Trend Inflation in Advanced Economies*

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We derive estimates of trend inflation for fourteen advanced economies from a framework in which trend shocks exhibit stochastic volatility. The estimated specification allows for time variation in the degree to which longer-term inflation expectations are well anchored in each economy. Our results bring out the effect of changes in monetary regime (such as the adoption of inflation targeting in several countries) on the behavior of trend inflation.

Our estimates represent an expansion of those in the previous literature along several dimensions. For each country, we employ a multivariate approach that pools different inflation series in order to identify their common trend. In addition, our estimates of the inflation gap (that is, the difference between trend and observed inflation) are allowed to exhibit considerable persistence—a treatment that affects the trend estimates to some extent. A forecast evaluation based on quasi-real-time estimates registers sizable improvements in inflation forecasts

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at different horizons for almost all countries considered. It remains the case, however, that simple random-walk forecasts of inflation are difficult to outperform by a statistically significant amount.

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1. Introduction

Measures of trend inflation play an important role in the study of inflation in many countries. In the context of policy analysis, the level and variability of trend inflation can be viewed as summaries of the degree to which inflation expectations in a particular country have remained anchored over time. Application of New Keynesian analysis to inflation data over long samples may also benefit from the availability of estimates of trend inflation, as the New Keynesian approach to the Phillips curve typically specifies inflation dynamics in terms of the deviation of inflation from a steady-state or trend-inflation rate, with this trend rate possibly varying over time (see, for example, Cogley and Sbordone 2008). Furthermore, an estimate of trend inflation can serve as a useful centering point in the construction of inflation forecasts at various horizons. Still another reason for interest in estimates of trend inflation is the fact that the existing literature has found that a substantial portion of the observed persistence of international inflation data is accounted for by variations in trend inflation, which are in turn often related to changes in monetary regimes; see, for example, Levin and Piger (2004), Cecchetti et al. (2007), Ireland (2007), Stock and Watson (2007, 2010), Wright (2011), and Morley, Piger, and Rasche (2015).

In this paper, we provide estimates of the level of, and time-varying uncertainty of, trend inflation for fourteen advanced economies. The estimates are derived from a multivariate statistical model that pools information from different inflation series for each country. The model is applied on a country-by-country basis.[1] Our motivation for this choice is twofold. First, the country-by-country approach is most amenable to a comparison, for the full sample

[1]This approach contrasts with the procedure of pooling information across countries, as in Mumtaz and Surico (2009, 2012), Ciccarelli and Mojon (2010), and Mumtaz, Simonelli, and Surico (2011), for example.

period as well as for subsamples, of alternative models of trend. Second, although there are clearly some cross-country co-movements in overall inflation, the reason that inflation rates move together across countries does not appear to lie solely in common behavior of the trend component of a "trend/cycle" decomposition—a complication that is underscored in, for example, Ciccarelli and Mojon's (2010) analysis of cross-country inflation behavior. On this score, our results confirm that there do, in fact, tend to be considerable differences across countries in estimates of trend inflation, very likely reflecting country-specific developments in monetary regimes.

Formally, we adopt the definition of trend inflation as the infinite-horizon forecast of inflation. This trend definition corresponds to the Beveridge-Nelson (1981) concept. This concept has been applied to inflation data in a number of studies, including Cecchetti et al. (2007), Stock and Watson (2007, 2010), Clark and Doh (2011), Cogley, Sargent, and Surico (2014), Cogley and Sargent (2015), and Morley, Piger, and Rasche (2015), with variants of the approach also employed by Cogley and Sargent (2005), Cogley, Primiceri, and Sargent (2010), and Kozicki and Tinsley (2012).[2] Our multivariate model incorporates the assumption that, for any particular country, different inflation measures share the same common trend. Specifically, we consider percentage changes in core and headline CPI, as well as percentage changes in the GDP deflator, proceeding throughout on the premise that the deviations that these inflation series exhibit from the common trend are dynamically stable.

Our multivariate model, designated the "MVSV" model, nests the popular unobserved-components model with stochastic volatility, designated the "UCSV" model, of Stock and Watson (2007, 2010) that has been applied to inflation data for the G7 countries by Cecchetti et al. (2007). The application of a multivariate extension of the UCSV model to different countries, and the comparison between UCSV and MVSV models across these economies, constitute specific

[2]Cogley and Sargent (2005) and Cogley, Primiceri, and Sargent (2010) derive their measure of trend inflation from a non-linear function of time-varying VAR coefficients, a measure that approximately corresponds to a Beveridge-Nelson (1981) trend. Kozicki and Tinsley (2012) refer to their measure as the "shifting endpoint of inflation expectations." In a similar spirit, Levin and Piger (2004) relate time variation in inflation persistence to structural breaks in the coefficients of autoregressive time-series representations of inflation.

contributions of this paper. The multivariate model used in this paper represents an extension of the UCSV approach along two dimensions. The first dimension pertains to our reliance on multiple inflation series: as in Mertens (2011), the model extracts its trend estimates from a set of inflation series, instead of drawing information from a single inflation measure. The second dimension pertains to the treatment of the difference between trend and observed inflation—the *inflation gap*, in the terminology of Cogley, Primiceri, and Sargent (2010). Inflation-gap fluctuations are assumed to be serially uncorrelated in the UCSV model. In contrast, we allow the inflation gap to exhibit considerable persistence, while constraining the gap fluctuations to be dynamically stable, governed by a vector autoregression with time-invariant parameters. In this way, we allow for the possibility of persistence in the inflation gap, as in, for example, Kang, Kim, and Morley (2009) or Cogley, Primiceri, and Sargent (2010). Unlike these authors, however, we do not permit inflation-gap persistence to vary over time. The more parsimonious approach to the treatment of inflation-gap persistence that we adopt has advantages that we discuss below.

As in the UCSV model, we keep track of different measures of stochastic volatility that affect different components of the inflation process: one for trend shocks and one capturing changes in gap volatility for each of the different inflation measures used in our multivariate model. Although we allow for time-varying persistence in each inflation measure by letting the magnitude of shocks to the inflation trend and gap vary, we have also chosen to keep the coefficients governing inflation-gap persistence constant in order to limit time variation in model parameters. Such a restriction is especially warranted in view of the fact that we lack observations on several input series in the earlier part of our sample.

In the spirit of the UCSV model, our procedure does not involve taking a stand on the issue of potential statistical linkages between inflation and other economic variables. For example, we do not investigate connections between persistent behavior of inflation and persistence in resource slack (such as those considered in, for example, Morley, Piger, and Rasche 2015). In so limiting the scope of our analysis, we in no way deny that such linkages are of economic interest. On the contrary, real/nominal interactions are crucial to monetary policy analysis. But trend estimates of the kind we derive are closely related to a model's forecasting properties, and the contribution that real variables make to the forecasting of inflation has frequently been established to be modest—as documented in, for example, Stock and Watson (2009) and Faust and Wright (2013). Accordingly, attention is confined here to models of the inflation process that do not draw upon data other than on inflation. Therefore, when we speak of our estimates being "multivariate," we mean that we use multiple measures of inflation in estimation; we do not use series other than inflation to inform our estimates.[3]

Because our estimation relies on state-space methods and involves a limited number of time-varying parameters, we can handle cases in which observations are missing for particular inflation series. Throughout our estimation, we use data beginning in 1960. Associated with this early start date for the sample is the fact that, for some countries, subsets of the series used may have missing observations, reflecting a later initial date for those series or other data-availability problems. In addition to providing estimates that take this data issue into account, we also consider estimates that are conditioned on data sets for which available observations on inflation have been discarded for certain dates for judgmental reasons. These reasons reflect our concern that the variations in inflation recorded in certain periods arose from "price shifts," with the latter attributable to non-market factors—such as outright governmental price controls or tax changes that bore directly on measured inflation. In taking this approach regarding price shifts, we expand on a number of earlier studies, including Gordon (1983), Levin and Piger (2004), Neiss and Nelson (2005), and Morley, Piger, and Rasche (2015), to name only a few.[4] In comparing estimates with and without allowance for price shifts, we find that the shifts tend to have an

[3]We believe, however, that the approach to the data we take here has elements that could be usefully applied to the study of the inflation/resource slack connection. For example, our concern with controlling for episodes in which price controls distorted measured inflation series is highly relevant for the task of obtaining valid estimates of the inflation trend in a context in which resource-slack series are among the variables used in the computation of the trend.

[4]Cecchetti et al. (2007, p. 14) adjusted their data on real GDP growth for France for a strike-affected observation. In so doing, they recognized the principle that disruptions to market activity should not be permitted to affect trend estimates. They did not, however, apply this principle to their estimation of trend inflation.

appreciable effect on trend estimates—especially so for the UCSV model—a result that suggests that the shift-affected inflation observations should be excluded when estimating trend inflation. On the other hand, our multivariate estimates of the inflation trend show signs of being more robust to the inclusion of such periods in the estimation.

Finally, we compare the forecast performance of our multivariate model with that of the UCSV model and (as in Atkeson and Ohanian 2001) random-walk forecasts of inflation, in a context of quasi-real-time forecasts from 1985 through 2013. Across forecast horizons ranging from one quarter to sixteen quarters ahead, our multivariate extensions generally deliver lower root mean squared errors (RMSEs) for predictions of inflation, in some cases by 20 percent or more. The improvements are, however, statistically significant in only a few instances—perhaps most notably in the case of medium-term inflation forecasts for the United States.

The remainder of this paper proceeds as follows. Section 2 describes our data set of fourteen industrialized countries. Section 3 lays out the empirical models used throughout the paper. Section 4 presents estimates for level and variability of trend inflation derived from univariate and multivariate models. Section 5 reviews periods in which price shifts occurred and their influence on the estimates. Section 6 evaluates quasi-real-time estimates of trend inflation derived from the UCSV model and our preferred MVSV alternative, and section 7 analyzes the forecast performance of our model in "quasi-real time." Section 8 concludes the paper.

2. International Inflation Data

Our data set consists of quarterly inflation series for fourteen developed countries from 1960:Q1 through 2013:Q4. Whenever data availability permits, we have used three different inflation measures for each country: headline CPI, core CPI, and the GDP deflator, all computed as annualized quarterly log-differences. Details on the available data for each country are provided in table 1. CPI data, including the core CPI series (typically defined as the CPI excluding prices of food and energy) are obtained from the Main Economic

Table 1. Data Overview

Inflation Rates			
Country	**Headline CPI**	**Core CPI**	**GDP Deflator**
Australia	1960:Q1	1976:Q3	1960:Q1
Belgium	1960:Q1	1976:Q3	1980:Q1
Canada	1960:Q1	1961:Q1	1960:Q1
France	1960:Q1	1960:Q1	1960:Q1
Germany	1960:Q1	1962:Q1	1960:Q1
Ireland	1960:Q2	1976:Q1	1980:Q1
Italy	1960:Q1	1960:Q1	1960:Q1
Japan	1960:Q1	1970:Q1	1960:Q1
New Zealand	1960:Q1	1969:Q1	1987:Q2
Spain	1960:Q1	1976:Q1	1970:Q1
Sweden	1960:Q1	1970:Q1	1980:Q1
Switzerland	1960:Q1	1960:Q1	1970:Q1
United Kingdom	1960:Q1	1970:Q1	1960:Q1
United States	1960:Q1	1960:Q1	1960:Q1

Inflation Goals		
Country	**Inflation Goal**	**Dates**
Australia	2.0–3.0	1993:Q2[a]–EOS
Canada	2.0	1991:Q1–EOS
Euro Area[b]	2.0	1998:Q2–EOS
New Zealand	3.0–5.0	1990:Q1–1990:Q4
	1.5–3.5	1991:Q1–1991:Q4
	0.0–2.0	1992:Q1–1996:Q4
	0.0–3.0	1997:Q1–2001:Q4
	1.0–3.0	2002:Q1–EOS
Spain	3.0	1994:Q4–1998:Q1
Sweden	2.0 ± 1	1993:Q1–EOS
Switzerland	< 2.0	2003:Q3–EOS
United Kingdom	2.5	1992:Q4–2003:Q3
	2.0	2003:Q4–EOS
United States	2.0	2012:Q1–EOS

[a]Some sources (for example, Bernanke et al. 1999) give a later date for the inception of inflation targeting in Australia.
[b]Belgium, France, Germany, Ireland, Italy, and Spain have all been euro-area countries since the currency area's inception.
Notes: The model uses quarterly observations from 1960:Q1 through 2013:Q4. Countries with inflation goals continuing through the end of the sample are marked with "EOS." All inflation series are annualized and expressed as log-changes. Section 2 provides more information on the data sources.

Indicators database produced by the OECD.[5] With a few exceptions, GDP deflator data are obtained from the International Financial Statistics (IFS) electronic database maintained by the International Monetary Fund.[6]

Following Faust and Wright (2013), we applied the X-12-ARIMA filter, maintained by the U.S. Census Bureau, to each inflation series analyzed in this paper.[7] The GDP deflator data tended to display strong seasonal components—notwithstanding the label "seasonally adjusted."[8] As a precaution, therefore, we ran the filter over these series.

We have also obtained results with an alternative CPI series for the United States, the "Consumer Price Index Research Series Using Current Methods" (CPI-U-RS). In common with the standard CPI measure for the United States, this alternative series has been constructed by the Bureau of Labor Statistics. In contrast to the regular CPI, whose values do not undergo historical revisions as official measurement procedures change, the CPI-U-RS applies current methods backward to 1978. We use the latest available version at the time of our study, giving us data through the end of 2013. Our trend estimates for the United States are not appreciably altered by the use of this series, and we defer a summary of our results using the CPI-U-RS to section 7.

For many countries, our estimation sample encompasses periods over which recorded price series were likely significantly distorted

[5]The only exception pertains to the data for Ireland's headline CPI, which were compiled from the International Monetary Fund's International Financial Statistics electronic database.

[6]In the case of Sweden, the source is the OECD's Main Economic Indicators. GDP deflators for Italy and Japan in IFS exhibited rebasing problems, so deflator series from Stock and Watson (2003) starting in 1960:Q1 were spliced together with IFS data from 2000:Q1 to 2013:Q4. Conefrey Thomas and Stefan Gerlach kindly supplied us with data for Ireland's GDP deflator for the period 1980–1997, a sample that precedes the series' commencement in the IFS database.

[7]Complete documentation on the X-12-ARIMA seasonal adjustment program can be found in "X-12-ARIMA Reference Manual, Version 0.3, February 28, 2011" at http://www.census.gov/srd/www/x12a/. The filter is implemented in IRIS (an open-source toolbox for MATLAB), which can be obtained from http://www.iris-toolbox.com.

[8]Stock and Watson (2003, p. 803) report the same phenomenon in their study of international data.

by non-market forces, like government-imposed price controls and major changes in indirect taxes.[9] We discuss these episodes, and their effects on our estimates, in detail in section 3. An overview of these "price-shift" dates is given in table 2.

3. Model Description

Our paper uses two different statistical models for the estimation of measures of trend levels and variability and to construct inflation forecasts. Both models rest on time-series approaches that deploy the same trend concept. The models mainly differ in the data on which their estimates are conditioned. The first model is the univariate, "UCSV," model of Stock and Watson (2007, 2010), which is applied to data for each country's CPI inflation (i.e., the headline rate). The second model is a variant of the multivariate common-trend model of Mertens (2011), which we estimate using data on three inflation series for each country, employing headline and core CPI inflation as well as percentage changes in the GDP deflator. Both models utilize the trend concept of Beveridge and Nelson (1981), as discussed presently, and both allow for time-varying volatility in trend shocks. The UCSV model embeds the assumption that deviations between actual inflation and trend have no persistence. In contrast, the multivariate model uses a (time-invariant) vector autoregression to describe the dynamics of deviations between inflation and its trend.

Throughout this paper, we employ a statistical "trend/cycle" decomposition of inflation into a trend level, τ_t, and inflation gap, $\tilde{\pi}_t$. In the tradition of Beveridge and Nelson (1981), the inflation trends that we consider correspond to long-run—that is to say, distantly

[9] Some dates were excluded only from the GDP deflator series because of rebasing errors. The series for Belgium, Canada, Germany, Italy, Spain, and Switzerland all included large, discrete escalations in the price level that are not present in corresponding data reported in other studies such as Stock and Watson (2003). These data points are not included in any of the estimation results below. The dates for which observations are omitted from all estimations are 1966:Q1 (Italy), 1981:Q1 (Spain), 1991:Q1 (Germany), 1995:Q1 (Canada), and 1999:Q1 (Belgium and Spain).

