore, in what follows I can proceed as if swap/futures contracts are like forward contracts in that prices are available for any given maturity at any date.

Having noted an analogy between swaps and futures, let me provide a refresher on the language of futures markets. I do this because things routinely done in the futures/forward market literature will turn out to be useful in the context of inflationary expectations. Let S_t be the spot price on date t and $F_{t,M}$ be the futures price on date t for delivery M periods hence, on date $t+M$ (as just argued, we can pretend that for any given maturity M there are futures contracts traded on date t). The *excess return*, the *risk premium*, and the *basis* are defined as

$$\text{(Excess Return)} \qquad ER_{t,t+M} \equiv \frac{(S_{t+M} - F_{t,M})/F_{t,M}}{M}, \qquad (1)$$

$$\text{(Risk Premium)} \qquad RP_{t,M} \equiv \mathrm{E}_t(ER_{t,t+M}), \qquad (2)$$

$$\text{(Basis)} \qquad Basis_{t,M} \equiv \frac{(S_t - F_{t,M})/F_{t,M}}{M}, \qquad (3)$$

where, as usual, E_t is the expectations operator conditional on information available at t. The deflation by M is for stating the quantities per unit of time. Equation (1) defines the return from taking a long position in the futures market, expressed as a fraction of the notional $F_{t,M}$. It is an excess return because taking a position requires no cash. Equation (2) shows that the ex post risk premium is the excess return.

Using the log-linear approximation (that $\log(1 + x) \approx x$) and writing natural logs in lowercase letters (e.g., $s_t \equiv \log(S_t)$), we can derive the following equations:

$$\text{(Excess Return)} \quad ER_{t,t+M} = \frac{1}{M}(s_{t+M} - f_{t,M}), \tag{4}$$

$$\text{(Risk Premium)} \quad \frac{1}{M}\mathrm{E}_t(s_{t+M}) = \frac{1}{M} f_{t,M} + RP_{t,M}, \tag{5}$$

$$\text{(Basis)} \quad Basis_{t,M} = \frac{1}{M}(s_t - f_{t,M}), \tag{6}$$

$$\text{(ER Decomposition)} \quad ER_{t,t+M} = \frac{1}{M}[s_{t+M} - \mathrm{E}_t(s_{t+M})] + RP_{t,M}. \tag{7}$$

The last equation, equation (7), can be derived by eliminating $f_{t,M}$ from (4) and (5). Equation (5) emphasizes that the expected price differs from the futures price by the risk premium. In the context of foreign exchange forward contracts, s_t is the spot exchange rate and $f_{t,M}$ is the M-period forward rate, so the forward premium (the difference between the spot and forward exchange rates) is the basis.

The familiar efficient market hypothesis for foreign exchange markets is that the expected spot rate equals the forward rate, namely that the risk premium is zero. As the large literature on the forward premium puzzle shows, the existence of the risk premium has long been suspected for the foreign exchange markets. Recent work on other markets reports that the risk premium is sizable in commodity futures (Gorton and Rouwenhorst 2006) and in emerging-market currencies (Gilmore and Hayashi 2008).

The risk premium is also suspected to be time dependent because it appears to be related to observable variables. A standard way to document this is to run the *excess return regression* in which the ex post risk premium, which is the excess return, is regressed on the basis:

$$ER_{t,t+M} = \alpha + \beta Basis_{t,M} + \text{error}. \tag{8}$$

This is a very familiar regression in disguise. Substituting (4) and (6) into this regression and rearranging, one obtains

$$\frac{1}{M}(s_{t+M} - s_t) = \alpha + (1-\beta)\left(\frac{1}{M}(f_{t,M} - s_t)\right) + \text{error}. \tag{9}$$

This is none other than the "Fama regression" in the forward premium puzzle literature: the dependent variable is the spot return and the regressor is the forward premium. The forward premium puzzle is that typically the coefficient $1-\beta$ in the Fama regression is *negative* and sometimes less than -1, implying that the basis coefficient in the excess return regression is far greater than unity.

Getting back to the equivalence between zero-coupon inflation swaps and futures, set $S_{t+M} = CPI_{t+M}/CPI_t$ (where CPI_t is the level of the CPI at t) and define $y_{t,M}$ by the relation $\exp(y_{t,M}M) = F_{t,M}$ (i.e., $y_{t,M} \equiv \frac{1}{M} f_{t,M}$). This $y_{t,M}$, called the *break-even inflation rate* over M periods, is the expected inflation measure one can infer from swaps. The four equations, (4)–(7), can be written as

$$ER_{t,t+M} = \underbrace{\frac{1}{M}[\log(CPI_{t+M}) - \log(CPI_t)]}_{\text{actual inflation over } M \text{ years}} - y_{t,M}, \tag{10}$$

$$\underbrace{\mathrm{E}_t\left[\frac{1}{M}(\log(CPI_{t+M}) - \log(CPI_t))\right]}_{\text{expected inflation rate over } M \text{ years}} = y_{t,M} + RP_{t,M}, \tag{11}$$

$$Basis_{t,M} = -y_{t,M}, \tag{12}$$

$$ER_{t,t+M} = \underbrace{\frac{1}{M}[\log(CPI_{t+M}) - \mathrm{E}_t(\log(CPI_{t+M}))]}_{\text{unexpected inflation over } M \text{ years}} + RP_{t,M}. \tag{13}$$

Equation (10) shows how the excess return is calculated from data on the CPI and the break-even inflation rate. That the expected inflation measure (i.e., the break-even inflation rate) differs from the true expected inflation by the risk premium is emphasized this time by (11). Equation (12) shows that (the negative of) the break-even inflation rate is the basis.

