# Liquidity Hoarding and Inefficient Abundant Funding*

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This paper studies banks' choice between building liquidity buffers and raising funding ex post to deal with reinvestment shocks. We uncover the possibility of an inefficient liquidity squeeze equilibrium when ex post funding is abundant. In the model, banks typically build larger liquidity buffers when they expect funding to be expensive. However, when banks hold larger liquidity buffers, pledgeable income is larger and they hence can raise more funding, which in the aggregate raises the funding cost. This feedback loop between liquidity hoarding and the cost of ex post funding yields multiple equilibria, one being an inefficient liquidity squeeze equilibrium where banks do not build any liquidity buffer. Comparative statics show that this inefficient equilibrium is more likely when the supply of ex post funding is large. Last, in this equilibrium, a "borrower"-of-last-resort policy can improve social welfare if drying up ex post funding restores bank incentives to hold liquidity ex ante.

JEL Codes: D53, D82, D86.

## 1. Introduction

Financial crises usually find their roots in boom periods that tend to precede them. The 2008–9 financial crisis is no exception in this

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respect: significant vulnerabilities developed in the run-up to the crisis. For example, liquidity buffers, e.g., cash, claims on the central bank, and claims on the government, which still accounted for around 10 percent of U.S. banks' total assets in the late 1990s, dropped down to around 5 percent in 2007, at the onset of the financial crisis.[1] This undoubtedly made U.S. banks more vulnerable to financial distress, given that holding liquidity buffers is key for funding during adverse conditions.[2] Yet, a key question is, why did banks decide to reduce their liquidity buffers so much? One answer is easy funding conditions: banks were indeed able to raise funding, irrespective of the size and quality of their balance sheet. Yet, when funding became suddenly more expensive and scarce, U.S. banks lacked the relevant assets as financial distress rose.[3]

Notwithstanding the role that unexpected changes in funding conditions can play in triggering financial crises, this paper focuses on an alternative mechanism which highlights the externalities at play in liquidity buffer holdings and how they relate to funding conditions. More specifically, we provide an analytical model to show that agents rationally do not hold enough liquidity buffers compared with the social optimum when the funding supply is sufficiently large. In this model, the economy paradoxically runs short of liquidity because funding is abundant, not because it suddenly becomes very scarce.

We build on the seminal paper by Holmström and Tirole (1998), where banks build liquidity buffers to self-insure against shocks affecting illiquid projects. Illiquid projects typically display a higher yield but are subject to shocks and then require reinvestment.[4]

[1]Source: International Monetary Fund's International Financial Statistics. Liquid assets are the sum of reserves at the central bank (line 2:20), other claims on monetary authorities (line 2:20:N), claims on the central government (line 2:22:A), and claims on state and local governments (line 2:22:B). Total assets are the sum of liquid assets (defined as above), foreign assets (line 2:21), claims on the private sector (line 2:22:D), and claims on other financial corporations (line 2:22:G).

[2]For example, claims on the government can be sold against cash when need be, a reason why they are also called "safe-haven" assets.

[3]Financial institutions' inability to evaluate current funding conditions at that time as abnormal was due to an inherent myopia of banks and/or to the perception that public authorities would support banks if such conditions were to evaporate.

[4]Reinvestment risk works here as a rollover risk, since getting the final payoff once a reinvestment shock has happened requires raising fresh funds.

To carry out reinvestment, banks can either use liquidity buffers or they can raise funds ex post. Yet, raising funds faces limits because future income streams are not fully pledgeable. Hence, besides providing self-insurance, liquidity buffers also help banks raise more funds. To this framework we add an exogenous supply of funding and ask how it affects the banks' decision to build liquidity buffers.

The main result of the model is that there can be multiple equilibria. The economy can coordinate on a "liquidity squeeze" equilibrium where banks do not build liquidity buffers and are unable to meet reinvestment needs when a shock occurs. Multiple equilibria result from a positive externality of aggregate liquidity buffers on the funding cost. Let us detail the mechanism. First, banks typically choose to hold large liquidity buffers if they expect funding to be costly. Second, when future income streams are not fully pledgeable, banks holding large liquidity buffers can raise a large amount of funding. Large liquidity buffers in the aggregate hence lead to a high demand for funding, which drives up the equilibrium funding cost. And with a higher funding cost, banks are willing to build large liquidity buffers. Larger *aggregate* liquidity buffers therefore raise *individual* incentives to hold liquidity buffers through the positive effect on funding cost. This externality yields two possible equilibria, one where banks build large liquidity buffers and another one where banks do not build any liquidity buffer. Importantly, the first equilibrium, where banks prefer to build liquidity buffers, always dominates. This is because illiquid projects deliver a high return even if they face a shock owing to large reinvestment, while liquidity buffers provide a high return owing to the high funding cost.[5]

Next, we show that the positive feedback loop between liquidity buffer holdings and the funding cost is more likely when the exogenous funding supply is large. Indeed, when banks hold large liquidity buffers, those banks that need to reinvest can raise a large amount of funding—because holding liquidity raises pledgeable income and relaxes the borrowing constraint—but those that face no reinvestment need can also supply a large amount of funding. When the exogenous funding supply is large, changes in banks' liquidity holdings have a second-order effect on the *total* funding supply,

[5] This mechanism illustrates the view that abundant and easy funding can lead to over-investment in illiquid projects, i.e., inefficient risk taking.

given that the latter is dominated by the large exogenous supply. As a result, the relative increase in the demand for funding dominates and the equilibrium funding cost needs to go up to balance the market.[6] The positive feedback loop is therefore more likely when the exogenous supply of funding is large.

Last, we focus on policy options to improve social welfare. The inefficiency in the equilibrium where banks do not hold liquidity is due to the cost of funding being too cheap. Hence a lender-of-last-resort policy which contributes to further cutting the cost of ex post funding cannot bring any solution. On the contrary, a borrower-of-last-resort policy looks more natural to combat this inefficiency, as such a policy would consist in making funding more expensive.[7] To do so, the central bank can, for example, issue bonds. This would raise the demand for funding, and to the extent that the excess supply of funding disappears, the increase in the funding cost would preclude the equilibrium without liquidity holdings.