Table 2. Omitted Price-Shift Dates

Country	Date	Event
Australia	1975:Q3	Universal Health Insurance[a]
	1975:Q4	Sales Tax Increase[a]
	1976:Q4	Removal of Universal Health Insurance[a]
	1984:Q1	Medicare Introduction[a]
	2000:Q3	GST Introduction[b]
Canada	1991:Q1	GST Introduction[b,c]
	1994:Q1–1994:Q2	Cigarette Tax Change[b,c]
France	1963:Q3–1963:Q4	Price Freeze and Strict Controls[d]
	1969:Q3–1969:Q4	Price Freeze[d]
	1973:Q1	VAT Decrease[d]
	1976:Q4	Price Freeze[d]
	1977:Q1	VAT Decrease[d]
	1995:Q3	VAT Increase[d]
	2000:Q2	VAT Decrease[d]
Germany	1991:Q1–1991:Q4	Reunification[b]
	1993:Q1	VAT Increase[b]
Ireland	1975:Q3	Indirect Tax Cut[e]
	2012:Q1	VAT Increase[f]
Japan	1997:Q2	Consumption Tax Increase[b]
New Zealand	1982:Q3–1984:Q3	Price Controls[e]
	1986:Q4	GST Introduction[b]
	2010:Q4	GST Introduction[e]
Spain	2012:Q3	VAT Increase[f]
Sweden	1990:Q1	VAT Increase[b]
	1991:Q1	VAT Increase[b]
United Kingdom	1972:Q4–1974:Q2	Price Controls[a]
	1979:Q3	VAT Increase[a]
	1990:Q2	Poll Tax Introduction[b]
	1991:Q2	VAT Increase[g]
United States	1971:Q3–1974:Q2	Nixon Price Controls[h]

[a]Neiss and Nelson (2005).
[b]Levin and Piger (2004, table A2).
[c]We do not include Canada's controls program of 1975–8 among our price-shift dates, on the grounds that that regime was primarily one of wage control (see Braun 1986, pp. 48, 244).
[d]Our dates for France price control are derived from the accounts in Berstein (1993, p. 119), Braun (1986, p. 43), Salin and Lane (1977, p. 577), and Ungerer (1997, p. 61).
[e]From our own analysis of news records.
[f]Klitgaard and Peck (2014).
[g]Debelle and Wilkinson (2002).
[h]Gordon (1983).

far-ahead—forecasts for the level of inflation.[10] As described below, the two models used in this paper differ in their implied dynamics for the inflation gap. In both models, the long-run forecast of inflation corresponds to the Beveridge-Nelson trend concept:

$$\pi_t = \tau_t + \tilde{\pi}_t \qquad \tau_t = \lim_{k\to\infty} E_t \pi_{t+k}. \tag{1}$$

As the trend is defined as a martingale, its law of motion is a random-walk process that cumulates (the current and past values of) serially uncorrelated disturbances $\overline{e_t}$:

$$\tau_t = \tau_{t-1} + \overline{e_t}. \tag{2}$$

This specification necessarily imparts a random-walk component to inflation. Whether this non-stationary component has appreciable effects on observed inflation dynamics depends on the relative magnitude of fluctuations in the inflation trend and the inflation gap. In this connection, we seek estimates that are well suited to environments in which inflation expectations are well anchored and trend changes are near zero, *as well as* episodes in which expectations became unhinged and trend changes were large. To that end, the random-walk disturbances are assumed to have stochastic volatility, with drifting log-variances, following the specification used, for example, by Cogley and Sargent (2005) as well as Stock and Watson (2007). That is,

$$\overline{e_t} \sim N\left(0, \overline{\sigma}_t^2\right) \qquad \log \overline{\sigma}_t^2 = h_t = h_{t-1} + \varphi_{\overline{h}} \xi_t \qquad \xi_t \sim N(0,1). \tag{3}$$

This trend definition is then embedded into two models of inflation dynamics, to which we now turn.

[10] In contrast to the original Beveridge-Nelson decomposition—and in keeping with the approach of Stock and Watson (2007)—our trend estimates are derived in the context of an unobserved-components model. In this class of models, the distinction between filtered and smoothed trend estimates—that is, the distinction between estimates that condition only on a *subset* of observations and those that condition on the full data sample—becomes highly relevant. For further discussion see, for example, Harvey (1989, ch. 6) and Morley (2011).

3.1 Univariate UCSV Model

The UCSV model of Stock and Watson (2007) takes the inflation gap as exhibiting no persistence and also embeds the principle that the gap is itself governed by a separate process for stochastic volatility. That is,

$$\tilde{\pi}_t \sim N\left(0, \tilde{\sigma}_t^2\right) \qquad \log \tilde{\sigma}_t^2 = \tilde{h}_t = \tilde{h}_{t-1} + \varphi_{\tilde{h}} \tilde{\xi}_t \qquad \tilde{\xi}_t \sim N(0,1). \quad (4)$$

Disturbances to the inflation trend and to the inflation gap, as well as the shocks to stochastic volatility, are assumed to be serially and mutually uncorrelated. Stock and Watson (2007) fix the volatility of shocks to the log-variance processes in gap and trend, $\varphi_{\bar{h}}$ and $\varphi_{\tilde{h}}$, to constant values—equal to 0.20 for both parameters, which is close to typical estimates obtained for U.S. data. We, however, estimate these two parameters, using a relatively loose prior as our starting point.[11]

3.2 Multivariate Model (MVSV)

As an alternative to the univariate UCSV model, we also study trend estimates derived from a multivariate model with stochastic volatility (MVSV), which jointly conditions on three inflation measures for each country. A variant of the model has been applied by Mertens (2011) to U.S. data. The model incorporates time-varying volatility in both the trend and the gap component of inflation; accordingly, it nests the UCSV case. In our application, the model uses observations on inflation in headline CPI, core CPI, and the GDP deflator—all stacked into a vector, Y_t—and applies a "trend/cycle" decomposition, along the lines of the UCSV model described above:

$$Y_t = \tau_t + \tilde{Y}_t \qquad \tau_t = \lim_{k \to \infty} E_t Y_{t+k}. \quad (5)$$

The key assumption underlying the multivariate model is that all variables in Y_t share the same common trend, with their trend

[11]Specifically, we specify an inverse-Wishart prior for each parameter with a mean equal to the Stock-Watson value of 0.2; for the gap and trend parameter, we use three and thirty degrees of freedom, respectively.

levels differing only up to a constant.[12] Crucially, trend changes in all three inflation measures are driven by a single shock, for which the stochastic-volatility behavior applies as in equation (4) above.

In contrast with the UCSV model, and in keeping with the more recent literature on estimation of inflation trends, inflation gaps are permitted to be persistent in the multivariate model, subject to the condition that the law of motion governing the inflation gap has convergent dynamics. Specifically, the inflation gaps are assumed to follow a dynamically stable VAR with constant parameters and constant correlations and a common volatility factor. That is,

$$A(L)\tilde{Y}_t = \tilde{e}_t \qquad \tilde{e}_t \sim N(0, \Sigma_t) \qquad \Sigma_t = L\,diag\widetilde{(\sigma_t^2)}\,L \tag{6}$$

$$\log \tilde{\sigma}_t^2 = \tilde{h}_t = (I - 0.951)^{-1}\tilde{h} + 0.95\tilde{h}_{t-1} + \Theta_{\tilde{k}}\tilde{\xi}_t$$

$$\tilde{\xi}_t \sim N(0,1), \tag{7}$$

where L is a lower triangular matrix of constant parameters and every element of the vector of log-variances $\tilde{h}_t$ follows a highly persistent AR(1) process, each with an autoregressive coefficient equal to 0.95, as indicated, but with correlated shocks.[13] The AR(1) specification for the variances was chosen over the random walk in light of the existence of extended periods, in the earlier part of our sample, of missing data for some of our input series; estimates obtained from a random walk would quickly lead to unbounded variance estimates over those periods.[14] Importantly, shocks to the

[12]Within the Y_t vector, *average* levels of trend inflation are allowed to differ in recognition of discrepancies across the various inflation series in the average rate (for example, the existence of a different mean rate for CPI inflation from that for GDP deflator inflation).

[13]The diagonal elements of L are normalized to unity, and the lower triangular elements have been assigned standard normal priors. Analogously to the UCSV model, the variance-covariance matrix of the stochastic volatility shocks has an invariant-Wishart prior with mean equal to $0.2^2 \cdot I$ and five degrees of freedom—this value for the degrees of freedom is the lowest possible value that ensures the existence of a prior mean for a 3x3 matrix of random variables, drawn from the inverse-Wishart distribution.

[14]Grassi and Proietti (2010) modify the UCSV model of Stock and Watson (2007) to permit an AR(1) specification for stochastic volatility, doing so in part on *a priori* grounds of the unattractiveness of the unboundedness associated with the random-walk model. Clark and Ravazzolo (2014) compare the forecasting performance of different specifications for stochastic volatility—including the cases of a random walk and an AR(1) process—for various macroeconomic variables.

individual gap volatilities are allowed to be correlated with one another. In many cases, our estimates will imply generally high levels of such cross-correlation. It will emerge, however, that, notwithstanding the substantial co-movement in gap volatilities, there are also notable episodes of *idiosyncratic* changes in volatility of a particular inflation-gap series. This phenomenon reflects the behavior of individual inflation measures, most particularly the GDP deflator inflation rate, which would not be adequately captured had we assumed a uniform pattern of behavior for the volatilities of the different inflation series for a particular country.

As in the UCSV model, shocks to the volatility of trend and gap components are assumed to be uncorrelated. The roots of the VAR polynomial $A(L)$ are required to lie outside the unit circle, thereby ensuring that the gaps exhibit convergent dynamic properties.[15] Shocks to the gap levels are allowed to be mutually correlated. However, in our baseline specification, all gap shocks are assumed to be *uncorrelated* with trend shocks.[16] The multivariate model therefore nests the UCSV model, at the same time extending it to multiple input series and persistent gap dynamics. Missing observations in Y_t are handled by casting the model in state-space form with (deterministic) time variation in measurement loadings. Instances of missing observations lead to the appropriate elements of Y_t being assigned a value of zero, and the same is true of their loadings on the model's states. See, for example, Mertens (2011) for details.

[15]The VAR coefficients have been assigned a prior distribution that is multivariate normal (subject to the stationarity constraint) and that is centered on a prior mean of zero. For the variances, we have experimented with several relatively small values. This is in order that most of the prior mass of the vector of VAR coefficients satisfies the stationarity constraint and to ensure convergence of the model estimates across all countries and all of the quasi-real-time estimation samples analyzed in sections 6 and 7 below. Results shown here were obtained from a multivariate normal prior for the VAR coefficients with mean zero, zero covariances, and prior volatilities equal to 0.20 for own-lag coefficients and 0.10 for all other coefficients. Although this prior is quite tightly centered on zero, our posterior estimates of the VAR coefficients imply substantial inflation-gap persistence, as shown below. Largely similar results were also obtained when the scale of prior volatilities was doubled.

[16]Mertens (2011) allows the shocks to trend and gap to be correlated in the MVSV case. For simplicity, however, we impose orthogonality between the two classes of shocks for both our UCSV and MVSV estimates.

3.3 Alternative Specifications of the Multivariate Model

We also considered several alternative specifications of the MVSV model. In its baseline version, described above, the MVSV model embeds the assumptions that shocks to inflation trend and gaps are uncorrelated and that the VAR dynamics of the gaps are time invariant. We separately relax each assumption. To model correlation between shocks to the inflation trend and gaps, we rewrite the gaps' equation as

$$A(L)\tilde{Y}_t = e_t \qquad e_t = \bar{\beta}\bar{e}_t + \tilde{e}_t, \tag{8}$$

where $\tilde{e}_t$ is specified as before.[17]

We have also considered time variation in the VAR coefficients, of a kind that implies an inflation-gap equation of $A_t(L)\tilde{Y}_t = \tilde{e}_t$. The VAR coefficients are modeled as drifting random walks, subject to the stationarity condition for each polynomial $A_t(L)$ with correlated shocks.[18]

As a third alternative, we explicitly incorporate information regarding a country's inflation goal in the data set used for conditioning our model estimates. This version of the model will also be labeled "MSVS-T." With the exception of Japan, each country in our data set had by 2013 (the end of our sample) introduced some form of explicit inflation goal. (See table 1.) For these countries, we have augmented the measurement equation of the MVSV model with a fourth variable that is equal to each country's inflation goal—or

[17] The choice of the prior for $\bar{\beta}_t$ turned out to be important for the convergence of the Markov chain Monte Carlo (MCMC) algorithm used in the estimation. Results reported below were generated from a standard normal prior, which led to satisfactory convergence for almost all countries considered. In the case of less informative normal priors with larger variances, the MCMC estimation typically failed to achieve convergence in our experience.

[18] In contrast to its application to the stochastic gap volatilities, the random-walk assumption for the VAR coefficients does not lead to unbounded posterior draws when there is missing data. This reflects the additional restriction that all draws of $A_t(L)$ must have all roots outside the unit circle. The variance/covariance matrix of random-walk shocks to the vector of VAR coefficients is given a vaguely informative inverse-Wishart prior with $N+2$ degrees of freedom, where N is the number of VAR coefficients, and the prior is given a mean of $0.05^2 \cdot I$. The scale of this prior reflects has been chosen to allow for considerable range of possible persistence, within the region of coefficient values that are consistent with a dynamically stable VAR structure.

the midpoint of its goal range—and that is treated as missing data in the absence of an official inflation objective. This variable will be interpreted as a direct reading of the trend level for headline CPI inflation.[19]

3.4 Estimation Methods

The models are estimated with Markov chain Monte Carlo methods, similar to those described in Mertens (2011). The algorithm yields not only estimates of the latent factors. The sampling algorithm recovers the posterior distribution of missing data entries, conditional on the model and all observed data values. Convergence is assessed with scale-reduction tests (see Gelman and Rubin 1992), applied to the output of multiple chains that started from dispersed initial conditions.

4. Inflation Trends: Levels and Uncertainty

This section reports country-by-country estimates of inflation trends and gaps as well as their evolving variability, as generated from our application of the UCSV model of Stock and Watson (2007) and our MVSV model. In essence, these UCSV estimates complement and extend the results reported by Cecchetti et al. (2007), whose estimates are conditioned on the GDP deflator inflation rates for the G7 economies. The UCSV estimates reported below are conditioned on the CPI inflation headline rate. We report the inflation-gap estimates only for CPI (headline) inflation for the MVSV model, taking this measure of inflation as the one of greatest interest, particularly in the context of the targeting and forecasting of inflation. Generally speaking, the estimates reported below are conditioned on all available data from 1960:Q1 through 2013:Q4, the only major qualification being that we remove from estimation certain dates, specified in table 2, when price shifts occurred.[20]

[19]After the introduction of an inflation goal, trend changes are treated as deterministic by the MVSV-T model. No country in our data set has abandoned its inflation goals after inception, except for changes in the goal's value.

[20]The effects of these price shifts on our estimates are discussed in section 5.

A comparison of estimates from the UCSV model and the MVSV for each country indicates that while there are broad similarities, there also plainly exist notable differences. Estimates from both models capture very similar low-frequency movements. By and large, estimates of the inflation trend and its stochastic volatility from the two models are quite similar. That said, in several instances—especially around the time of the global financial crisis in 2008–9—the effects of the UCSV model's assumption of serially uncorrelated inflation gaps are also quite apparent. For example, in the cases of Belgium, France, Italy, Japan, Spain, Switzerland, the United Kingdom, and the United States, the UCSV estimates seem to be affected by transitory fluctuations in inflation—fluctuations from which the MVSV model essentially insulates its inflation-trend estimates.

A notable aspect of the results for the United States is that the trend-inflation estimate tracks actual inflation quite closely in the 1970s. In particular, the trend estimate reaches double digits in the mid-1970s. In the case of the UCSV model, this result, for CPI inflation, is similar to that obtained for U.S. GDP deflator inflation by Cecchetti et al. (2007). The fact that trend inflation closely matches actual inflation during the 1970s in the UCSV case is consistent with the notion that actual U.S. inflation behavior resembled that of a random walk during those years; it is therefore natural for the UCSV model, in which the trend rate corresponds to the predictable component of inflation, to attribute much of the observed fluctuations in inflation to variations in the trend rate.

In the case of the MVSV model, which allows for persistent inflation-gap dynamics, it may appear surprising that we find, once again, that trend inflation in the mid-1970s largely mirrors actual inflation. Our estimates differ on this score from those in Cogley, Sargent, and Primiceri (2010), who find that trend U.S. inflation was well below actual inflation in the mid-1970s.[21] One major reason for the difference in findings is that in Cogley, Sargent, and Primiceri (2010), a long-term interest rate was included among the variables with which inflation was assumed to share a trend. Long-term interest rate data in the United States in the mid-1970s implied

[21] However, Morley, Piger, and Rasche (2015, figure 1) find that trend inflation tracks actual CPI inflation quite closely in the United States during the 1970s.

longer-term inflation expectations far below actual inflation in the mid-1970s, and so inclusion of these interest rates in the analysis would point toward a conclusion that the surge in inflation during that period largely amounted to an increase in the inflation gap.[22] In our analysis, however, the variables with which we assume CPI inflation has a common trend do not include long-term interest rates but do include the GDP deflator inflation rate. In the mid-1970s, the GDP deflator inflation rate exhibited a rise that largely conformed to that of the CPI inflation rate, and so our assumption that these two inflation series have a common trend makes the MVSV model more likely to regard the mid-1970s rise in inflation as a rise in the inflation trend. In contrast, the late-1970s upsurge in inflation was much steeper for CPI inflation than for the GDP deflator rate. Consequently, our MVSV estimates imply a sharp rise in the CPI inflation gap for this period, as opposed to a surge in trend inflation: see figure 14.

As noted earlier, a great number of countries have introduced formal inflation goals during the sample period. In the majority of cases, estimated trend levels from both models tend to hover around the numerical value for the inflation goal. But there are some notable exceptions, as discussed below. In the wake of the formal introduction of an inflation target, the stochastic volatility of trend shocks—our measure, alongside the inflation-trend estimate itself, of the degree to which inflation expectations are anchored—decreases in many cases only after some time, about five to ten years. This result likely reflects the fact that our measure is conditioned solely on the realized inflation experience of a given country.