The Galati, Poelhekke, and Zhou paper assumes that the change in the risk premium is a linear function of a vector of observable variables, denoted $\mathbf{Z}_t$. Here, I assume that the *level* of the risk premium is a linear function of $\mathbf{Z}_t$:

$$RP_{t,M} = \boldsymbol{\gamma}'\mathbf{Z}_t. \tag{14}$$

Substituting this into (13) and denoting the first term on the right-hand side of (13), which is an expectation error, by $\varepsilon_{t,t+M}$, we obtain the following excess return regression:

$$ER_{t,t+M} = \boldsymbol{\gamma}'\mathbf{Z}_t + \varepsilon_{t,t+M}, \quad t = 1, 2, \ldots, T - M, \tag{15}$$

where T is the sample size and M (recall) is the maturity. By definition, the error term $\varepsilon_{t,t+M}$ is orthogonal to the regressor $\mathbf{Z}_t$ under rational expectations. Therefore, the coefficient $\boldsymbol{\gamma}$ can be consistently estimated by OLS (ordinary least squares).

The alternative methodology I suggest, then, is the following:

(i) Estimate the coefficient $\boldsymbol{\gamma}$ in the excess return regression (15) by OLS. This produces a consistent estimate $\hat{\boldsymbol{\gamma}}'\mathbf{Z}_t$ of the risk premium $RP_{t,M}$ where $\hat{\boldsymbol{\gamma}}$ is the OLS coefficient estimate. Given the prominence of the basis in the commodity futures and foreign exchange literature, the control variables $\mathbf{Z}_t$ should include the break-even inflation rate (i.e., the basis).

(ii) Calculate the *level* of the expected inflation rate (the left-hand side of (11)) as $y_{t,M} + \hat{\boldsymbol{\gamma}}'\mathbf{Z}_t$.

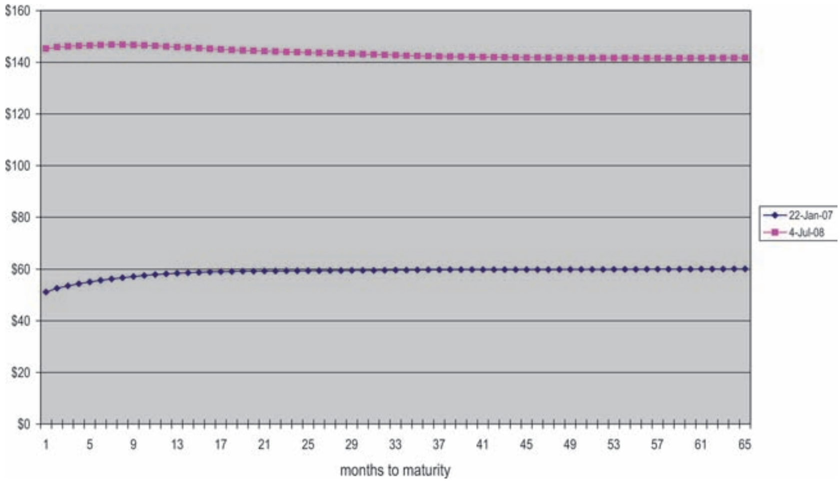
3. Accounting for the Persistence in Commodity Prices

I now turn to the paper by Erceg, Guerrieri, and Kamin on the recent run-up in commodity prices. My suggestion is to introduce inventories in the dynamic general equilibrium model used in the paper. The reason for my suggestion is that commodity prices exhibit persistent movements and inventories can account for that.

The persistence of commodity prices is apparent from the paper's figure 1, which plots the "spot" price of WTI (West Texas Intermediate) oil (I assume the "spot" price here is measured by the futures price of the nearest contracts, which is legitimate because futures prices converge to spot prices as the maturity approaches zero). Indeed, if one takes the monthly series on the WTI nearest futures price and regresses the log of the current price on the last month's log price, one gets a coefficient of 0.99. The oil price behaves like a random walk.

The market seems to have noticed the near random walk behavior of commodity prices. Looking at the paper's plot of the spot oil price, one notices the run-up stretching from \$51 on January 22, 2007 to \$145 on July 4, 2008, followed by the plunge to below \$40. This commentary's figure 1 shows the WTI forward curve (the plot of the WTI futures price against months to maturity for any given date)

Figure 1. WTI Forward Curves



on January 22, 2007 and July 4, 2008. Clearly, the market believed that the price would remain more or less the same. In particular, the market didn't anticipate the plunge after July 2008.

There is a well-established theory of commodity prices in Deaton and Laroque (1992). They assume risk-neutral inventory holders who enforce the usual arbitrage condition linking the current spot price to the expected future spot price. The arbitrage condition holds with equality when the level of inventory is positive. In the event of a stockout, the current spot price shoots up to equate demand to current output of the commodity. Their model can account for the near random walk behavior and occasional run-ups of the spot price. Although their model, with an exogenously given demand curve and with output assumed to be an exogenous endowment process, is partial equilibrium in nature, it should be feasible to introduce inventory holders into the dynamic general equilibrium model. It seems to me that all that is required is to add the inventory arbitrage condition and modify the market equilibrium condition for the commodity by adding the change in inventory to the amount supplied.

References

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