This paper contributes to our understanding of the mechanics of liquidity crises. As noted above, we build on the Holmström and Tirole (1998) approach in which banks use liquidity buffers to meet refinancing needs stemming from shocks affecting illiquid projects. We also closely follow Caballero and Krishnamurthy (2001, 2004), who look extensively at the problem of underinsurance against refinancing shocks in an open-economy context. A key difference with their approaches is that we do not get into the problem of refinancing non-tradable assets with limited tradable resources. Moreover, in our framework, inefficiencies stem from the abundance and not from an abrupt shortage of interim refinancing. This model also speaks to the recent literature on the macroeconomic effects of large capital flows that have been directed towards advanced economies (Bernanke 2005, 2009). While the recent theoretical and empirical literature has stressed some positive implications of these large flows (see Caballero, Farhi, and Gourinchas 2008 and Mendoza, Quadrini,

[6]On the contrary, when the exogenous funding supply is low, then the funding supply increases more than the funding demand when banks hold larger liquidity buffers. The funding cost then needs to go down to balance the market.

[7]In the second best, the amount of liquidity buffer is not contractible. The simple policy consisting in imposing a minimal liquidity ratio is therefore not possible, as that would boil down to assuming that the policymaker can contract on the amount of liquidity buffers.

and Ríos-Rull 2009 or Warnock and Warnock 2009), our model raises more skepticism, underscoring a possible detrimental implication of such flows.

We also build on Diamond and Dybvig (1983), Diamond and Rajan (2001), and Allen and Gale (2004) in which banks provide liquidity to depositors while investing in long-term assets, thereby facing a risk of bank run.[8] Bhattacharya and Gale (1987) extends the original Diamond-Dybvig framework and looks at how liquidity provided by the interbank market affects banks' willingness to hold liquidity. Bolton, Santos, and Scheinkman (2011) provides a model where agents' reliance on inside liquidity as opposed to outside liquidity—in our framework, liquidity buffer holdings vs. ex post funding—can affect the timing of trades on the market for liquidity. This model also features a multiple equilibria mechanism. An important difference, however, is that outside liquidity is efficient in their framework. On the contrary, our model highlights abundant funding as a potential source of inefficiency. It should also be clear that many different and important aspects relating to the notion of liquidity—see, for instance, Gorton and Pennacchi (1990) for an information-based approach to liquidity—are simply not covered in this paper. Finally Acharya, Shin, and Yorulmazer (2011) is also related: this paper looks at how foreign bank entry reduces domestic banks' incentives to hold liquid assets. The focus there, however, relates to fire sales.

The paper is organized as follows. The following section lays down the main assumptions of the model. Section 3 describes the decentralized equilibrium and its main properties. Section 4 investigates possible policy options to improve welfare. Conclusions are drawn in section 5.

## 2. Timing and Technology

We consider a single good economy populated with a unit mass continuum of banks and a unit mass continuum of investors.

[8]Note, however, that in the standard Diamond-Dybvig framework (1983), multiple equilibria relate to depositors' behavior for a given allocation between illiquid investments and liquidity buffers. In our framework, it is this allocation which can be subject to multiple equilibria.

The economy lasts for three dates: $0, 1$, and 2. All agents are risk neutral and derive utility from profits at date 2. They can freely store capital at any date $t$ with a unit return at date $t + 1$. A bank storing capital will be said to hold liquidity buffers, and $L$ will denote a bank's liquidity buffer holdings.

### *2.1 Banks*

Each bank starts with a unit endowment at date 0 and can invest an amount $I \geq \kappa$ in an illiquid project, $\kappa$ being the minimum size requirement for illiquid projects $(0 < \kappa < 1)$. At date 1, banks experience an idiosyncratic shock with a probability $\alpha$. In the absence of a shock—banks are then said to be *intact*—the project returns $\rho I$ at date 2. But if the shock hits—banks are then said to be *distressed*—the project yields no return at date 2. Yet, *distressed* banks have a reinvestment opportunity: reinvesting an amount $J$ of fresh resources at date 1 in the project returns $\rho_1 J$ at date 2, with $\rho_1 > (1 - \alpha)\rho > 1$. Last, banks can only pledge a fraction $\phi$ of illiquid projects' output and $\phi\rho_1 < 1$. Banks facing a reinvestment shock therefore have a reinvestment opportunity whose output is relatively difficult to pledge. This imperfect pledgeability will introduce a positive relationship between liquidity buffer holdings $L$ at date 0 and reinvestment $J$ at date 1.[9,10]

### *2.2 Investors*

Investors are willing to provide liquidity to banks or raise liquidity from banks at date 1. Their net liquidity supply is denoted $\lambda$, which can be either positive or negative.[11]

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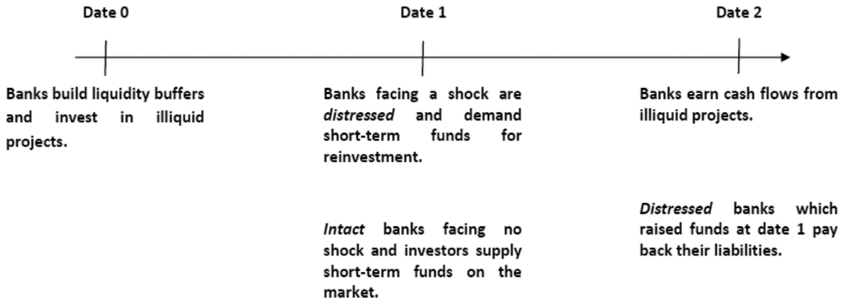
[9]We refer to agents investing in illiquid projects as banks because they are those in the model who face a liquidity mismatch.

[10]The pledgeability constraint for banks facing a shock is hence binding since their pledgeable return $\phi\rho_1$ is lower than the opportunity cost of capital (equal to 1 here).

[11]The assumption that investors take actions only at date 1 is a matter of simplification. If investors had on top of that an endowment at date 0, there would be two further issues that would either reinforce or complement the mechanism described here. First, banks could issue claims at date 0. This possibility—introducing leverage for banks at date 0—would actually reinforce the multiple equilibria property described later. Second, investors could decide strategically

**Figure 1. Timing of the Model**




## *2.3 Frictions*

Banks face two frictions. First, idiosyncratic shocks are *not contractible* and therefore cannot be diversified. Banks hence need to self-insure through liquidity buffer holdings. Second, banks' allocation at date 0 between liquidity buffers and illiquid investment is *observable* but *not verifiable.* With this assumption, investors cannot charge to a bank an interest rate which would depend on the bank's individual allocation. The funding cost at date 1 will actually depend on the aggregate amount of liquidity banks hold, which will be the source of the pecuniary externality.