Among those cases in which countries have explicit inflation goals, the trend estimates for Sweden, shown in panels A and B of figure 11, stand out, as the trend has regularly moved below the Riksbank's inflation target of 2 percent by half a percentage point or more since the target was introduced in 1993—a finding that

[22]Likewise, in Mertens's (2011) estimates of trend inflation for the United States, both longer-term interest rates and inflation expectations survey data are assumed to have a common trend with inflation. As both expectations data and longer-term interest rates registered a much milder rise in the mid-1970s than actual inflation, their inclusion in the analysis held down the estimated peak of trend inflation in Mertens (2011).

is consistent with Svensson's (2015) characterization of the behavior of inflation expectations in Sweden. In the same vein, late in the sample the inflation-trend estimates for Germany and France, joined by Ireland, Italy, and Spain, exhibit inflation-trend estimates somewhat below the European Central Bank's target rate of "close to but below 2 percent."

A noteworthy comparison between the MVSV and UCSV estimates is offered by the case of the United Kingdom, estimates for which are displayed in figure 13. For several years late in the sample period, U.K. inflation often persistently exceeded the Bank of England's 2 percent target, and these overshoots influence our estimates in varying degrees. In particular, the UCSV estimates of trend inflation tend to increase in the final years of the sample, with the estimate moving up to levels near 4 percent, and the 90 percent range for the estimate of trend inflation barely includes the target rate of 2 percent. In contrast, the MVSV model implies a much more limited increase in trend inflation for the United Kingdom, because the persistence embedded in the model's specification of inflation-gap dynamics separates the phenomenon of sustained overshoots of the inflation target from the phenomenon of a shift up in trend inflation.

The estimated trend levels of inflation for Japan (shown in figure 8) are, for the latter part of the sample, among the lowest for the countries we study. Both the MVSV and UCSV estimates put trend inflation for Japan at levels generally below zero for the last decade; in particular, the trend estimate derived from the MVSV model has been below zero, and even the upper bound of the 90 percent credible set for the trend barely covers values above zero from about 2000 through 2011. Concerns about elevated risks of deflation are also raised by our trend estimates for Switzerland, shown in figure 12, which have steadily been falling, and even moved briefly below zero, over the last few years, after having remained stable near 2 percent for most of the prior fifteen years.

For most countries, very similar trend estimates are also obtained if the MVSV model is replaced by a variant that allows for correlation between shocks to inflation trend and gaps, in the manner described in section 3. However, for some countries, this alternative specification generated noticeably different trend estimates. This has been the case for Germany, Sweden, and Switzerland, results for which are depicted in figure 15. For each of these three countries, the assumption of correlation in the shocks to trend and gaps generates trend estimates that are somewhat less volatile than in the baseline case—at least when judged by the paths for the pointwise posterior means. At the same time, uncertainty around these estimates, as measured by the width of the 90 percent confidence sets, is considerably wider than in the baseline case, as can be seen from comparison of figure 15 with the top-left panels in figures 5, 11, and 12.

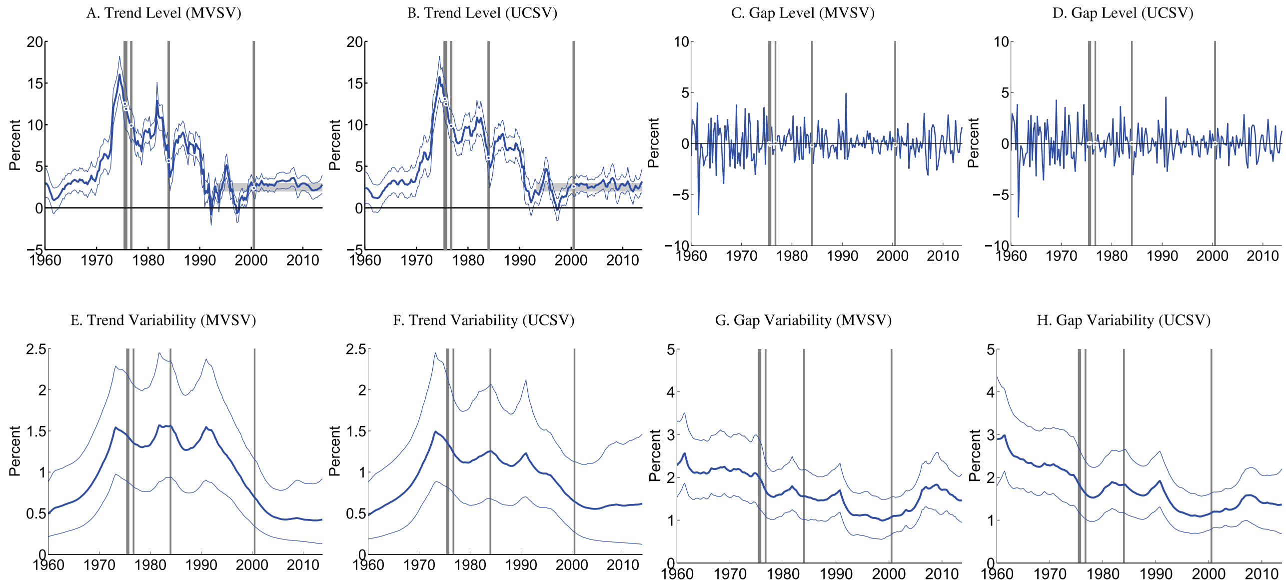
For each country, we also derived trend estimates from a further variant of the MVSV model, one featuring time variation in the VAR parameters that govern the evolution of the inflation gaps. The results for this variant are very similar to the baseline estimates shown in figures 1–14. For brevity, the estimates for this variant are not shown here. The MVSV model with time-varying VAR coefficients does, however, generate sizable variation in the estimated degree of gap persistence, a result brought out in figure 16. For each country, gap persistence is measured by the largest absolute eigenvalue of the gap VAR's companion form. There is no uniform pattern in the changes of gap persistence implied by these estimates. For some countries, like Canada, New Zealand, and Japan, gap persistence seems to have decreased over the latter part of our sample. For other countries, such as France, Ireland, Italy, Sweden, and Switzerland, gap persistence has rather increased.

Trend estimates from the MVSV-T model are very similar to our baseline estimates, except for the periods when the official inflation goal was different from the baseline trend estimates as shown in figures 1–14 (and not shown separately). The MVSV-T model will be discussed further in section 7 in the context of forecast evaluation.

5. The Effects of Price-Shift Dates on Trend Estimates

In general, the estimates presented in the previous section are derived from data sets that excluded the observations associated with dates at which major price-level shifts occurred due to nonmarket factors. The results shown in figures 1 to 14 were generated from inflation data for which periods of price shifts are treated as

Figure 1. Australia



Notes: Solid, thick lines show posterior means, and thinner lines depict 90 percent confidence sets derived from the model's posterior distribution conditional on all data. All levels are measured in annualized percentage points. Variability is measured by the standard deviation of a quarterly trend shock. Data sources are as listed in table 1, using all available data since 1960. Dark gray shading marks dates in which data were excluded from computation due to shifts in the price index at that time. All country-specific price-shift dates for input measures are listed in table 2. For those periods, estimated inflation gaps, shown in panels C and D, are marked by white circles. When there are no price-shift dates, the gap estimates are identical to the difference between actual inflation and the trend estimates, shown in panels A and B. In panels A and B, the light gray area marks the range assigned for an officially stated inflation goal.

Figure 2. Belgium

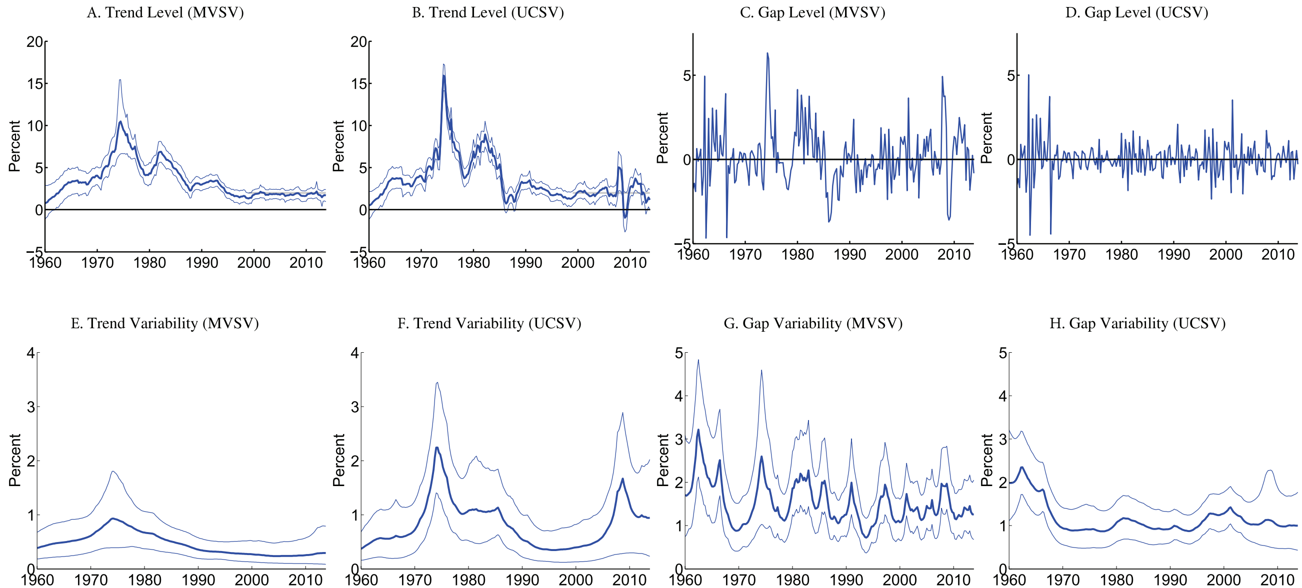


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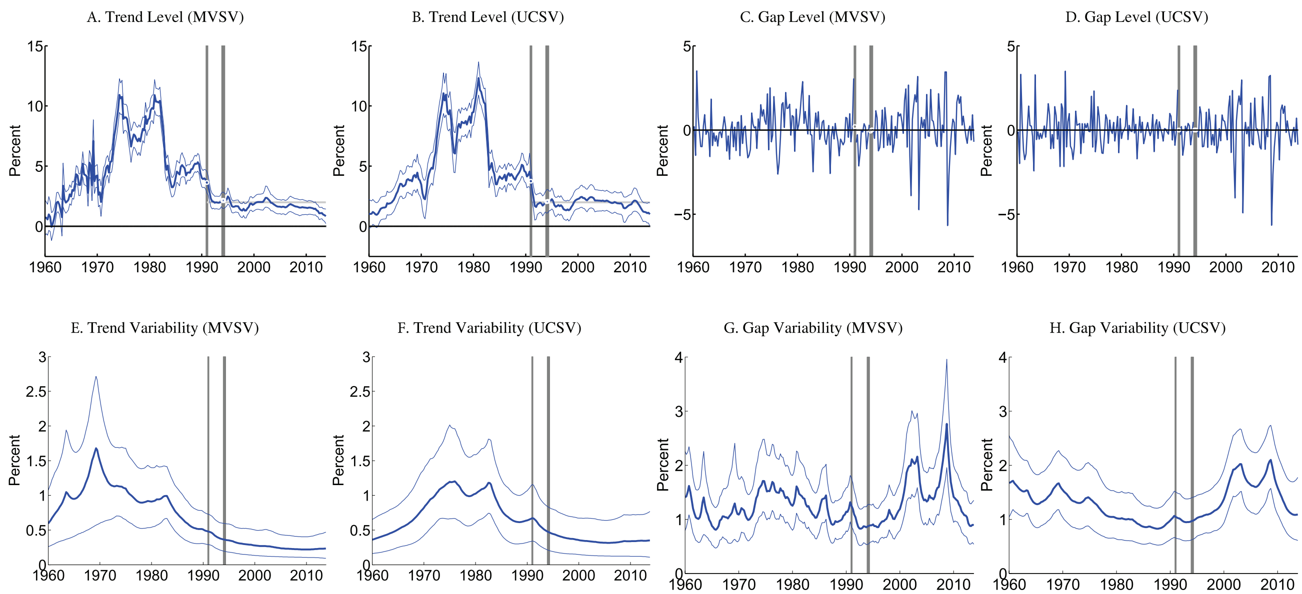
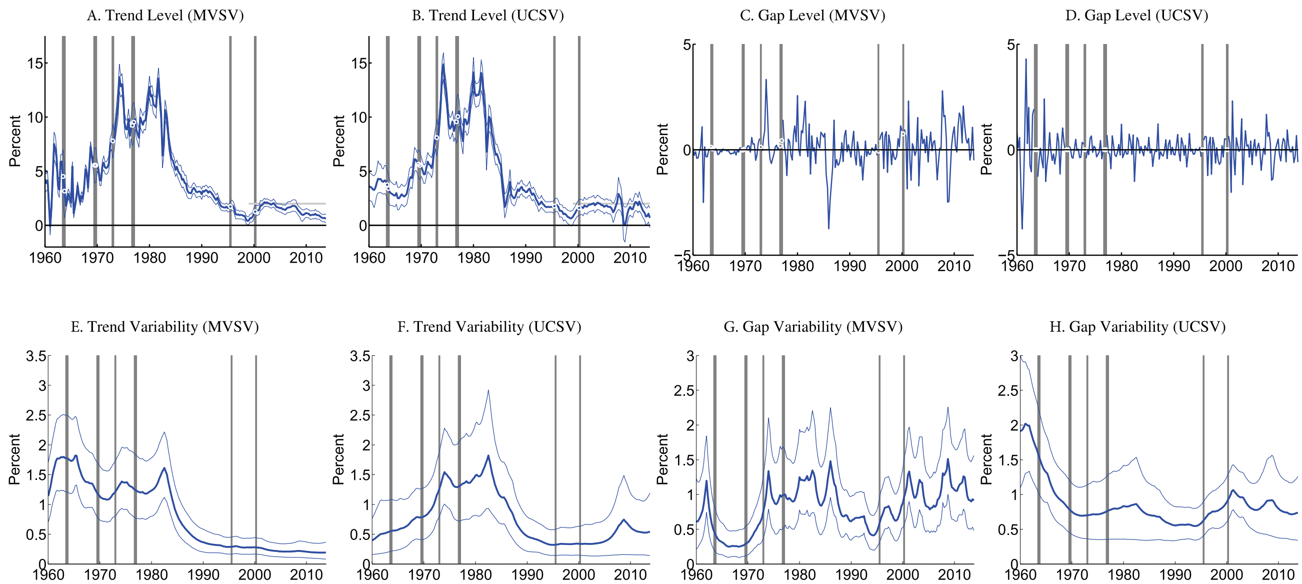
Figure 3. Canada



Notes: Solid, thick lines show posterior means, and thinner lines depict 90 percent confidence sets derived from the model's posterior distribution conditional on all data. All levels are measured in annualized percentage points. Variability is measured by the standard deviation of a quarterly trend shock. Data sources are as listed in table 1, using all available data since 1960. Dark gray shading marks dates in which data were excluded from computation due to shifts in the price index at that time. All country-specific price-shift dates for input measures are listed in table 2. For those periods, estimated inflation gaps, shown in panels C and D, are marked by white circles. When there are no price-shift dates, the gap estimates are identical to the difference between actual inflation and the trend estimates, shown in panels A and B. In panels A and B, the solid, light gray line marks the level assigned for an officially stated inflation goal.

Figure 4. France



Notes: Solid, thick lines show posterior means, and thinner lines depict 90 percent confidence sets derived from the model's posterior distribution conditional on all data. All levels are measured in annualized percentage points. Variability is measured by the standard deviation of a quarterly trend shock. Data sources are as listed in table 1, using all available data since 1960. Dark gray shading marks dates in which data were excluded from computation due to shifts in the price index at that time. All country-specific price-shift dates for input measures are listed in table 2. For those periods, estimated inflation gaps, shown in panels C and D, are marked by white circles. When there are no price-shift dates, the gap estimates are identical to the difference between actual inflation and the trend estimates, shown in panels A and B. In panels A and B, the solid, light gray line marks the level assigned for an officially stated inflation goal.

Figure 5. Germany

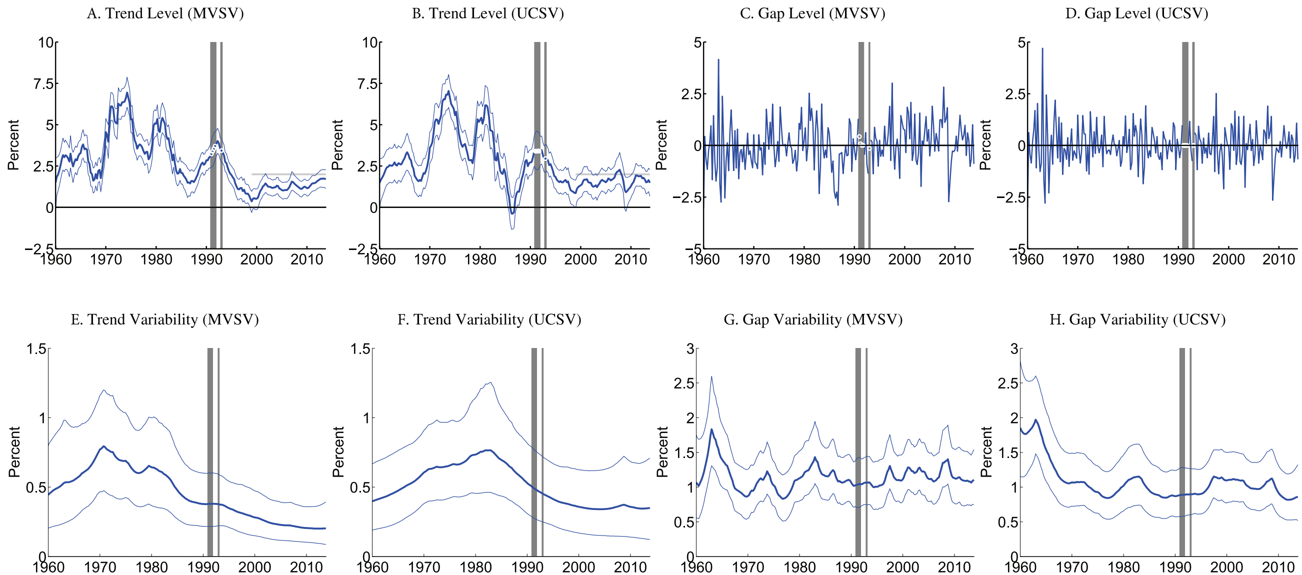


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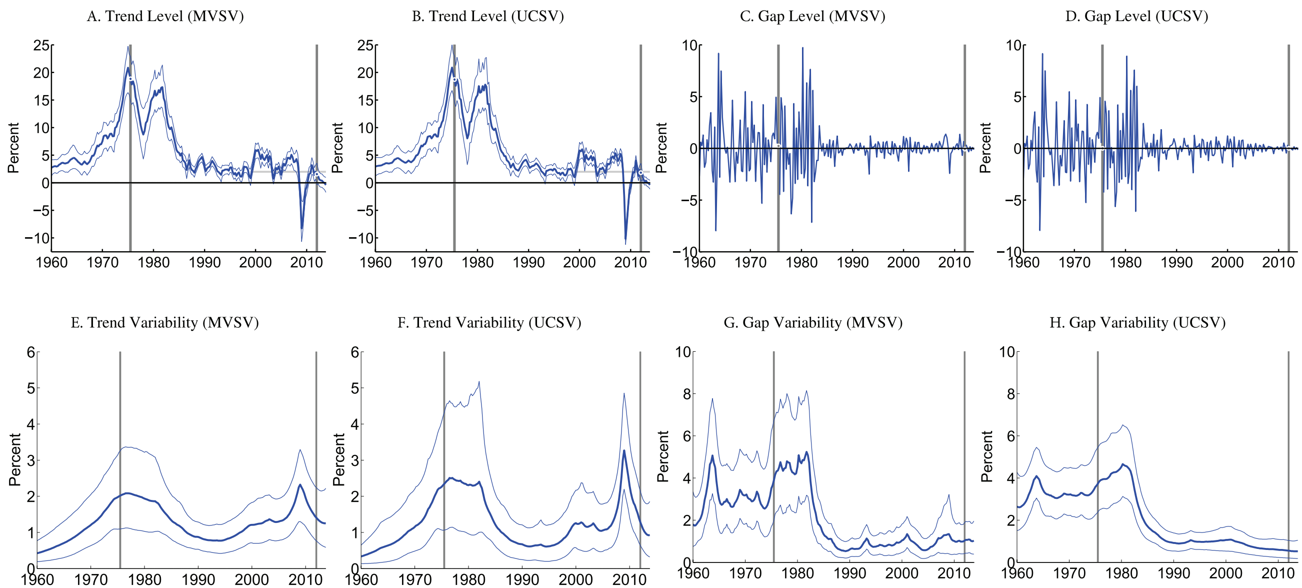
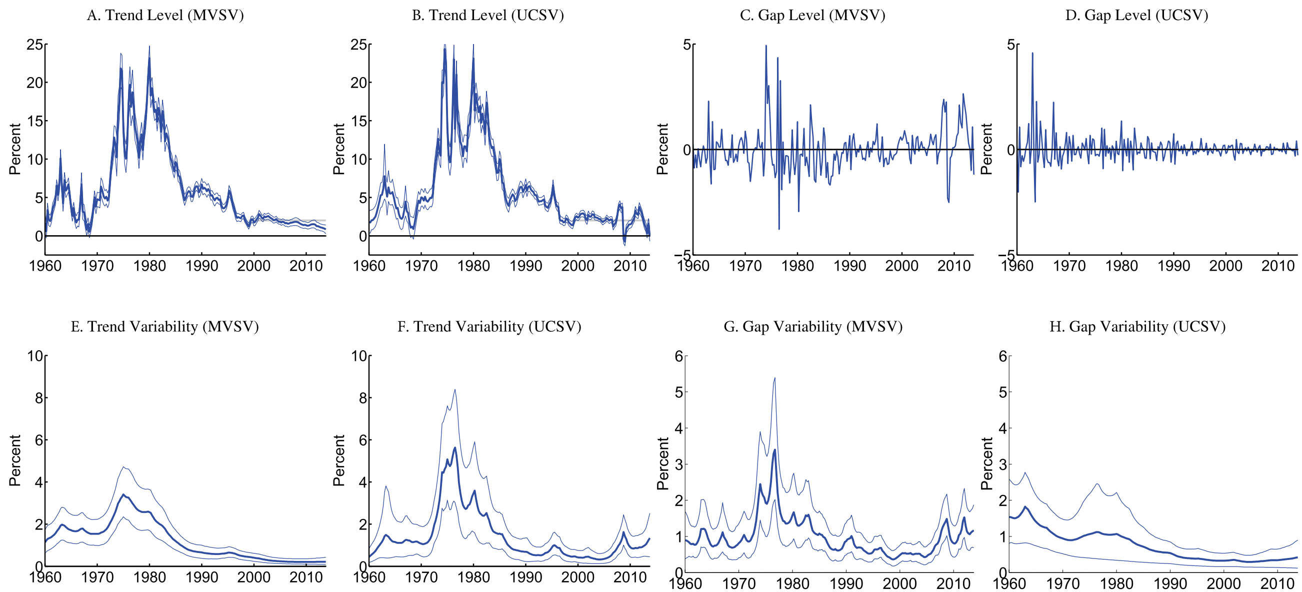
Figure 6. Ireland



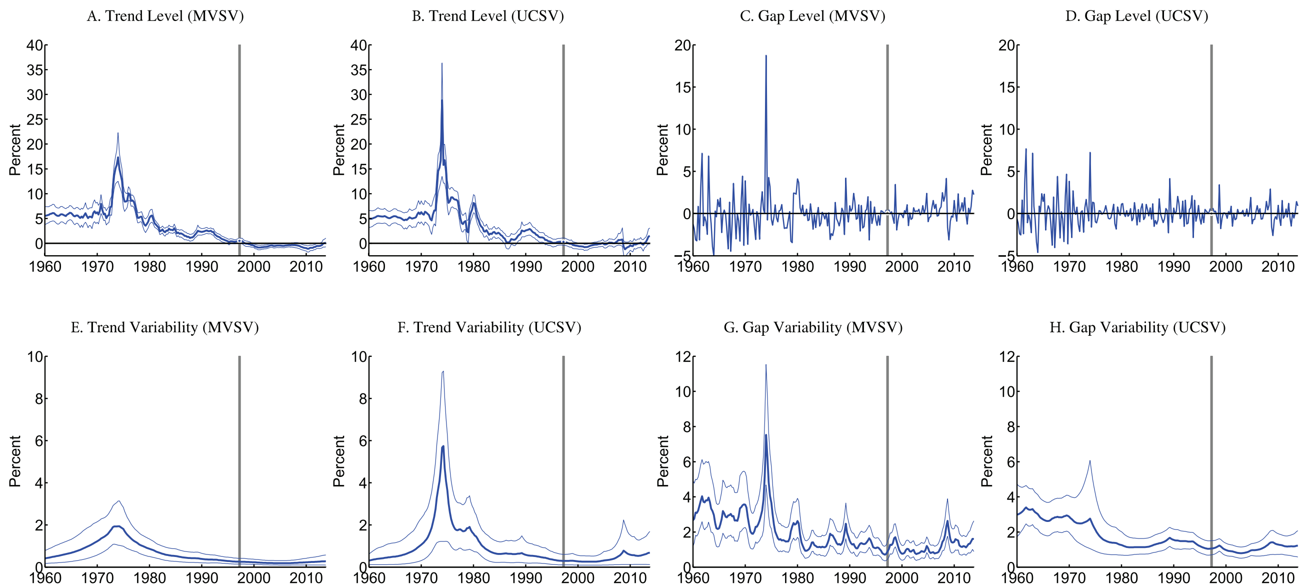
Notes: Solid, thick lines show posterior means, and thinner lines depict 90 percent confidence sets derived from the model's posterior distribution conditional on all data. All levels are measured in annualized percentage points. Variability is measured by the standard deviation of a quarterly trend shock. Data sources are as listed in table 1, using all available data since 1960. Dark gray shading marks dates in which data were excluded from computation due to shifts in the price index at that time. All country-specific price-shift dates for input measures are listed in table 2. For those periods, estimated inflation gaps, shown in panels C and D, are marked by white circles. When there are no price-shift dates, the gap estimates are identical to the difference between actual inflation and the trend estimates, shown in panels A and B. In panels A and B, the solid, light gray line marks the level assigned for an officially stated inflation goal.

Figure 7. Italy



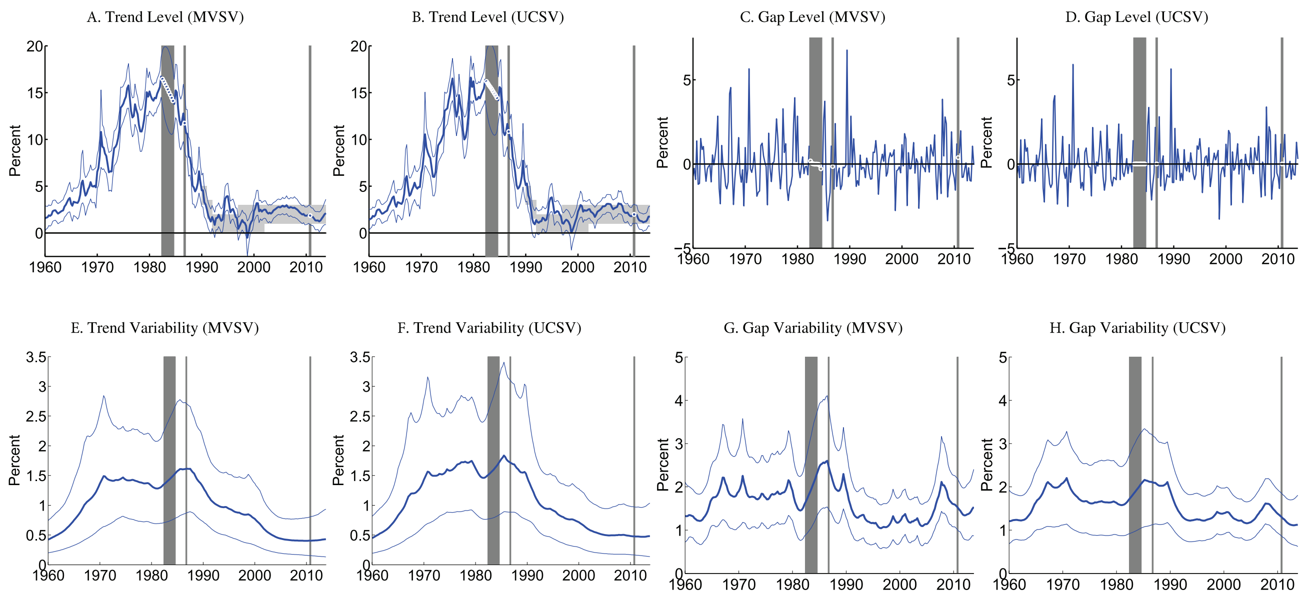
Notes: Solid, thick lines show posterior means, and thinner lines depict 90 percent confidence sets derived from the model's posterior distribution conditional on all data. All levels are measured in annualized percentage points. Variability is measured by the standard deviation of a quarterly trend shock. Data sources are as listed in table 1, using all available data since 1960. In panels A and B, the solid, light gray line marks the level assigned for an officially stated inflation goal.

Figure 8. Japan



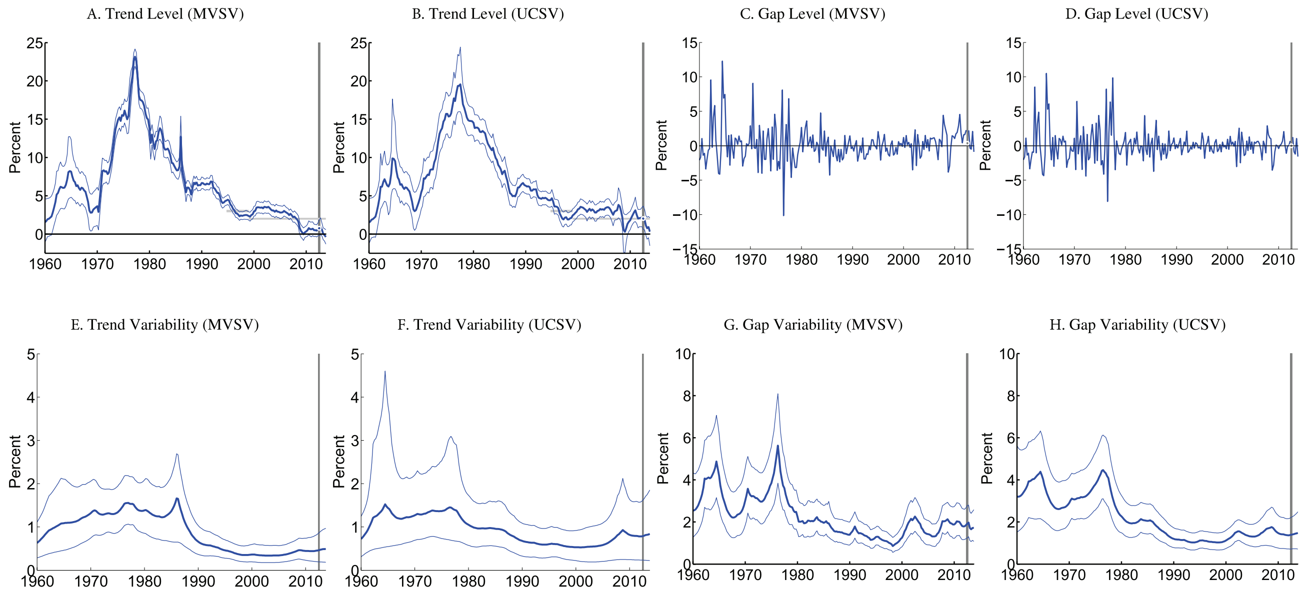
Notes: Solid, thick lines show posterior means, and thinner lines depict 90 percent confidence sets derived from the model's posterior distribution conditional on all data. All levels are measured in annualized percentage points. Variability is measured by the standard deviation of a quarterly trend shock. Data sources are as listed in table 1, using all available data since 1960. Dark gray shading marks dates in which data were excluded from computation due to shifts in the price index at that time. All country-specific price-shift dates for input measures are listed in table 2. For those periods, estimated inflation gaps, shown in panels C and D, are marked by white circles. When there are no price-shift dates, the gap estimates are identical to the difference between actual inflation and the trend estimates, shown in panels A and B.

Figure 9. New Zealand



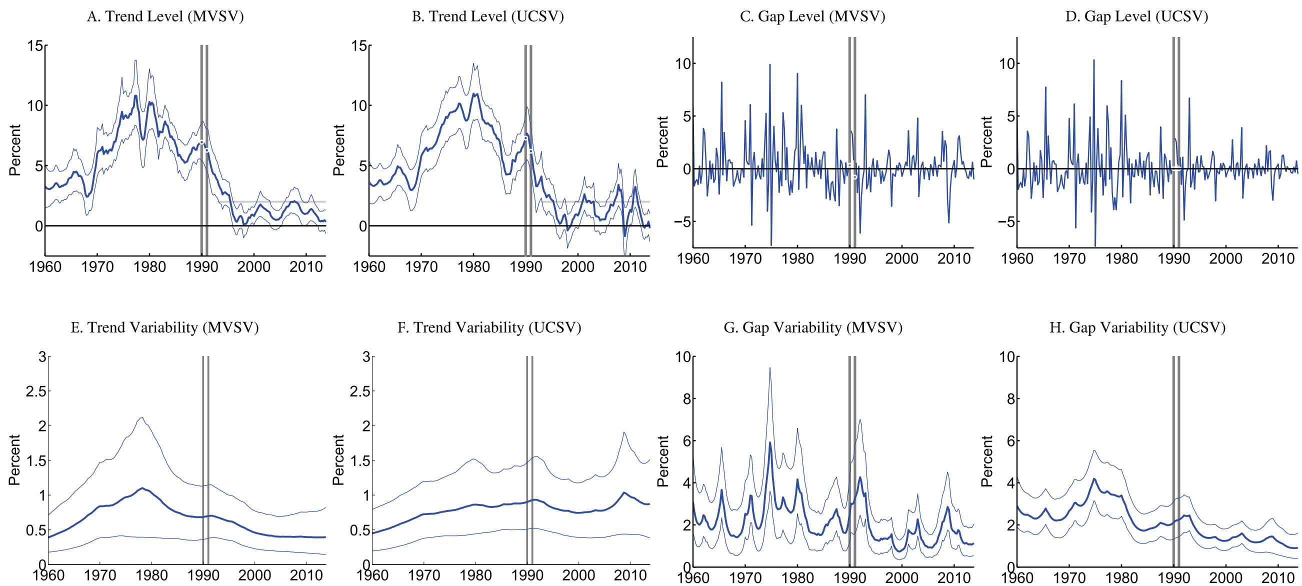
Notes: Solid, thick lines show posterior means, and thinner lines depict 90 percent confidence sets derived from the model's posterior distribution conditional on all data. All levels are measured in annualized percentage points. Variability is measured by the standard deviation of a quarterly trend shock. Data sources are as listed in table 1, using all available data since 1960. Dark gray shading marks dates in which data were excluded from computation due to shifts in the price index at that time. All country-specific price-shift dates for input measures are listed in table 2. For those periods, estimated inflation gaps, shown in panels C and D, are marked by white circles. When there are no price-shift dates, the gap estimates are identical to the difference between actual inflation and the trend estimates, shown in panels A and B. In panels A and B, the light gray area marks the range assigned for an officially stated inflation goal.