## *2.4 Timing*

Figure 1 summarizes the timing of the model. At date 0, banks choose how much to invest in illiquid projects and how much liquidity to hold. At date 1, a fraction $\alpha$ of banks are *distressed.* These banks can then use their liquidity buffers and raise funds to carry out reinvestment. Investors and *intact* banks can then lend to *distressed* banks. They may also prefer to store their liquidity holdings if this is more profitable than lending. Finally, at date 2, banks pay back their liabilities, if any, and consume.

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to provide their funds at date 0 or at date 1, depending on the return attached to each of these options. Under some conditions, this possibility to choose strategically the timing of the funding supply can itself be a source of multiple equilibria, independently of the mechanism highlighted here.

At the heart of the model is a trade-off for banks which have to compromise between the initial investment $I$ on the one hand and the reinvestment $J$ in the event of a shock on the other hand. Maximizing initial investment $I$ requires minimizing liquidity buffer holdings $L$ and thereby pledgeable income. This in turn cuts reinvestment $J$ in the event of a shock. Conversely, maximizing liquidity holdings $L$ to mitigate shocks requires sacrificing initial investment $I$.

## 3. Liquidity, Funding Cost, and the Decentralized Equilibrium

The decentralized equilibrium is based on two building blocks. The first is banks' choice at date 0 between liquidity holdings and illiquid investment. This allocation decision depends primarily on the funding cost at date 1. The second is the equilibrium of the market at date 1 where *distressed* banks raise funding from investors and *intact* banks. The equilibrium cost of funding then depends on banks' allocation decision at date 0 and, specifically, on how much in aggregate liquidity buffers they hold. We start by setting up the banks' allocation problem and the equilibrium of the market for funding. Then, we derive the different equilibria.

### *3.1 Banks' Optimal Liquidity Buffer*

Consider a bank which holds an amount $L$ of liquidity and invests $I = 1 - L \geq \kappa$ in an illiquid project at date 0. Then if the bank is *intact*, the project returns $(1 - L)\rho$ at date 2. In addition, the bank has an amount $L$ of available funds at date 1 which can either be stored with a unit return or lent to *distressed* banks with a return denoted $r$. For simplicity and without loss of generality, we will restrict to cases where the funding cost, denoted $r$, satisfies $1 < r < \rho_1$ so that lending is always preferred to storing. An *intact* bank therefore reaps $Lr$ at date 2. The *intact* bank's final profit $\pi_g$ writes as

$$\pi_g(L) = (1 - L)\rho + Lr. \tag{1}$$

When $r < \rho$, the bank's profit $\pi_g$ decreases with liquidity buffers $L$. Building liquidity buffers is therefore costly for a bank which does not face any shock. If now the bank is *distressed*, the project yields no output at date 2. But the bank can reinvest. To do so, it can rely on liquidity buffers $L$; it can also raise an amount $D$ of funds—from investors and *intact* banks—against the promise to pay back $rD$ at date 2. Reinvestment hence yields a cash flow $(L+D)\rho_1$ so that the *distressed* bank's final profit $\pi_b$ writes as

$$\pi_b\left(D,L\right) = \left(L+D\right)\rho_1 - rD. \tag{2}$$

Moreover, *distressed* banks face a pledgeability constraint: repayments should not exceed pledgeable final output, $rD \leq \phi(L+D)\rho_1$. *Distressed* banks hence choose to raise $D^*$ at date 1 such that

$$D^*\left(r,L\right) = \begin{cases} \frac{\phi\rho_1}{r-\phi\rho_1}L & \text{if } r \leq \rho_1 \\ 0 & \text{otherwise,} \end{cases} \tag{3}$$

and *distressed* banks' final profits $\pi_b$ write as

$$\pi_b\left(L\right) = \rho_1 L + \left(\rho_1 - r\right)\frac{\phi\rho_1 L}{r-\phi\rho_1}. \tag{4}$$

A *distressed* bank with larger liquidity holdings $L$ enjoys larger profits when the pledgeability constraint binds, as it can raise more funds—$D^*(L)$ increases with $L$. This is the self-insurance aspect of holding liquidity buffers.

Now that we have determined banks' profits when *intact* and when *distressed*, we can turn to the bank's allocation decision at date 0. The probability of distress being $\alpha$, the problem for a bank consists in choosing liquidity $L$ which maximizes expected profits:

$$\begin{gathered} \max_L \pi = \left(1-\alpha\right)\pi_g\left(L\right) + \alpha\pi_b\left(L\right) \\ \text{s.t. } 0 \leq L \leq 1-\kappa. \end{gathered} \tag{5}$$

Banks face a simple trade-off: holding liquidity implies forgoing profits when *intact* but contributes to higher profits when *distressed*. The funding cost $r$ at date 1 affects this trade-off in two ways. First, a larger funding cost reduces profits for *distressed* banks and hence incentives to hold liquidity. Second, a larger funding cost raises the return on liquidity buffers for *intact* banks; it hence provides incentives to build larger liquidity buffers. When this second effect dominates, banks choose to hold more liquidity at date 0 if they expect a larger funding cost $r$ at date 1. We will indeed focus on this case in what follows: When the funding cost is $r = \rho_1$, banks prefer to hoard liquidity, i.e., $\partial\pi/\partial L|_{r=\rho_1} > 0$, and this holds if $\rho_1 > (1-\alpha)\rho$, which we assumed is always true. Conversely, when the funding cost is $r = 1$, banks prefer illiquid investments, i.e., $\partial\pi/\partial L|_{r=1} < 0$, and this holds when $\phi$ is sufficiently low.[12] Let us now denote $\widehat{r}$ the funding cost such that banks are indifferent between liquidity buffers and illiquid investments, i.e., $\partial\pi/\partial L|_{r=\widehat{r}} = 0$. Then, under the assumption that $\phi$ is sufficiently low, there is a unique funding cost $\widehat{r}$ which satisfies $1 < \widehat{r} < \rho_1$, and it writes as

$$\widehat{r} = \frac{\rho + \psi\rho_1}{2} + \sqrt{\left(\frac{\rho + \psi\rho_1}{2}\right)^2 - \rho\phi\rho_1}, \tag{6}$$

where $\psi = \frac{\phi-\alpha}{1-\alpha}$. As is shown in figure 2, when the funding cost $r$ is below $\widehat{r}$, banks do not build any liquidity buffer, $L = 0$, but when the funding cost $r$ is above $\widehat{r}$, they are better off holding liquidity and choose $L = 1 - \kappa$.[13]

### 3.2 The Equilibrium Funding Cost

Let us now turn to the market for reinvestment which opens at date 1. On this market, *distressed* banks can raise funding from investors and *intact* banks to finance reinvestment. A fraction $\alpha$ of banks face distress, and the individual demand for funding from each *distressed* bank is $D^*(r, L)$. The aggregate demand for funding is therefore $\alpha D^*(r, L)$. Conversely, a fraction $1 - \alpha$ of banks end up *intact* and each bank holds an amount $L$ of liquidity buffers. Aggregate funding supply from banks is $(1-\alpha)L$ and investors' net funding supply

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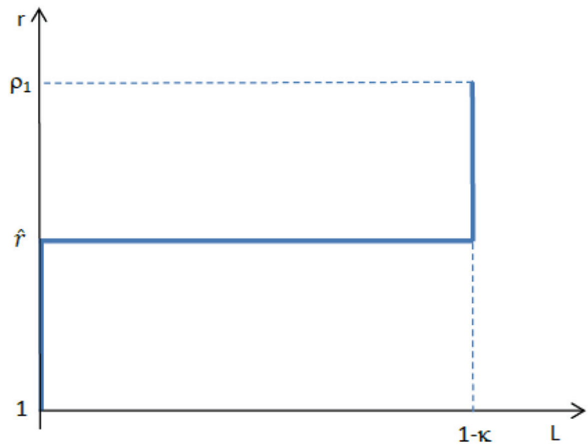
[12] Specifically, the condition $\partial\pi/\partial L|_{r=1} < 0$ simplifies as $\phi\rho_1 < \frac{\beta-\rho_1}{\beta-1}$ with $\beta = \frac{1-\alpha}{\alpha}(\rho - 1)$.

[13] Under the assumption that $\partial\pi/\partial L|_{r=1} < 0$ , i.e., $\phi\rho_1 < \frac{\beta-\rho_1}{\beta-1}$, the negative root of the equation $\partial\pi/\partial L = 0$ always lies below 1 and is hence irrelevant. This is why we focus on the positive root in (6). To put it differently, the assumption $\phi\rho_1 < \frac{\beta-\rho_1}{\beta-1}$ ensures that when $1 < r < \rho_1$, $\partial\pi/\partial L$ is monotonically increasing in the funding cost $r$, moving from negative to positive numbers, once and only once.

**Figure 2. Banks' Optimal Liquidity Buffers**



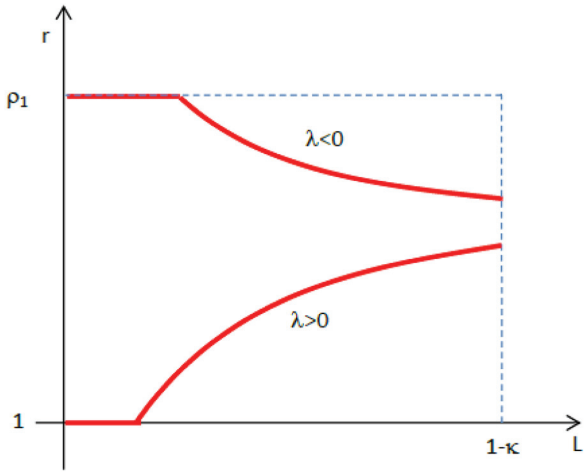

is $\lambda$. The equilibrium of the market for funding at date 1 therefore writes as

$$\alpha D^* \left(r, L\right) = \lambda + \left(1 - \alpha\right) L. \tag{7}$$

Note that this expression is only valid if the funding cost $r$ satisfies $1 \leq r \leq \rho_1$ so that *intact* banks and investors are willing to lend and *distressed* banks are willing to borrow. The equilibrium is indeed trivial if either $\lambda \geq \alpha \frac{\phi \rho_1}{1-\phi \rho_1}(1-\kappa)$ or $\lambda \leq -(1-\alpha)(1-\kappa)$. In the first case, the supply of funding $\lambda + (1-\alpha)L$ is always larger than the demand for funding $\alpha D^*(r, L)$, while in the second case it is the demand for funding $\alpha D^*(r, L)$ which is always larger than the supply of funding $\lambda + (1-\alpha)L$. We therefore exclude these two possibilities in what follows and focus on the case where investors' net funding supply $\lambda$ satisfies $-(1-\alpha)(1-\kappa) < \lambda < \alpha \frac{\phi \rho_1}{1-\phi \rho_1}(1-\kappa)$.

When banks choose to hold larger liquidity buffers $L$ at date 0, the supply of funding goes up because *intact* banks have more funds available. However, the demand for funding also goes up, as *distressed* banks can also demand more funds. This is because the pledgeability constraint—which limits the amount of funds *distressed* banks can raise—is relaxed. The change in the equilibrium cost of funding is therefore a priori ambiguous. To determine which of these

**Figure 3. Equilibrium of the Market for Funds**



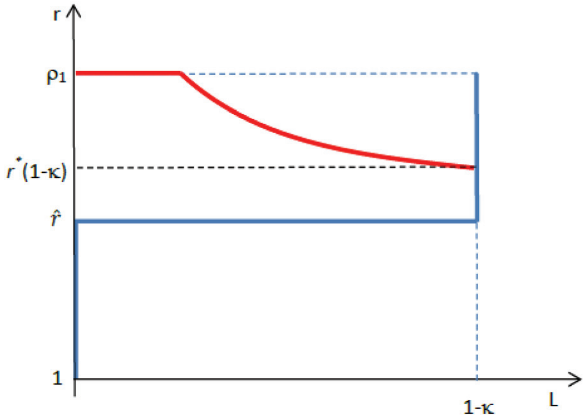

two effects dominates, we can use (7) to write the expression for the equilibrium funding cost as

$$r^{*}\left(L\right)=\phi\rho_{1}+\frac{\alpha\phi\rho_{1}L}{\lambda+\left(1-\alpha\right)L}. \tag{8}$$

When investors' net funding supply $\lambda$ is positive, an increase in banks' liquidity holdings $L$ raises the equilibrium funding cost $r^*$ because the aggregate demand for funding increases more than the aggregate supply of funding, $\partial r^*/\partial L|_{\lambda>0} > 0$. On the contrary, when investors' net funding supply $\lambda$ is negative, an increase in banks' liquidity holdings $L$ reduces the equilibrium funding cost $r^*$ because the aggregate supply of funding increases more than the aggregate demand for funding, $\partial r^*/\partial L|_{\lambda<0} < 0$. These two cases are represented in figure 3.

Investors' net funding supply $\lambda$ affects the equilibrium funding cost in two ways: a level and a slope effect. When investors' net funding supply $\lambda$ goes up, this reduces the *level* of the equilibrium funding cost but also, importantly, raises the *slope* of the equilibrium funding cost with regard to aggregate liquidity holdings (which can turn from negative to positive as in figure 3).