Figure 10. Spain



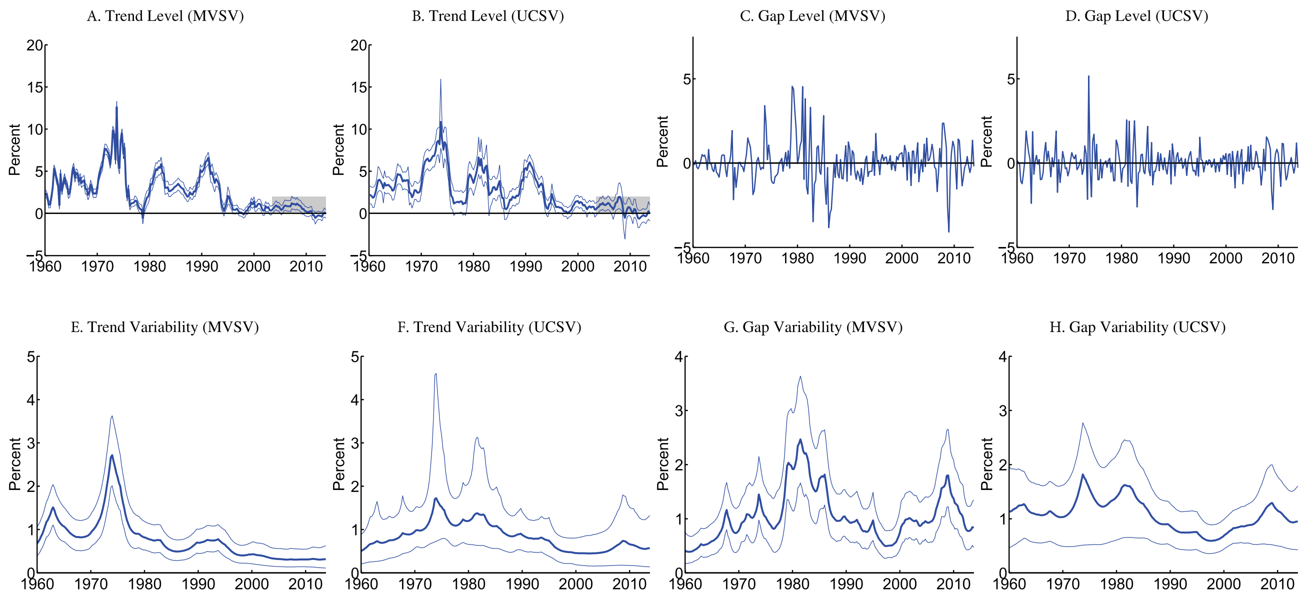
Notes: Solid, thick lines show posterior means, and thinner lines depict 90 percent confidence sets derived from the model's posterior distribution conditional on all data. All levels are measured in annualized percentage points. Variability is measured by the standard deviation of a quarterly trend shock. Data sources are as listed in table 1, using all available data since 1960. Dark gray shading marks dates in which data were excluded from computation due to shifts in the price index at that time. All country-specific price-shift dates for input measures are listed in table 2. For those periods, estimated inflation gaps, shown in panels C and D, are marked by white circles. When there are no price-shift dates, the gap estimates are identical to the difference between actual inflation and the trend estimates, shown in panels A and B. In panels A and B, the solid, light gray line marks the level assigned for an officially stated inflation goal.

Figure 11. Sweden



Notes: Solid, thick lines show posterior means, and thinner lines depict 90 percent confidence sets derived from the model's posterior distribution conditional on all data. All levels are measured in annualized percentage points. Variability is measured by the standard deviation of a quarterly trend shock. Data sources are as listed in table 1, using all available data since 1960. Dark gray shading marks dates in which data were excluded from computation due to shifts in the price index at that time. All country-specific price-shift dates for input measures are listed in table 2. For those periods, estimated inflation gaps, shown in panels C and D, are marked by white circles. When there are no price-shift dates, the gap estimates are identical to the difference between actual inflation and the trend estimates, shown in panels A and B. In panels A and B, the solid, light gray line marks the level assigned for an officially stated inflation goal.

Figure 12. Switzerland



Notes: Solid, thick lines show posterior means, and thinner lines depict 90 percent confidence sets derived from the model's posterior distribution conditional on all data. All levels are measured in annualized percentage points. Variability is measured by the standard deviation of a quarterly trend shock. Data sources are as listed in table 1, using all available data since 1960. In panels A and B, the light gray area marks the range assigned for an officially stated inflation goal.

Figure 13. United Kingdom

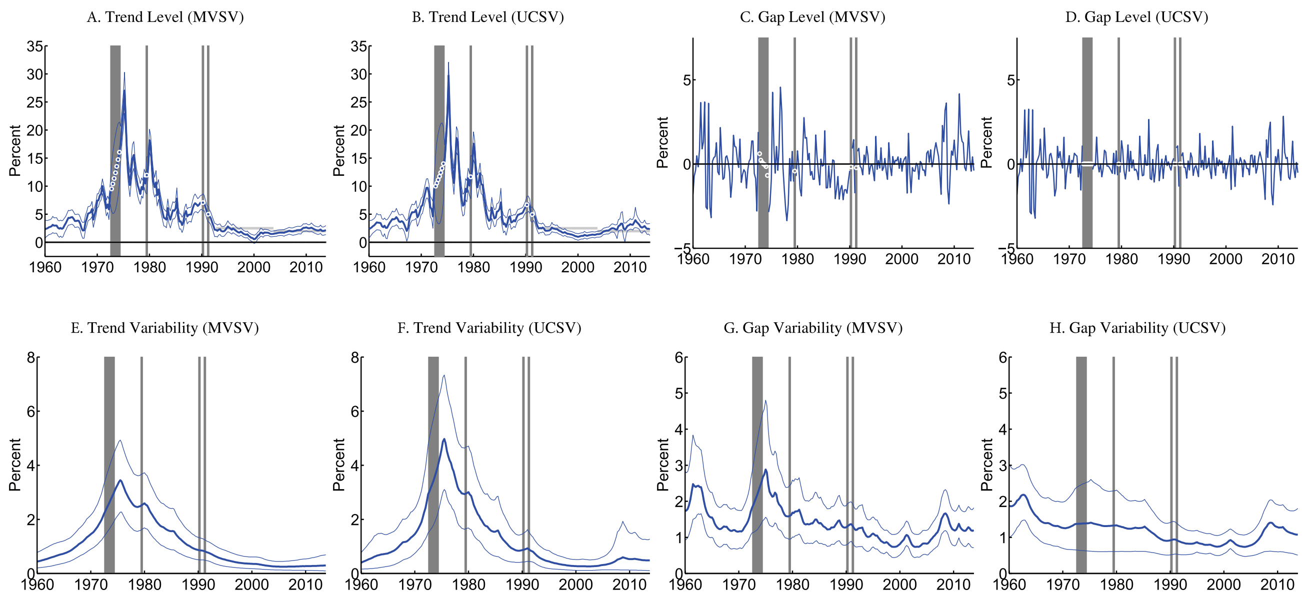


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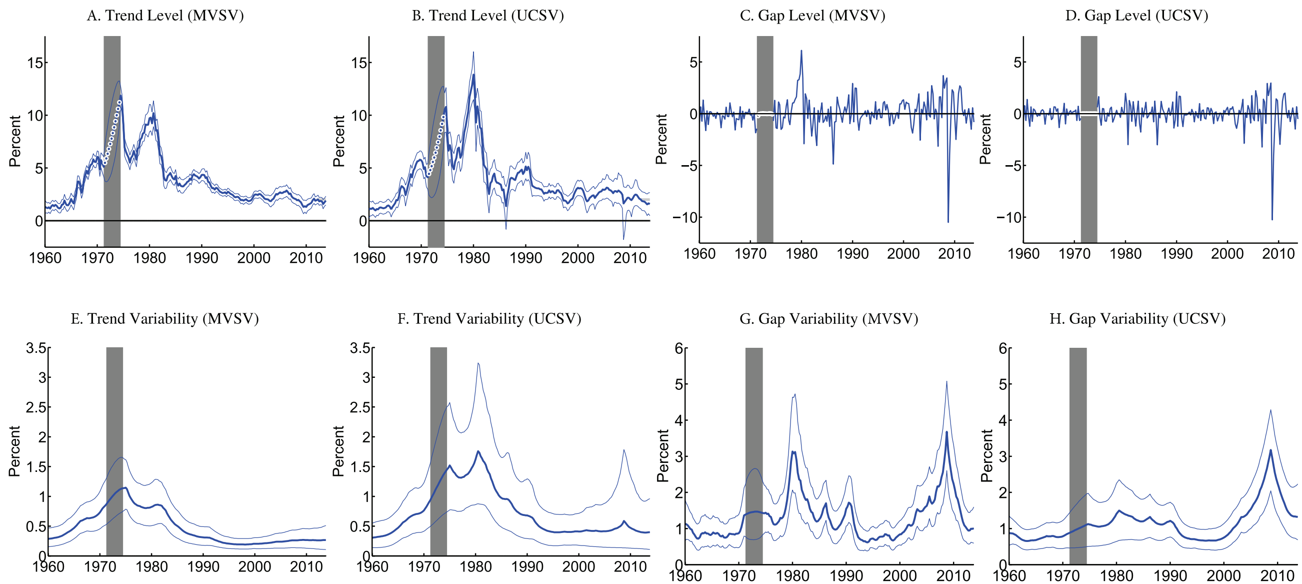
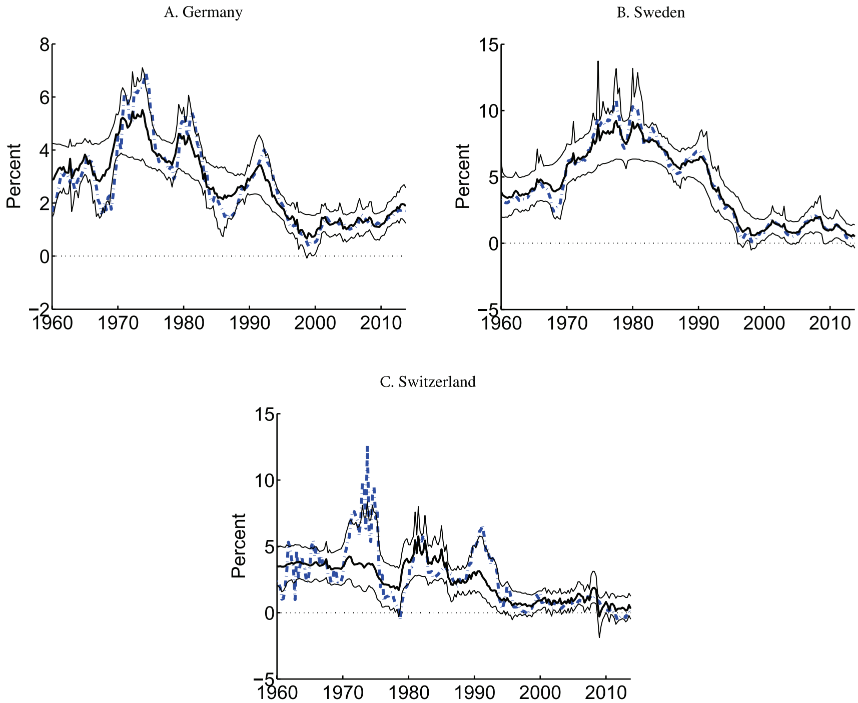
Figure 14. United States



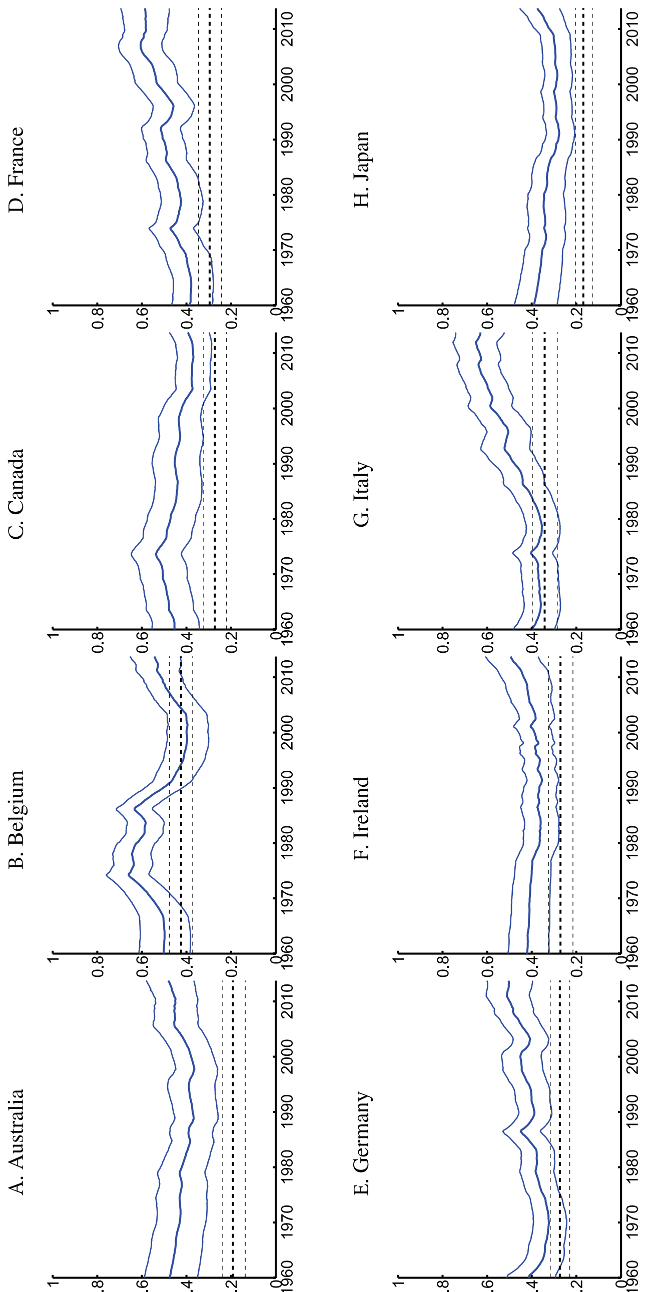
Notes: Solid, thick lines show posterior means, and thinner lines depict 90 percent confidence sets derived from the model's posterior distribution conditional on all data. All levels are measured in annualized percentage points. Variability is measured by the standard deviation of a quarterly trend shock. Data sources are as listed in table 1, using all available data since 1960. Dark gray shading marks dates in which data were excluded from computation due to shifts in the price index at that time. All country-specific price-shift dates for input measures are listed in table 2. For those periods, estimated inflation gaps, shown in panels C and D, are marked by white circles. When there are no price-shift dates, the gap estimates are identical to the difference between actual inflation and the trend estimates, shown in panels A and B.