**Figure 4. The Equilibrium with Liquidity Buffers**




### *3.3 The Decentralized Equilibrium*

We can now determine the decentralized equilibrium using the two relations described above: the problem (5) determines the optimal liquidity buffer $L^*(r)$ depending on the funding cost $r$, and the relationship (7) determines the equilibrium funding costs $r^*(L)$ as a function of aggregate liquidity buffers.

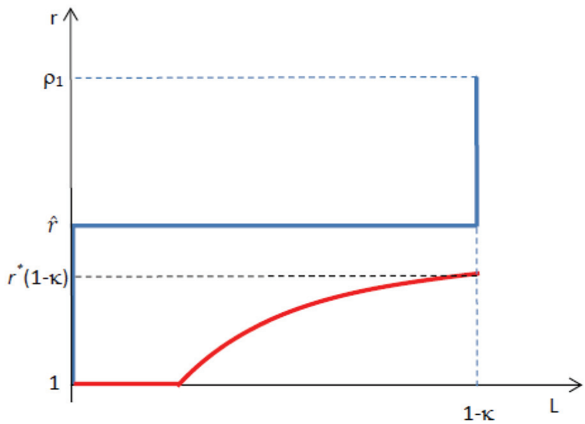
PROPOSITION 1. *The decentralized equilibrium is such that*

*(i) banks hold maximum liquidity buffers* $L^* = 1 - \kappa$ *when* $\lambda \leq 0$ *and* $r^*(L = 1 - \kappa) \geq \widehat{r}$,
*(ii) banks are indifferent between holding liquidity buffers and investing in illiquid projects when* $\lambda \leq 0$ *and* $r^*(L = 1 - \kappa) < \widehat{r}$,
*(iii) banks hold no liquidity buffer* $L^* = 0$ *when* $\lambda > 0$ *and* $r^*(L = 1 - \kappa) < \widehat{r}$, *and*
*(iv) banks face multiple equilibria: they may either hold maximum liquidity buffers* $L^* = 1 - \kappa$ *or no liquidity buffer* $L^* = 0$ *when* $\lambda > 0$ *and* $r^*(L = 1 - \kappa) \geq \widehat{r}$.

*Proof.* Cf. the appendix.

Consider first the equilibrium where banks prefer to hold liquidity buffers—represented in figure 4. This equilibrium arises when investors' net funding supply $\lambda$ is relatively low: Denoting

**Figure 5. The Equilibrium without Liquidity Buffers**

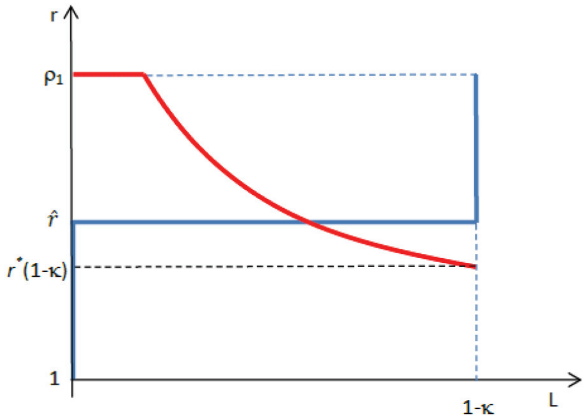



$\mu = \frac{\phi\rho_1-(1-\alpha)\widehat{r}}{\widehat{r}-\phi\rho_1}(1-\kappa)$, the conditions $\lambda \leq 0$ and $r^*(L = 1-\kappa) \geq \widehat{r}$ can be simplified as $\lambda \leq \min\{0;\mu\}$. The intuition for this equilibrium is pretty simple: Irrespective of how much liquidity banks decide to hold, a relatively low net funding supply $\lambda$ ensures a large funding cost $r$. And when the funding cost is large, banks are better off holding liquidity buffers. This equilibrium is also more likely to hold if the probability $\alpha$ of distress is larger. The need for banks to build liquidity buffers naturally increases if distress is more likely.

Turning now to the equilibrium where banks prefer illiquid investments and do not build any liquidity buffers, it arises when $\lambda > 0$ and $r^*(L = 1-\kappa) < \widehat{r}$, i.e., when investors' net funding supply $\lambda$ satisfies $\lambda > \max\{0;\mu\}$. Figure 5 provides a graphical representation of this equilibrium. The intuition for this equilibrium is very similar to that of the previous equilibrium, where banks prefer to hold liquidity buffers: Irrespective of how much liquidity banks decide to hold, a relatively large net funding supply $\lambda$ ensures a low funding cost $r$. And when the funding cost $r$ is low, banks are better off investing in illiquid projects and not holding any liquidity buffer. This equilibrium is also less likely to hold if the probability $\alpha$ of distress is larger. The need for banks to build liquidity buffers naturally decreases if distress is less likely.

Then we are left with two cases to examine. When $\mu < 0$, there is an equilibrium where banks are indifferent between holding

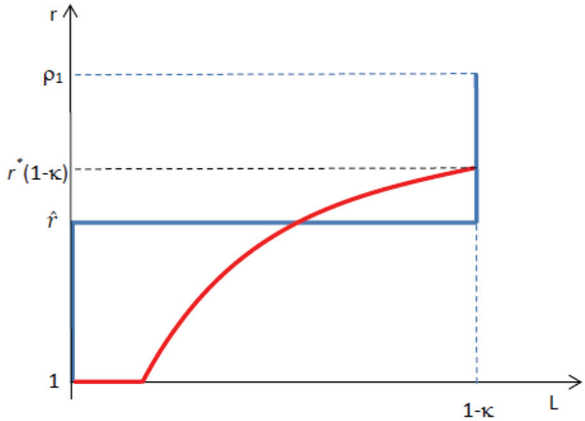
**Figure 6. The Mixed-Strategy Equilibrium**




liquidity buffers and investing in illiquid projects when $\lambda \leq 0$ and $r^*(L = 1 - \kappa) < \widehat{r}$, which simplifies as $\mu < \lambda \leq 0$. When investors' net funding supply is negative, the relationship between the equilibrium funding cost and the amount of liquidity banks decide to hold is negative. As is shown in figure 6, if banks decide to hold a large amount of liquidity, then the equilibrium cost of funding will be low and banks will not have incentives to hold liquidity. Similarly, if banks decide to invest in illiquid projects and not hold any liquidity, then the equilibrium cost of funding will be high and banks will not have incentives to invest in illiquid projects. As a result, there is no pure-strategy equilibrium. The equilibrium funding cost is then such that banks are indifferent between holding liquidity and investing in illiquid projects and the amount of liquidity banks hold just balances the market for funding.

On the contrary, when $\mu > 0$, the economy faces multiple equilibria when $\lambda > 0$ and $r^*(L = 1-\kappa) \geq \widehat{r}$, which simplifies as $0 < \lambda \leq \mu$. Banks may either hold maximum liquidity buffers $L^* = 1 - \kappa$ or no liquidity buffers $L^* = 0$. When investors' net funding supply is positive, the relationship between the equilibrium funding cost and the amount of liquidity banks decide to hold is positive. Hence if banks decide to hold liquidity $L = 1 - \kappa$, then the equilibrium cost of funding is relatively high, which provides incentives to banks to hold liquidity. Similarly, if banks decide to invest in illiquid projects,

**Figure 7. Multiple Equilibria**




$L = 0$, then the equilibrium cost of funding is relatively low, which provides incentives to banks to invest in illiquid projects. There are hence multiple equilibria. Figure 7 provides a graphical example.

Three further remarks can be made here. First, the equilibrium where banks do not hold liquidity can be described as a "liquidity squeeze" since *distressed* banks have a profitable reinvestment opportunity which they are unable to exploit, as they lack pledgeable income. Interestingly, this equilibrium is not related to an abrupt or sudden reduction in investors' funding supply $\lambda$, as is the case with sudden stops or capital flow reversals. On the contrary, it emerges because investors' large funding supply $\lambda$ reduces the funding cost $r$ and thereby the profits banks can reap from holding liquidity at date 0. Banks are therefore better off investing in illiquid projects. Second, while the funding cost $r$ is low in the equilibrium where banks do not hold liquidity buffers, the shadow cost of capital is rather large, as *distressed* banks are not able to exploit their reinvestment option. The low funding cost $r$ rather reflects here the scarcity of banks' pledgeable income. Third, multiple equilibria require the condition $\mu > 0$. To understand the intuition for this condition, it is useful to note that when the economy faces multiple equilibria, there is a mixed-strategy equilibrium on top of the two pure-strategy equilibria described above. Yet, the condition $\mu > 0$ ensures that this mixed-strategy equilibrium is unstable, i.e., not robust to small perturbations. To see this, we can note that the condition $\mu > 0$ can be written as $\alpha\frac{\phi\rho_1}{\widehat{r}-\phi\rho_1} > 1-\alpha$. Then if banks coordinate on the mixed-strategy equilibrium, the funding cost would be $r = \widehat{r}$, and the condition $\alpha\frac{\phi\rho_1}{\widehat{r}-\phi\rho_1} > 1-\alpha$ then states that if banks decide to hold marginally more liquidity, the marginal change in the demand for funding $\alpha\frac{\phi\rho_1}{\widehat{r}-\phi\rho_1}$ would be larger than the marginal change in the supply of funding $1-\alpha$. As a result, the funding cost would need to increase to balance the market, and this increase in the funding cost would encourage banks to further increase their liquidity holdings, etc. The economy would eventually end up in the equilibrium where banks are better off holding liquidity. The condition $\mu > 0$ therefore ensures that banks' liquidity holdings have a positive pecuniary externality on the cost of funding. This is why it is required for multiple equilibria. This is also why we can focus on the two pure-strategy equilibria and disregard the mixed-strategy equilibrium. Last, using the expression for $\widehat{r}$, the condition $\mu > 0$ can be simplified as $\rho_1 > (1-\alpha)\rho$, which holds by assumption.[14]

### *3.4 Social Welfare*

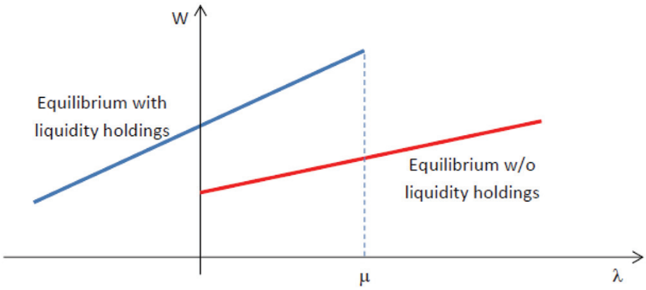
We consider social welfare $W$ as the sum of banks' and investors' profits and investigate which of the equilibria described above is socially optimal.

$$W = (1-\alpha)\,\pi_g\,(L^*) + \alpha\pi_b\,(L^*) + \lambda r^*\,(L^*) \tag{9}$$

Let us start with the equilibrium where banks prefer to invest in illiquid projects and do not hold any liquidity. This equilibrium holds when investors' funding supply satisfies $\lambda > 0$. Banks then enjoy expected profits $(1-\alpha)\rho$ while investors reap a unit return on their funding supply $\lambda$. Social welfare $W$ therefore writes as $W = W^i(\lambda) = (1-\alpha)\rho+\lambda$. Let us turn now to the equilibrium where banks are better off holding liquidity. This equilibrium holds when investors' funding supply satisfies $\lambda < \mu$. In this equilibrium, each bank holds an amount of liquidity $1-\kappa$, and banks enjoy expected

[14]The condition $\mu > 0$ is equivalent to $\widehat{r} < \frac{\phi\rho_1}{1-\alpha}$. Then, given that $\partial\pi/\partial L$ is increasing in $r$ and that $\partial\pi/\partial L = 0$ when $r = \widehat{r}$, the condition $\mu > 0$ holds if and only if we have $\partial\pi/\partial L > 0$ when $r = \frac{\phi\rho_1}{1-\alpha}$, which simplifies as $\rho_1 > (1-\alpha)\rho$.

**Figure 8. Social Welfare and Investors' Funding Supply**




profits $(1-\alpha)\pi_g(1-\kappa)+\alpha\pi_b(1-\kappa)$ while investors reap a return on $r^*(1-\kappa)$ on their funding supply $\lambda$. In this case, after some algebra, social welfare $W$ writes as $W = W^l(\lambda) = (1-\alpha)\kappa\rho + (1-\kappa+\lambda)\rho_1$. The social welfare function can hence be summarized as

$$W = \begin{cases} W^i(\lambda) & \text{if } \lambda > 0 \\ W^l(\lambda) & \text{if } \lambda \leq \mu. \end{cases} \tag{10}$$

PROPOSITION 2. *With multiple equilibria, the equilibrium in which banks hold liquidity ensures higher social welfare.*

*Proof.* Cf. the appendix.