Figure 15. Trend Estimates with Correlation between Shocks to Trend and Gaps



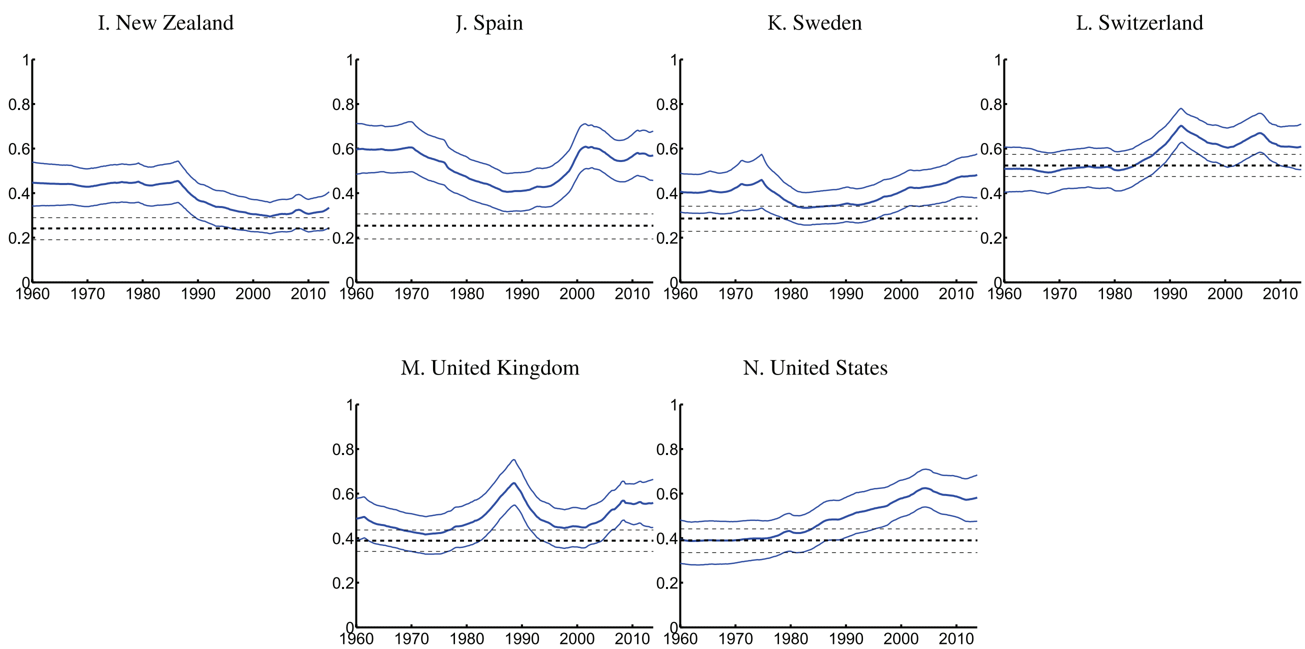
Notes: In each panel, solid lines depict estimates derived from a version of the MVSV model that allows for correlation between the shocks to trend and gap inflation. Dashed lines show the corresponding estimates from a baseline specificton where shocks to trend and gaps are assumed to be uncorrelated. Estimates are obtained from data that treat the price-shift dates listed in table 2 as missing observations. Solid, thick lines show posterior means, and thinner lines depict 90 percent confidence sets derived from the model's posterior distribution conditional on all data. All series are measured in annualized percentage points (as approximated by log-changes).

Figure 16. Gap Persistence



(continued)

Figure 16. (Continued)



Notes: Thick solid lines depict posterior means of the largest, absolute eigenvalue of the companion matrix associated with the VAR for gap inflation in a version of the MVSV model with time-varying VAR parameters; thin lines depict the interquartile range. Corresponding estimates obtained from our baseline specification with constant VAR parameters are depicted by dashed lines. Estimates are obtained from data that treat the price-shift dates listed in table 2 as missing observations.

missing values in each model's estimation.[23] The relevance of these episodes for our estimates, as brought out by a comparison with estimates conditional on all data, is the subject of this section.

In all, we consider fifteen price-shift episodes affecting seven out of the fourteen countries in our sample; all are listed in table 2. Most episodes are related to increases in taxes on goods and services and similar administrative imposts; in these instances, only a single quarterly observation is omitted from the data. The rationale for excluding inflation observations for these specified dates is that the price level shifted in the period in question not as a reflection of monetary policy or of private-sector-initiated behavior, but because of a non-monetary governmental measure whose effect was essentially to rescale the price level. Only four episodes stretched beyond a year or more: the periods of official price controls in the United States (1971–4), the United Kingdom (1972–4), and New Zealand (1982–4), as well as the transition period in the wake of German reunification (1991).[24] Again, the shift in the price level in these dates corresponded either to a movement away from market determination of prices (in the case of the price-control periods) or a major redefinition of the area covered by the price index (as when the former East Germany was brought into the Federal Republic of Germany).[25]

[23]In the case of price controls, this procedure amounts to interpolating between the final value of inflation recorded *before* the imposition of price controls and the first observation on inflation occurring *after* the period in which controls were lifted. An alternative interpolation procedure would have involved constraining inflation during the omitted quarters to be equal to the average value of inflation observed over those quarters. This alternative procedure would have captured the idea that, when price controls are lifted, the price level catches up to the value it would have reached in the absence of controls. However, following this alternative procedure would have meant treating price controls in a different way from other price shifts that we consider, such as changes in indirect taxes.

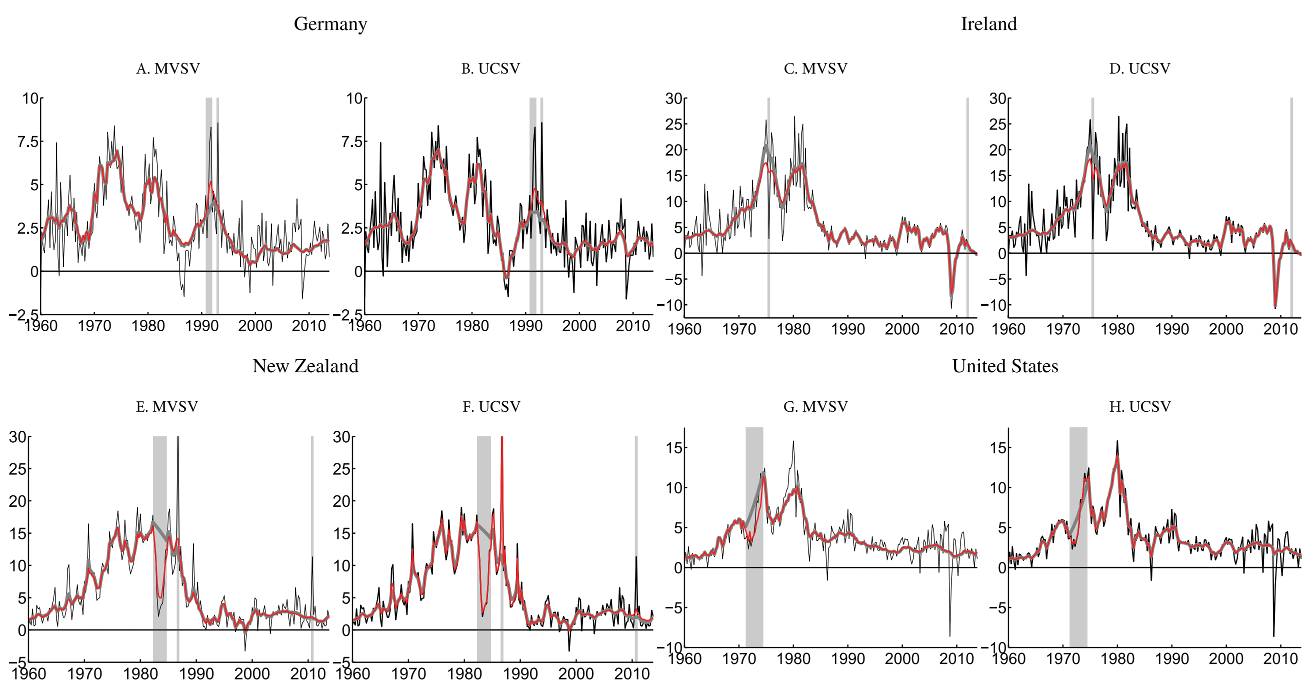
[24]We treat the whole of 1991 as a price shift for Germany, following Levin and Piger (2004).

[25]Gordon (1983) and Staiger, Stock, and Watson (1997) figure among previous studies that allow for the effects of price controls in their study of inflation dynamics, while Levin and Piger's (2004) study of international inflation dynamics allows for major changes in national sales taxes. In addition, the exclusion of control and tax periods from the estimates represents a step in the direction of incorporating historical information about individual countries' experiences into the study of inflation dynamics, as recommended by Cecchetti et al. (2007).

Reflecting their typically short duration, the price-shift dates are not associated with a great impact on trend estimates for many countries. This is not invariably the case, however. Figure 17 presents trend estimates for four countries—Germany, Ireland, New Zealand, and the United States—for which the inclusion of price-shift dates has different effects on trend estimates. The figure presents a comparison of trend estimates discussed in the previous section with estimates that condition on the entire data, including inflation data recorded during the price-shift episodes. For each country, the effects of including price-shift dates on trend estimates from the MVSV model are qualitatively similar to those on the UCSV estimates.

The price-shift episodes of longer duration evidently can have quite sizable effects on trend estimates. For example, estimates of trend inflation in the United States—whether derived from the UCSV or the MVSV model—peak well above 10 percent in the mid-1970s, when conditioned on all observations, whereas for the case in which the price-shift episode is treated as a period of missing data, the estimated inflation trend rises only gradually from about 5 to just below 10 percent. This correction may well, of course, be considered excessive, as it leads to much of the mid-1970s rise in inflation being classed as transitory. The fact that the rise in inflation in the United States in the mid-1970s was preceded by a lengthy and substantial monetary expansion points instead to the possibility that a good deal of the rise in inflation amounted, instead, to an increase in the trend rate of inflation. But even our price-shift-adjusted model estimates largely attribute the mid-1970s peak of inflation to a rise in trend. In particular, and as earlier noted, the MVSV model's characterization of the rise in CPI inflation in the mid-1970s as largely comprising an increase in trend inflation in good part reflects the fact that the increase in GDP deflator inflation over the same period basically confirmed the picture provided by CPI inflation behavior. For both of the U.S. inflation-trend estimates (that is, MVSV and UCSV), the effect of allowing for the 1971–4 price-control episode is not to lower substantially the rise in trend inflation, but instead to remove the decline in trend inflation that is registered in the controls-affected year of 1972—when measured inflation exhibited a decline that was likely spurious.

Figure 17. Trend Estimates and Price-Shift Dates



Notes: In each panel, thick gray lines depict estimates conditioned on data sets, where price-shift dates have been removed, whereas dark solid lines depict estimates conditioned on all inflation data. For each country, separate panels display results from the MVSV model and the UCSV model. Gray shading marks dates in which data were excluded from computation due to shifts in the price index at that time. All country-specific price-shift dates for input measures are listed in table 2. Thin lines denote the actual data for the headline CPI index. All levels are measured in annualized percentage points (as approximated by log-changes).

6. Trend Estimates in Quasi-Real Time

The trend estimates described in the previous two sections have been conditioned on full-sample data—with or without price shifts. Such estimates are typically labeled "smoothed" estimates, as distinct from "quasi-real-time" estimates, to use the terminology familiar from Orphanides and van Norden (2002). These real-time estimates, which we shall also refer to as "filtered" estimates, generate the inflation trend for time t solely on the basis of data observations up to and including time t. We now derive such quasi-real-time estimates by reestimating each for each quarter from 1984:Q4 through 2013:Q4, using all available data from 1960:Q1 onwards. The difference between quasi-real-time and smoothed estimates reflects the effects of reestimating the model's hyperparameters like φ_h, governing the volatility of shocks to the stochastic log-variances, or the coefficients $A(L)$ of the gap-based VAR. Our analysis abstracts from data revisions as a source of difference between real-time estimates of trend inflation and our inflation data. Rather, the data we use throughout are from what is essentially a single vintage that we collected in 2014.[26]

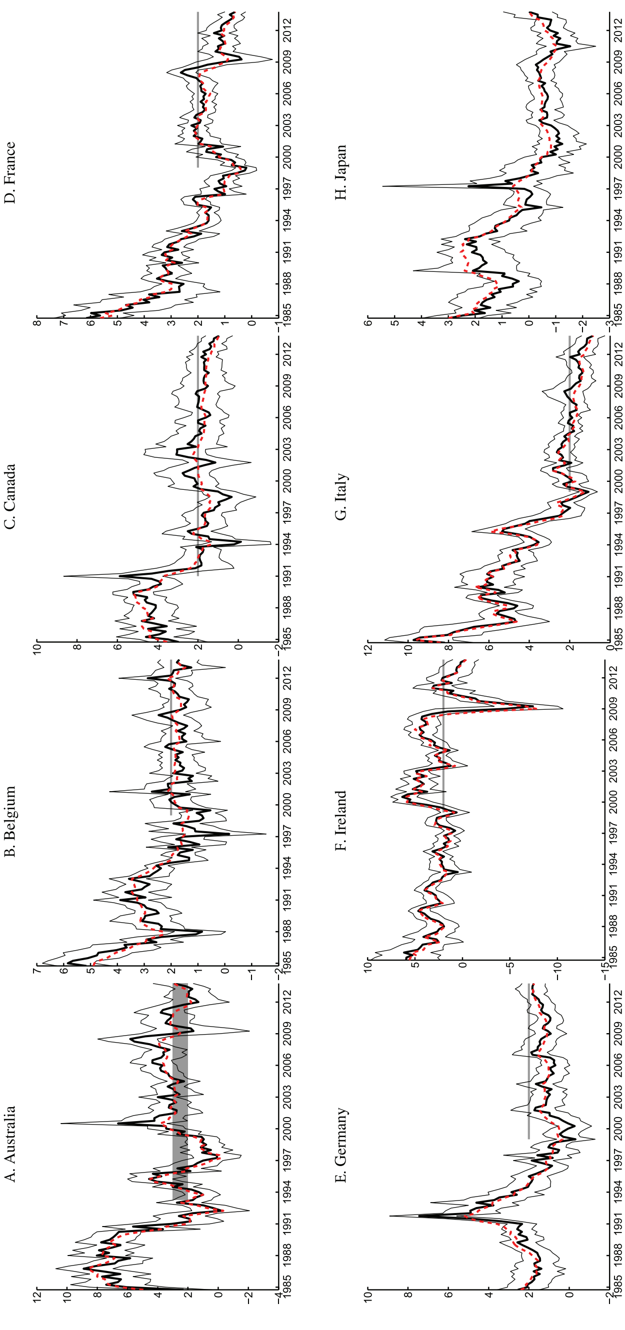
Ahead of our analysis in section 7 of each model's forecast performance based on this quasi-real-time analysis, this section provides a comparison of smoothed and filtered estimates of trend inflation from the MVSV model, as well as the difference between filtered estimates of trend inflation between the UCSV and the MVSV model.

Two results stand out from this comparison. We discuss each in turn.

First, filtered trend estimates from the MVSV model are fairly close to their smoothed counterparts, as can be seen in figure 18. Overall, as is to be expected, the smoothed estimates are a little less variable than their quasi-real-time counterparts. Smoothed estimates are designed to be more precise estimates of the underlying inflation trend, and they benefit from knowledge regarding the subsequent behavior of realized inflation. For this reason, they may not

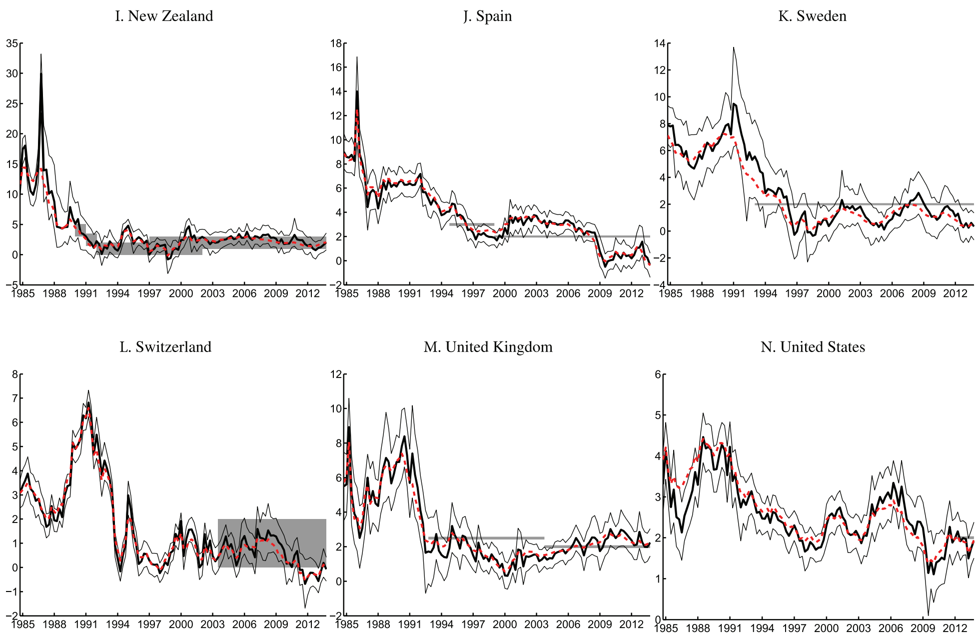
[26] An alternative notion of "filtered" estimates, not pursued here, would be one in which the model's hyperparameters were taken as given—as would be the case, for example, if values estimated based on the full sample of data were used—so that only the values of the model's latent states (like the level of trend inflation and the stochastic volatility in trend and gaps) needed to be estimated.

Figure 18. Smoothed and Filtered Trend Estimates (MVSV model)



(continued)

Figure 18. (Continued)



Notes: Each panel depicts quasi-real-time trend estimates (thick solid lines) from the MVSV model and their 90 percent confidence sets (thin solid lines). Each quasi-real-time estimate for a given quarter has been generated by a separate model estimation, using data from 1960:Q1 through the indicated quarter. The dashed line depicts the corresponding smoothed trend estimates, which are conditioned on the entire sample period through 2013:Q4. Gray-shaded areas and solid gray lines mark the range (or levels) assigned for an officially stated inflation goal.