Figure 8 represents social welfare $W$ as a function of investors' funding supply $\lambda$ for the two possible equilibria. It shows that social welfare under the equilibrium where banks hold liquidity is higher for any possible value of $\lambda$. This result relates to the pecuniary externality of aggregate liquidity buffers on the funding cost. In the equilibrium in which banks prefer to build liquidity buffers, illiquid projects display a high return because *distressed* banks are able to exploit their reinvestment opportunity while *intact* banks enjoy a high return both on illiquid projects and on liquidity buffers. Moreover, investors also enjoy a relatively high return. Hence aggregate output and social welfare are both relatively high. By contrast, in the equilibrium in which banks do not build liquidity buffers, *distressed* illiquid projects are relatively unproductive—their return is actually zero—because there is no reinvestment and investors only reap the minimal return on storage. As a result, aggregate output and social welfare are relatively low. The equilibrium in which banks do not build liquidity buffers hence provides lower welfare.

## 4. Policy Options to Restore Efficiency

In the previous section, we established that in the presence of multiple equilibria, the equilibrium in which banks do not hold any liquidity buffer is dominated. We now investigate whether there are any welfare-improving policies conditional on a given equilibrium. To do so, we consider a central bank which can act either as a lender of last resort or as a "borrower" of last resort. In the former case, the central bank lends $\lambda_{cb} > 0$ to *distressed* banks, while in the latter case, the central bank issues an amount $-\lambda_{cb}$ of bonds ($\lambda_{cb} < 0$) and sells such bonds to *intact* banks and investors. The central bank, having no endowment, needs to borrow to act as a lender of last resort. We assume it does so at a cost $\rho_{cb}$ which satisfies $1 < \rho_{cb} < \rho_1$. Similarly, the central bank, having no investment opportunity, can only park the funds raised when acting as borrower of last resort at the storage facility for a unit return. Last, we assume perfect commitment: the central bank always delivers on the announced policy $\lambda_{cb}$.

### *4.1 Central Bank Intervention and Social Welfare*

In the equilibrium where banks do not hold liquidity, i.e., when the total funding supply satisfies $\lambda + \lambda_{cb} > 0$, *distressed* banks cannot borrow—$D^*(L = 0) = 0$—and the funding cost is $r = 1$, as investors park their funds at the storage facility. A central bank acting as a lender of last resort therefore cannot affect the funding cost, which cannot go below 1. Banks and investors are hence unaffected. Social welfare net of intervention costs $W^n$ therefore writes as $W^n = W^i(\lambda) - (\rho_{cb} - 1)\lambda_{cb}$. Similarly, when the central bank issues bonds at date 1, if the bond issuance does not affect banks' liquidity holdings, then the equilibrium funding cost is still $r = 1$. Banks and investors are hence still unaffected. Social welfare net of intervention costs $W^n$ then writes as $W^n = W^i$ given that the central bank does not bear any cost (it borrows and lends at the same rate $r = 1$).

Let us now turn to the equilibrium where banks hold liquidity, i.e., the total funding supply satisfies $\lambda + \lambda_{cb} \leq \mu$. Now a central bank acting as a lender/borrower of last resort affects the funding cost. A positive funding supply $\lambda_{cb}$ reduces the funding cost for *distressed* banks and allows for larger borrowing. The central bank then earns the return $r^*$—the equilibrium funding cost—but borrows the funds lent at a cost $\rho_{cb}$. It hence undergoes a cost $(\rho_{cb} - r^*)\lambda_{cb}$. After some algebra, social welfare net of central bank intervention costs writes as $W^n = W^l(\lambda) + (\rho_1 - \rho_{cb})\lambda_{cb}$.[15] Now when the central bank issues bonds at date 1, it does so at the prevailing funding cost $r^*$ and parks the funds raised at the storage facility. It hence faces a cost $-(r^* - 1)\lambda_{cb}$ (remember that $\lambda_{cb} < 0$ in the case of "borrower"-of-last-resort policies). Social welfare net of central bank intervention costs then writes as $W^n = W^l + (\rho_1 - 1)\lambda_{cb}$. Wrapping up these results, social welfare $W^n$ under central bank intervention writes as

$$W^n(\lambda_{cb}) = \begin{cases} W^i(\lambda) - (\rho_{cb} - 1)\lambda_{cb}\mathbf{1}_{[\lambda_{cb} > 0]} & \text{if } \lambda + \lambda_{cb} > 0 \\ W^l(\lambda) + \left(\rho_1 - \rho_{cb}\mathbf{1}_{[\lambda_{cb} > 0]} - \mathbf{1}_{[\lambda_{cb} < 0]}\right)\lambda_{cb} & \text{if } \lambda + \lambda_{cb} \leq \mu. \end{cases} \quad (11)$$

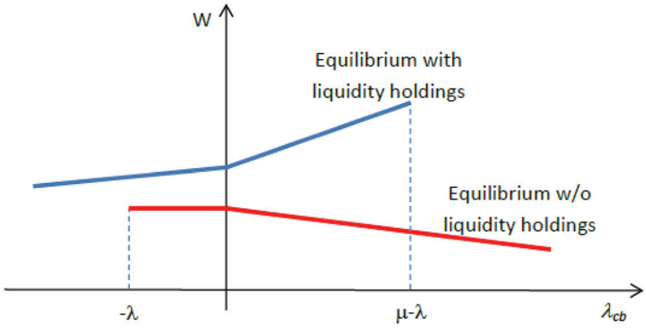
PROPOSITION 3. *The socially optimal policy for the central bank consists in announcing it will act as a lender of last resort, lending an amount $\lambda^*_{cb} = \mu - \lambda$ to distressed banks if the economy coordinates on the equilibrium where banks hold liquidity. Conversely, the socially optimal policy for the central bank consists in announcing it will act as a "borrower" of last resort, issuing an amount of bonds $-\lambda^*_{cb} = \lambda$ if the economy were to coordinate on the equilibrium where banks do not hold liquidity.*

*Proof.* Cf. the appendix.

Figure 9 represents social welfare $W$ as a function of the central bank funding supply $\lambda_{cb}$. It shows two properties: (i) social welfare is

[15] Note that the expression for $W^n$ is very intuitive. It writes as the sum of two terms: social welfare in the absence of any central bank intervention $W^l$ and the social gain to central bank intervention $(\rho_1 - \rho_{cb})\lambda_{cb}$, given that the funds $\lambda_{cb}$ are borrowed at a cost $\rho_{cb}$ and reinvested with a return $\rho_1$.

**Figure 9. Social Welfare and Central Bank Intervention**




always higher under the equilibrium where banks hold liquidity, and (ii) social welfare under the equilibrium where banks hold liquidity is increasing in $\lambda_{cb}$, while social welfare under the equilibrium where banks do not hold liquidity is weakly decreasing in $\lambda_{cb}$. The intuition for these results is relatively straightforward. In the equilibrium where banks hold liquidity, the cost of funding is relatively high, and reducing funding helps *distressed* banks, raising more funds and thereby increasing reinvestment. This is why social welfare under the equilibrium where banks hold liquidity is increasing in $\lambda_{cb}$. Yet the central bank's action faces a limit, as the reduction in the funding cost can affect banks' incentives to hold liquidity ex ante. As a result, the socially optimal policy consists for the central bank in lending an amount $\lambda_{cb}^* = \mu - \lambda$ to *distressed* banks. By contrast, in the equilibrium where banks do not hold liquidity, the opposite holds: the cost of funding is relatively low, and raising it through a borrower-of-last-resort policy can be helpful insofar as this changes banks' incentives to hold liquidity ex ante (remember that social welfare is higher in the equilibrium where banks hold liquidity). Yet such a policy is costly under the equilibrium where banks hold liquidity. However, this cost is always dominated by the welfare gain the economy enjoys when switching equilibriums. This is why drying up liquidity is then optimal, i.e., $\lambda_{cb}^* = -\lambda$.

A lender-of-last-resort policy can be helpful to improve social welfare but only insofar as *distressed* banks have some pledgeable income. In the absence of pledgeable income, a “borrower”-of-last-resort policy is needed to restore incentives to hold liquidity.

## 5. Conclusion

The model derived in this paper provides a framework to analyze banks' decision to build liquidity buffers ex ante as opposed to raising funding ex post. In particular, the model illustrates that a positive feedback loop can emerge between liquidity buffers and the funding cost when the economy faces a positive funding supply from investors. As a result, the economy can coordinate on an inefficient equilibrium in which banks do not build liquidity buffers and are unable to cope with shocks. Next, the paper investigates how policy can improve on social welfare and highlights that lender-of-last-resort policies, by further reducing funding costs, are no help in raising incentives to hold liquidity. However, a borrower-of-last-resort policy which aims at raising the funding cost and thereby changes incentives for liquidity holding can be helpful. Yet to be successful, it needs to be sufficiently massive to dry up liquidity on the market.

## Appendix

### *Proof of Proposition 1*

According to previous results, banks' optimal liquidity buffer $L^*(r)$ satisfies

$$L^*(r) = \begin{cases} 0 & \text{if } 1 \leq r < \widehat{r} \\ 1-\kappa & \text{if } \widehat{r} \leq r \leq \rho_1. \end{cases} \tag{12}$$

Turning now to the market for funding, and denoting $r^*(1-\kappa) = \frac{1-\kappa+\lambda}{(1-\alpha)(1-\kappa)+\lambda}\phi\rho_1$, the equilibrium funding cost $r^*(L)$ satisfies

$$r^*(L) = \begin{cases} \mathbf{1}[\lambda > 0] + \rho_1 \mathbf{1}[\lambda \leq 0] & \text{if } L = 0 \\ r^*(1-\kappa) & \text{if } L = 1-\kappa. \end{cases} \tag{13}$$

When banks do not hold liquidity buffers, i.e., $L^* = 0$, then there is an excess supply of funding if $\lambda > 0$ and an excess demand for funding when $\lambda \leq 0$. Conversely, when banks hold liquidity buffers $L^* = 1-\kappa$, then following (7), the equilibrium funding cost $r$ solves $\alpha\frac{\phi\rho_1}{r-\phi\rho_1}(1-\kappa) = (1-\alpha)(1-\kappa)+\lambda$, which simplifies as $r = r^*(1-\kappa)$.

Using (12) and (13), we can now derive the decentralized equilibrium.

When $\lambda \leq 0$, the equilibrium funding cost is $r^* = \rho_1$ if $L = 0$, while it is $r^* = r^*(1-\kappa)$ if $L = 1-\kappa$. Hence, $L = 0$ cannot be an equilibrium and we are left with two cases: if $r^*(1-\kappa) \geq \widehat{r}$, then $L^* = 1-\kappa$ is the unique equilibrium—banks are better off holding maximum liquidity buffers. But if $r^*(1-\kappa) < \widehat{r}$, then neither $L = 0$ nor $L = 1-\kappa$ can be an equilibrium. There is hence a mixed-strategy equilibrium and banks are indifferent between holding liquidity and investing in illiquid projects.

Now when $\lambda > 0$, the equilibrium funding cost is $r^* = 1$ if $L = 0$, while it is $r^* = r^*(1-\kappa)$ if $L = 1-\kappa$. As a result, $L = 0$ is always an equilibrium and we are left with two cases: if $r^*(1-\kappa) < \widehat{r}$, $L = 0$—banks are better off holding no liquidity buffers—is the unique equilibrium. Conversely, if $r^*(1-\kappa) > \widehat{r}$, then $L = 0$ and $L = 1-\kappa$ are both equilibria: banks may either hold maximum liquidity buffers or no liquidity buffers at all.

*Proof of Proposition 2*

Given the assumption that $\rho_1 > (1-\alpha)\rho > 1$ and given that $\lambda > 0$ is a necessary condition for multiple equilibria, social welfare is always larger in the equilibrium where banks hold liquidity.

*Proof of Proposition 3*

In the equilibrium where banks hold liquidity, i.e., $\lambda + \lambda_{cb} \leq \mu$, expression (11) for social welfare $W^n$ increases with central bank lending $\lambda_{cb}$. The optimal policy is then to maximize central bank lending, i.e., $\lambda + \lambda_{cb}^* = \mu$. Similarly, in the equilibrium where banks do not hold liquidity, i.e., $\lambda + \lambda_{cb} > 0$, expression (11) for social welfare $W^n$ decreases with central bank lending $\lambda_{cb}$. The optimal policy is then to minimize central bank lending, i.e., to have the central bank act as a borrower of last resort: $\lambda + \lambda_{cb}^* = 0$. In this case, the economy switches to the equilibrium where banks hold liquidity. Hence we still need to check that such policy is time consistent, i.e., that social welfare is still higher with this central bank policy once the economy has switched equilibrium. And indeed one can check that this is the case: given the assumption $\rho_1 > (1-\alpha)\rho$, we always have $W^n(\lambda_{cb} = -\lambda) > W^i(\lambda)$.

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