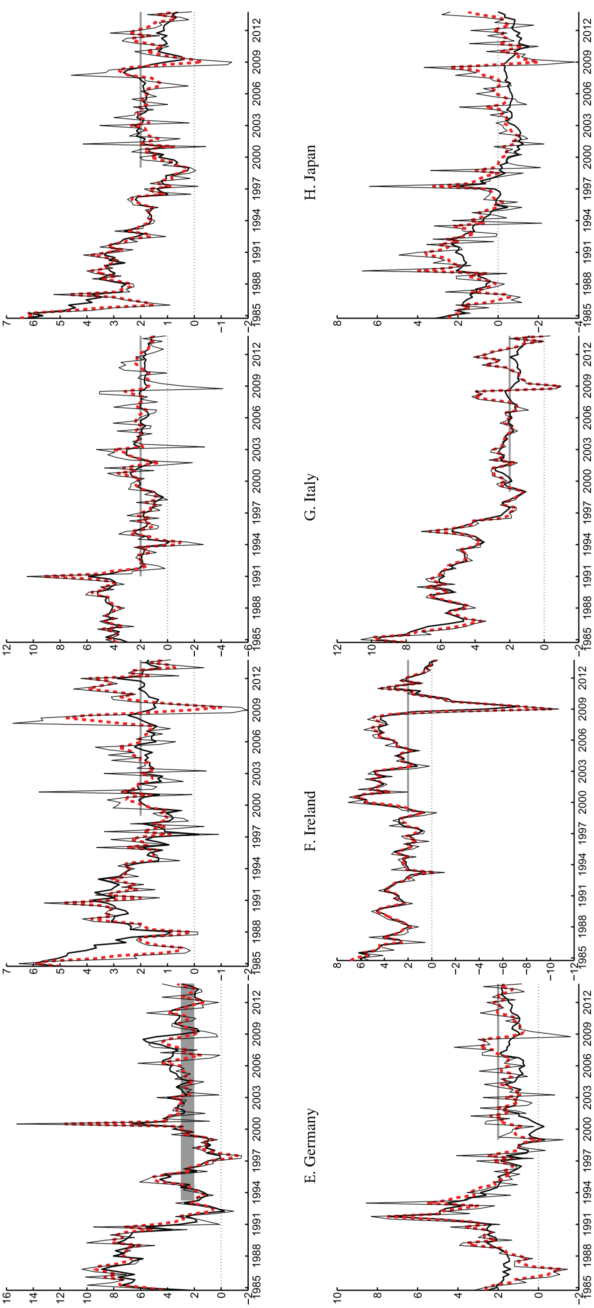
lend themselves to exercises such as determining the exact timing of events, such as the point at which an inflation target became credible. The filtered estimates might thus be more suitable for comparison against other measures of trend inflation derived from financial market indicators. But, at least in the case of the multivariate model, the differences between smoothed and quasi-real-time estimates appear to be fairly small. For example, as can be seen in figure 18, both estimates provide similar signals regarding the extent to which trend inflation is aligned with different countries' official inflation targets.

Second, filtered trend estimates from the UCSV model seem more prone to overreact to transitory changes in inflation than the MVSV model. A similar distinction has been noted previously in the context of smoothed estimates for both models. But the differences are especially striking in the case of the quasi-real-time estimates shown in figure 19. In particular, for the years 2006–12, considerable swings in commodity prices played a major role in observed fluctuations in inflation rates in many countries. And after 2009, persistent signs of disinflationary pressure are far more manifest in the MVSV model's quasi-real-time estimates of the inflation trend in a number of economies—notably the euro area, Japan, Sweden, the United Kingdom, and the United States—than in the UCSV estimates.

7. Forecast Evaluation

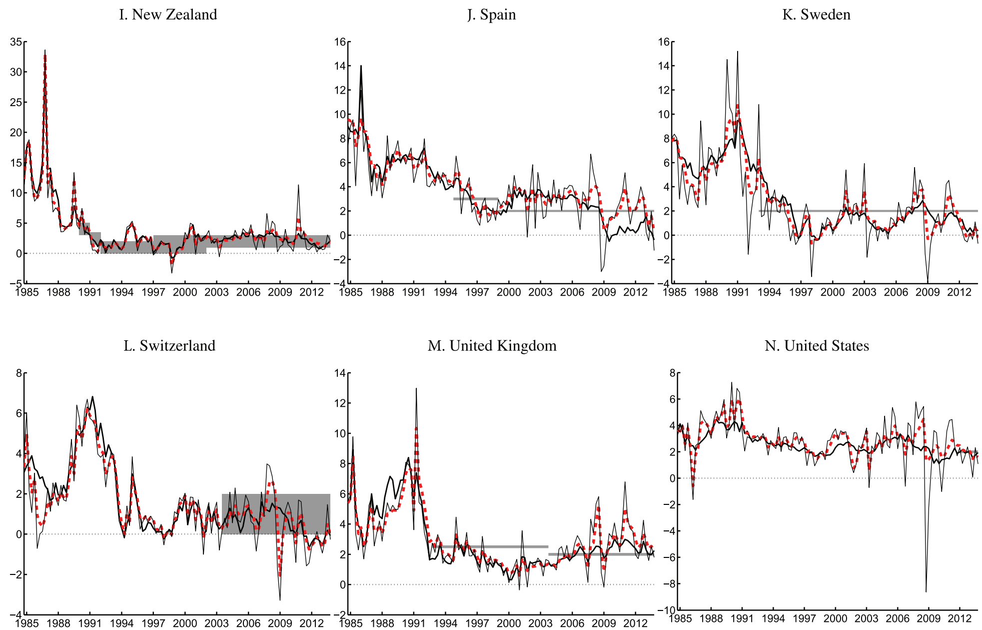
Trend inflation is a latent and unobservable variable. Some properties of the MVSV estimates documented in the previous sections might appear more appealing than their UCSV counterparts, but such a conclusion might well rely more on a subjective impression of what constitutes a "reasonable" estimate rather than a direct comparison between estimated and actual values of trend inflation—a comparison that by its very nature is infeasible. In the absence of such a direct comparison, an indirect way of assessing the validity, or usefulness, of different trend estimates is to evaluate inflation forecasts generated by each model at some finite horizons. The idea behind this approach is that, just as the Beveridge-Nelson trend is derived from solving a long-run forecasting problem, a model that allows explicitly for an evolving trend in the inflation rate should generate satisfactory inflation forecasts, and probably

Figure 19. Comparison of Filtered Trend Estimates



(continued)

Figure 19. (Continued)



Notes: Each panel shows quasi-real-time trend estimates from the MVSV model (solid) and the UCSV model (dashed). Each quasi-real-time estimate for a given quarter has been generated by a separate model estimation, using data from 1960:Q1 through that quarter. Thin solid lines depict CPI headline inflation data. Gray-shaded areas and solid gray lines mark the range (or levels) assigned for an officially stated inflation goal.

also at horizons shorter than the very long run. Evaluating the forecast performance of different trend-inflation models may be an enlightening basis on which to assess different trend estimates; it should also be relevant for researchers who are especially concerned with generating good inflation forecasts. As argued by Faust and Wright (2013), sound procedures for obtaining inflation forecasts likely include grounding those forecasts on an explicit measure of the trend-inflation rate.

This section evaluates forecasts of CPI headline inflation up to four years ahead derived from the UCSV and MVSV model for each country. In addition, we also consider forecasts motivated by the random-walk benchmark of Atkeson and Ohanian (2001). For this benchmark, inflation forecasts for all horizons are taken as equal to a four-quarter or, alternatively, a twelve-quarter moving average of lagged inflation. Inflation forecasts are generated in quasi-real time from 1985 onwards. The first forecast is therefore conditioned on model estimates obtained for data from 1960:Q1 through 1984:Q4, with an increasing estimation window as the forecast period is shifted forward (that is, as steadily more observations are used in the estimation sample). Every jumping-off date considered is associated with reestimation of each model.[27]

For each quarter considered, we generate inflation forecasts both for annual (that is, four-quarter) inflation rates (computed as the average of expected inflation rates over four consecutive quarters) and for quarterly changes at different horizons.[28] Annual inflation rates are forecast for the upcoming four quarters, one year ahead (quarters 5–8), two years ahead (quarters 9–12), three years ahead (quarters 13–16), and four years ahead (quarters 17–20). Quarterly inflation rates are forecast for the next quarter, then four, eight, twelve, and sixteen quarters ahead. Results are not particularly sensitive to the inclusion of the price-shift dates discussed in section 5—which mostly occurred prior to the 1985–2013 period spanned by our various forecast windows—and all results are

[27] As before, our analysis abstracts from discrepancies between real-time measures of inflation and our inflation data that might have arisen from data revisions.

[28] Stock and Watson (2009) also focus on forecasts of one-year or two-year percentage changes in the price level, whereas Faust and Wright (2013) study forecasts of quarterly inflation rates.

derived from data that include observed inflation for the price-shift dates.

In our application, we measure forecast accuracy using root mean squared errors ("RMSE") and average log-predictive scores, which are reported in table 3 for forecasts of annual inflation and table 4 for quarterly inflation rates. In both cases, inflation rates are expressed in annual percentage units.

Log-predictive scores measure the accuracy of a model's predictive density and are computed here for the UCSV, MVSV, and MVSV-T model.[29] As in Adolfson, Linde, and Villani (2007) and Clark and Ravazzolo (2014), we approximate the predictive density with a normal distribution and compute mean and variance of the predictive density by integrating over the draws generated by the MCMC sampler.[30] Denoting the predictive mean and variance at time t for inflation π_{t+h} by $\mu_{t+h|t}$ and $\sigma^2_{t+h|t}$, respectively, the log-predictive score at t is given by

$$l_{t+h|t} = -0.5 \left(\log(2 \cdot \pi) + \log(\sigma^2_{t+h|t}) + \frac{(\pi_{t+h} - \mu_{t+h|t})^2}{\sigma^2_{t+h|t}} \right)$$

and the average log-predictive score is computed by averaging $l_{t+h|t}$ across all forecasts t for a given forecast horizon h. Whereas the RMSE reflects only the quality of the mean of the predictive density, the normal approximation of the log-density score has the property that it also evaluates the squared errors in relation to forecast uncertainty as measured by the variance of the predictive density.

In tables 3 and 4, forecast performance of alternative models is measured by the ratio of each model's RMSE compared with the MVSV model as well as the difference between the average log-predictive scores (where applicable). A value below unity of the

[29] The moving averages generate only mean predictions without specifying a predictive density.

[30] Conditional on draws of model parameters and levels and volatilities of the inflation trend and gap, it is straightforward to compute the predictive means using standard formulas. Predictive variances can readily be computed by adapting formulas shown in Cogley and Sargent (2015). The laws of iterated expectations and total variances can then be used to compute the predictive mean and variance over all MCMC draws.

Table 3. Forecast Evaluation: Annual Inflation Rates

		Years Ahead				
		Next	**1 Year**	**2 Years**	**3 Years**	**4 Years**
Australia	RMSE for MVSV	1.74	2.28	2.56	2.77	2.71
	– Relative to MA(4)	0.89*	0.92	0.93*	0.95	0.95
	– Relative to MA(12)	0.86	0.98	1.04	1.07	0.99
	– Relative to MVSV-Trend	1.00	1.00	1.00	1.00**	1.00**
	– Relative to UCSV	0.90	0.92	0.92	0.94	0.93
	– Relative to MVSV-T	1.15**	1.22***	1.15**	1.13**	1.07*
	Predictive Score vs. UCSV	0.06	0.27***	1.02***	2.38***	4.16***
	– vs. MVSV-T	−0.08	−0.23**	−0.34***	−0.40**	−0.56***
Belgium	RMSE for MVSV	1.43	1.56	1.47	1.35	1.38
	– Relative to MA(4)	0.89	0.90	0.95	0.92	0.88
	– Relative to MA(12)	0.90	0.95	0.94	0.95	1.02
	– Relative to MVSV-Trend	0.98	1.00	1.00	1.00	1.00
	– Relative to UCSV	0.99	0.96	0.96	0.95	0.91
	– Relative to MVSV-T	1.09***	1.07**	1.01	1.03	1.07**
	Predictive Score vs. UCSV	−0.05	0.31***	0.88***	1.82***	3.10***
	– vs. MVSV-T	−0.09	−0.01	−0.02	−0.03	−0.08

(continued)

Table 3. (Continued)

		Years Ahead				
		Next	**1 Year**	**2 Years**	**3 Years**	**4 Years**
Canada	RMSE for MVSV	1.18	1.17	1.46	1.57	1.75
	– Relative to MA(4)	0.82**	0.78**	0.86	0.90	0.92
	– Relative to MA(12)	0.91	0.83	0.93	0.95	0.98
	– Relative to MVSV-Trend	1.02***	1.00**	1.00**	1.00	1.00**
	– Relative to UCSV	0.81*	0.80	0.84	0.87	0.90
	– Relative to MVSV-T	1.28**	1.19*	1.21	1.12**	1.12**
	Predictive Score vs. UCSV	0.17***	0.54***	1.32***	2.58***	4.17***
	– vs. MVSV-T	−0.05	−0.19***	−0.31**	−0.44***	−0.50***
France	RMSE for MVSV	0.91	1.03	1.08	1.05	1.15
	– Relative to MA(4)	0.83**	0.92	0.89**	0.94	0.90**
	– Relative to MA(12)	0.69	0.76	0.76	0.75	0.76
	– Relative to MVSV-Trend	0.99	1.00	1.00	1.00	1.00
	– Relative to UCSV	0.90**	0.97	0.93*	0.95	0.94**
	– Relative to MVSV-T	1.05	1.09	1.07	1.05	1.07
	Predictive Score vs. UCSV	0.16***	0.31***	0.75***	1.52***	2.61***
	– vs. MVSV-T	−0.03	0.02	0.03	−0.04	−0.11

(continued)

Table 3. (Continued)

		Years Ahead				
		Next	**1 Year**	**2 Years**	**3 Years**	**4 Years**
Germany	RMSE for MVSV	1.15	1.37	1.47	1.62	1.73
	– Relative to MA(4)	0.99	0.90	0.89	0.90	0.84
	– Relative to MA(4)	0.87	0.89	0.88	0.90	0.93
	– Relative to MVSV-Trend	0.99	1.00	1.00	1.00	1.00
	– Relative to UCSV	0.99	0.92	0.91	0.90	0.85
	– Relative to MVSV-T	1.08	1.10*	1.04	1.02	1.04
	Predictive Score vs. UCSV	0.01	0.17*	0.44***	0.90***	1.65***
	– vs. MVSV-T	−0.12	−0.19**	−0.17*	−0.20*	−0.27**
Ireland	RMSE for MVSV	2.18	2.95	3.03	2.84	2.52
	– Relative to MA(4)	0.90*	0.96	0.98	1.02	1.00
	– Relative to MA(4)	0.85**	1.06	1.12	1.12	0.92
	– Relative to MVSV-Trend	1.00	1.00	1.00**	1.00*	1.00**
	– Relative to UCSV	0.97	0.92	0.93	0.93	0.97
	– Relative to MVSV-T	1.03	1.30	1.31*	1.21*	1.08
	Predictive Score vs. UCSV	0.17	0.26***	0.68***	1.56***	2.86***
	– vs. MVSV-T	0.26	0.55	0.49	0.61	0.47

(continued)

Table 3. (Continued)

		Years Ahead				
		Next	**1 Year**	**2 Years**	**3 Years**	**4 Years**
Italy	RMSE for MVSV	1.10	1.42	1.45	1.46	1.49
	– Relative to MA(4)	0.88*	0.90**	0.90***	0.94**	0.91***
	– Relative to MA(12)	0.66*	0.73	0.77	0.78	0.74*
	– Relative to MVSV-Trend	0.99	1.00	1.00	1.00	1.00
	– Relative to UCSV	0.93	0.88*	0.87**	0.89***	0.89***
	– Relative to MVSV-T	1.04	1.03	1.01	1.01	1.02
	Predictive Score vs. UCSV	0.09	0.39***	0.99***	1.91***	3.11***
	– vs. MVSV-T	−0.07	0.01	0.00	−0.11	−0.19
Japan	RMSE for MVSV	1.10	1.29	1.30	1.25	1.16
	– Relative to MA(4)	0.93	0.89	0.80***	0.76**	0.73**
	– Relative to MA(12)	0.89	0.91	0.91	0.91	0.90
	– Relative to MVSV-Trend	0.99	1.00	1.00*	1.00*	1.00*
	– Relative to UCSV	0.92	0.91	0.83**	0.79**	0.74**
	Predictive Score vs. UCSV	0.08	0.65***	1.81***	3.48***	5.37***

(continued)

Table 3. (Continued)

		Years Ahead				
		Next	**1 Year**	**2 Years**	**3 Years**	**4 Years**
New Zealand	RMSE for MVSV	3.08	3.51	3.85	4.49	4.87
	– Relative to MA(4)	1.08	1.08	1.10	1.11	1.11
	– Relative to MA(12)	1.06	1.05	1.14	1.14	1.14
	– Relative to MVSV-Trend	1.01	1.00	1.00	1.00	1.00
	– Relative to UCSV	0.92	0.94*	0.96	0.97	0.99
	– Relative to MVSV-T	1.01	1.02	1.03	1.02	1.01
	Predictive Score vs. UCSV	0.14***	0.77***	1.98***	3.63***	5.53***
	– vs. MVSV-T	0.01	−0.28***	−0.46***	−0.63***	−0.78***
Sweden	RMSE for MVSV	2.03	2.37	2.71	2.97	3.13
	– Relative to MA(4)	0.90**	0.87	0.93	0.94	0.96
	– Relative to MA(12)	0.88***	0.90**	0.98	1.02	1.02
	– Relative to MVSV-Trend	0.99**	1.00	1.00	1.00	1.00
	– Relative to UCSV	0.96	0.89	0.96	0.96	0.98
	– Relative to MVSV-T	1.05	1.06	1.10*	1.09	1.08
	Predictive Score vs. UCSV	0.15***	0.27***	0.78***	1.88***	3.37***
	– vs. MVSV-T	−0.06	−0.13	−0.18*	−0.27**	−0.37***

(continued)

Table 3. (Continued)

		Years Ahead				
		Next	**1 Year**	**2 Years**	**3 Years**	**4 Years**
Spain	RMSE for MVSV	1.55	1.87	1.89	1.84	1.97
	– Relative to MA(4)	0.99	1.03	1.01	0.99	1.06
	– Relative to MA(12)	0.96	0.98	0.95	0.94	0.93
	– Relative to MVSV-Trend	0.99*	1.00	1.00	1.00	1.00
	– Relative to UCSV	1.06	1.08	1.03	1.01	1.05
	– Relative to MVSV-T	1.08	1.07	1.08	1.06	1.02
	Predictive Score vs. UCSV	0.01	0.26***	1.01***	2.26***	3.60***
	– vs. MVSV-T	0.00	−0.01	−0.13	−0.14**	−0.23***
Switzerland	RMSE for MVSV	1.20	1.59	1.94	2.06	2.14
	– Relative to MA(4)	0.90	0.99	0.99	0.93	0.96
	– Relative to MA(12)	0.84	0.91	1.00	1.01	1.05
	– Relative to MVSV-Trend	0.97	1.00	1.00	1.00	1.00**
	– Relative to UCSV	0.95	1.00	1.01	0.94	0.95
	– Relative to MVSV-T	0.92**	0.90*	0.94	0.94	0.95
	Predictive Score vs. UCSV	0.12*	0.33***	0.98***	2.31***	3.99***
	– vs. MVSV-T	0.15**	0.17*	0.21	0.21	0.26

(continued)

Table 3. (Continued)

		Years Ahead				
		Next	**1 Year**	**2 Years**	**3 Years**	**4 Years**
United Kingdom	RMSE for MVSV	1.21	1.45	1.86	2.15	2.27
	– Relative to MA(4)	0.90	0.85	0.97	1.03	1.07
	– Relative to MA(12)	0.85	0.86	1.02	1.13	1.12
	– Relative to MVSV-Trend	1.02	1.00	1.00	1.00	1.00
	– Relative to UCSV	0.85	0.83*	0.92	0.98	1.03
	– Relative to MVSV-T	0.93*	0.94	0.97	1.00	1.01
	Predictive Score vs. UCSV	0.04	0.53***	1.35***	2.69***	4.27***
	– vs. MVSV-T	0.11	−0.05	−0.16	−0.22*	−0.29**
United States	RMSE for MVSV	1.10	1.22	1.29	1.34	1.30
	– Relative to MA(4)	0.71**	0.75**	0.80***	0.79***	0.83*
	– Relative to MA(12)	0.84**	0.89	0.96	0.98	0.96
	– Relative to MVSV-Trend	1.02	1.00	1.00	1.00	1.00
	– Relative to UCSV	0.77**	0.83**	0.82***	0.81***	0.79**
	– Relative to MVSV-T	1.00	1.00	1.00	1.00	1.00
	Predictive Score vs. UCSV	0.32***	0.63***	1.27***	2.29***	3.57***
	– vs. MVSV-T	−0.00	0.00	−0.00	0.00	−0.00

(continued)

Table 3. (Continued)

		Years Ahead				
		Next	**1 Year**	**2 Years**	**3 Years**	**4 Years**
United States (CPI-U-RS)	RMSE for MVSV	1.14	1.27	1.32	1.38	1.38
	– Relative to MA(4)	0.69**	0.76**	0.80***	0.80**	0.87
	– Relative to MA(12)	0.82**	0.91	0.98	1.02	1.03
	– Relative to MVSV-Trend	0.99***	1.00	1.00	1.00	1.00
	– Relative to UCSV	0.84**	0.93*	0.93*	0.94	0.96
	– Relative to MVSV-T	1.00	1.00	1.00	1.00*	1.00
	Predictive Score vs. UCSV	0.24***	0.31***	0.70***	1.70***	3.08***
	– vs. MVSV-T	−0.00	0.00	−0.00	−0.00**	−0.00

Notes: For each country, root mean squared errors (RMSE) and average log-predictive scores are derived from out-of-sample forecasts that were generated from quasi-real-time estimates of each model from 1985:Q1 onwards; each model estimation is conditioned on data from 1960:Q1 until the beginning of each forecast period. Superscripts *, **, and *** denote statistically significant differences in squared forecast errors and log-predictive scores—as computed from the test by Diebold and Mariano (1995)—at the 10 percent, 5 percent, and 1 percent level, respectively.

Table 4. Forecast Evaluation: Quarterly Inflation Rates

		Years Ahead				
		Next	**1 Year**	**2 Years**	**3 Years**	**4 Years**
Australia	RMSE for MVSV	2.32	2.54	3.01	3.24	3.29
	– Relative to MA(4)	0.96	0.92*	0.96	0.98	0.97
	– Relative to MA(12)	0.92	0.92	1.02	1.07	1.04
	– Relative to MVSV-Trend	1.01	1.00	1.00	1.00	1.00
	– Relative to UCSV	0.96	0.92	0.95	0.96	0.96
	– Relative to MVSV-T	1.04	1.09**	1.13***	1.10**	1.07**
	Predictive Score vs. UCSV	0.06	0.03	0.06	0.12**	0.39***
	– vs. MVSV-T	−0.03	−0.06	−0.16***	−0.20***	−0.22**
Belgium	RMSE for MVSV	1.65	1.96	1.92	1.86	1.78
	– Relative to MA(4)	0.95	0.92	0.94	0.99	0.92
	– Relative to MA(12)	0.89*	0.95	0.99	0.96	0.99
	– Relative to MVSV-Trend	0.92*	1.00	1.00	1.00	1.00
	– Relative to UCSV	1.05	0.95	0.97	0.97	0.95
	– Relative to MVSV-T	1.04***	1.06***	1.05**	1.00	1.02
	Predictive Score vs. UCSV	−0.08	0.06	0.16**	0.22**	0.30**
	– vs. MVSV-T	−0.08**	−0.06*	0.00	−0.01	−0.05

(continued)

Table 4. (Continued)

		Years Ahead				
		Next	**1 Year**	**2 Years**	**3 Years**	**4 Years**
Canada	RMSE for MVSV	1.71	1.94	1.97	2.15	2.21
	– Relative to MA(4)	0.85***	0.96	0.91*	0.93	0.94
	– Relative to MA(12)	0.91**	0.98	0.95	0.98	0.96
	– Relative to MVSV-Trend	1.00	1.00***	1.00*	1.00***	1.00
	– Relative to UCSV	0.90**	0.91	0.90*	0.89	0.94
	– Relative to MVSV-T	1.04	1.11**	1.08**	1.10	1.05*
	Predictive Score vs. UCSV	0.10*	0.15***	0.24***	0.37***	0.66***
	– vs. MVSV-T	0.03	0.02	−0.07	−0.10	−0.16*
France	RMSE for MVSV	1.07	1.32	1.26	1.31	1.32
	– Relative to MA(4)	0.89**	0.92*	0.92*	0.93**	0.94
	– Relative to MA(12)	0.75*	0.81	0.82	0.81	0.81
	– Relative to MVSV-Trend	0.97	1.00	1.00	1.00	1.00
	– Relative to UCSV	0.99	0.91**	0.95	0.93**	0.97
	– Relative to MVSV-T	1.00	1.05	1.05	1.03	1.03
	Predictive Score vs. UCSV	0.06**	0.15***	0.19***	0.29***	0.49***
	– vs. MVSV-T	0.04	−0.06	−0.03	0.00	−0.06

(continued)

Table 4. (Continued)

		Years Ahead				
		Next	**1 Year**	**2 Years**	**3 Years**	**4 Years**
Germany	RMSE for MVSV	1.46	1.66	1.83	1.89	2.00
	– Relative to MA(4)	1.01	0.98	0.96	0.94	0.92
	– Relative to MA(12)	0.92	0.93	0.96	0.93	0.94
	– Relative to MVSV-Trend	0.99	1.00	1.00	1.00	1.00
	– Relative to UCSV	1.01	0.98	0.94	0.93	0.92
	– Relative to MVSV-T	1.02	1.06	1.05*	1.02	1.02
	Predictive Score vs. UCSV	−0.01	0.03	0.07	0.12***	0.15***
	– vs. MVSV-T	−0.03	−0.09**	−0.10**	−0.09*	−0.12*
Ireland	RMSE for MVSV	1.97	3.03	3.33	3.34	3.00
	– Relative to MA(4)	0.87**	0.96	0.98	1.00	1.00
	– Relative to MA(12)	0.73**	0.98	1.09	1.13	1.04
	– Relative to MVSV-Trend	1.00	1.00	1.00	1.00	1.00
	– Relative to UCSV	1.05	0.95	0.92	0.94	0.94
	– Relative to MVSV-T	0.88*	1.13	1.27	1.22*	1.10
	Predictive Score vs. UCSV	0.00	0.24	0.12	0.04	0.04
	– vs. MVSV-T	0.07	−0.14	−0.09	0.04	0.13

(continued)

Table 4. (Continued)

		Years Ahead				
		Next	**1 Year**	**2 Years**	**3 Years**	**4 Years**
Italy	RMSE for MVSV	1.06	1.49	1.60	1.64	1.58
	– Relative to MA(4)	0.89*	0.91*	0.90***	0.93***	0.95**
	– Relative to MA(12)	0.67***	0.75	0.78	0.81	0.80
	– Relative to MVSV-Trend	0.96**	1.00	1.00	1.00	1.00
	– Relative to UCSV	1.04	0.90	0.88*	0.89**	0.92***
	– Relative to MVSV-T	1.01	1.03	1.01	1.00	1.01
	Predictive Score vs. UCSV	0.03	0.12*	0.24***	0.39***	0.62***
	– vs. MVSV-T	−0.01	−0.07**	−0.02	0.00	−0.08
Japan	RMSE for MVSV	1.63	1.75	1.88	1.85	1.82
	– Relative to MA(4)	0.97	0.96	0.94	0.90*	0.83***
	– Relative to MA(12)	0.96	0.96	0.97	0.96	0.97
	– Relative to MVSV-Trend	0.99	1.00	1.00	1.00	1.00
	– Relative to UCSV	0.99	0.94	0.95	0.90**	0.86***
	Predictive Score vs. UCSV	−0.02	0.12*	0.30***	0.92***	2.03***

(continued)

Table 4. (Continued)

		Years Ahead				
		Next	1 Year	2 Years	3 Years	4 Years
New Zealand	RMSE for MVSV	3.53	4.29	4.60	4.36	4.82
	– Relative to MA(4)	0.96	1.13	1.08	1.06	1.09
	– Relative to MA(12)	0.96	1.08	1.09	1.12	1.11
	– Relative to MVSV-Trend	1.03	1.00*	1.00	1.00	1.00
	– Relative to UCSV	0.94	0.94*	0.97	0.96	0.98
	– Relative to MVSV-T	1.00	1.02**	1.02	1.02	1.02
	Predictive Score vs. UCSV	0.11***	0.11***	0.21***	0.42***	0.95***
	– vs. MVSV-T	0.03	−0.09	−0.24***	−0.37***	−0.43***
Sweden	RMSE for MVSV	2.46	2.87	3.07	3.36	3.50
	– Relative to MA(4)	0.97	0.93	0.93	0.95	0.96
	– Relative to MA(12)	0.90***	0.94*	0.95	1.01	1.02
	– Relative to MVSV-Trend	0.98	1.00	1.00	1.00	1.00
	– Relative to UCSV	1.00	0.95*	0.93	0.97	0.97
	– Relative to MVSV-T	1.03	1.03	1.05	1.07	1.06
	Predictive Score vs. UCSV	0.07	0.12***	0.11***	0.13***	0.18***
	– vs. MVSV-T	−0.00	−0.06	−0.10**	−0.15***	−0.19***

(continued)

Table 4. (Continued)

		Years Ahead				
		Next	**1 Year**	**2 Years**	**3 Years**	**4 Years**
Spain	RMSE for MVSV	1.87	2.17	2.33	2.22	2.21
	– Relative to MA(4)	0.98	1.00	1.02	0.97	1.02
	– Relative to MA(12)	0.94	1.00	0.98	0.96	0.94
	– Relative to MVSV–Trend	0.97***	1.00	1.00	1.00	1.00
	– Relative to UCSV	1.03	1.03	1.03	1.00	1.01
	– Relative to MVSV–T	1.03	1.04	1.04	1.07	1.02
	Predictive Score vs. UCSV	0.01	0.04	0.08*	0.31***	0.67***
	– vs. MVSV–T	0.00	−0.02	−0.04	−0.09*	−0.10***
Switzerland	RMSE for MVSV	1.26	1.71	1.90	2.22	2.29
	– Relative to MA(4)	0.87**	0.98	0.98	0.98	0.94
	– Relative to MA(12)	0.81**	0.94	0.93	1.02	1.02
	– Relative to MVSV–Trend	0.93*	1.00	1.00	1.00	1.00*
	– Relative to UCSV	0.95	0.96	0.98	0.99	0.95
	– Relative to MVSV–T	0.98	0.93*	0.92*	0.95	0.95
	Predictive Score vs. UCSV	0.07***	0.10**	0.18**	0.57***	1.52***
	– vs. MVSV–T	0.04	0.08*	0.12	0.18	0.13

(continued)

Table 4. (Continued)

		Years Ahead				
		Next	**1 Year**	**2 Years**	**3 Years**	**4 Years**
United Kingdom	RMSE for MVSV	1.65	1.75	2.06	2.24	2.53
	– Relative to MA(4)	0.97	0.95	0.93	0.98	1.06
	– Relative to MA(12)	0.92	0.91	0.99	1.03	1.13
	– Relative to MVSV-Trend	1.02	1.00	1.00	1.00	1.00
	– Relative to UCSV	0.96	0.89*	0.90**	0.96	0.99
	– Relative to MVSV-T	0.99	0.94**	0.99	0.98	1.00
	Predictive Score vs. UCSV	0.04	0.09*	0.21***	0.37***	0.72***
	– vs. MVSV-T	0.02	0.04	−0.12	−0.15*	−0.21**
United States	RMSE for MVSV	1.77	1.89	1.88	1.98	2.00
	– Relative to MA(4)	0.85**	0.88*	0.85*	0.91***	0.87**
	– Relative to MA(12)	0.92**	0.95	0.94	1.00	0.99
	– Relative to MVSV-Trend	0.98	1.00	1.00	1.00	1.00
	– Relative to UCSV	0.93*	0.87**	0.90**	0.91**	0.88***
	– Relative to MVSV-T	1.00	1.00	1.00	1.00	1.00
	Predictive Score vs. UCSV	0.13**	0.21***	0.23	0.29	0.37*
	– vs. MVSV-T	0.00	−0.00	−0.00	−0.00	−0.00

(continued)

Table 4. (Continued)

		Years Ahead				
		Next	**1 Year**	**2 Years**	**3 Years**	**4 Years**
United States (CPI-U-RS)	RMSE for MVSV	2.63	2.69	2.72	2.78	2.85
	– Relative to MA(4)	0.87***	0.91**	0.92	0.92***	0.92**
	– Relative to MA(12)	0.93***	0.94***	0.97**	0.98	0.99
	– Relative to MVSV-Trend	0.97	0.98***	0.99***	1.00***	1.00**
	– Relative to UCSV	0.91***	0.97**	0.99	0.99	1.00
	– Relative to MVSV-T	1.00	1.00	1.00	1.00	1.00*
	Predictive Score vs. UCSV	0.15***	0.02	0.04	0.02	0.07
	– vs. MVSV-T	0.00	−0.00	0.00	0.00	0.00

Notes: For each country, root mean squared errors (RMSE) and average log-predictive scores are derived from out-of-sample forecasts that were generated from quasi-real-time estimates of each model from 1985:Q1 onwards; each model estimation is conditioned on data from 1960:Q1 until the beginning of each forecast period. Superscripts *, **, and *** denote statistically significant differences in squared forecast errors and log-predictive scores—as computed from the test by Diebold and Mariano (1995)—at the 10 percent, 5 percent, and 1 percent level, respectively.

RMSE ratio indicates that the MVSV model has a lower RMSE, and conversely for values above unity. A positive value for the difference in the average log-predictive scores indicates a more accurate predictive density of the MVSV value, with the converse holding in the case of a negative value for the difference. The statistical significance of the differences in RMSE and log-predictive scores is assessed with the Diebold-Mariano (1995) test.[31]

Several results recur across both tables. First, with only a couple of exceptions, the MVSV model generates lower RMSE for almost each country and at almost each horizon than a simple random-walk forecast. Second, in most countries, the same is also true, but often to a lesser extent, when the MVSV forecasts are compared with those of the UCSV model. Third, although most of these differences are notable—in the neighborhood of several tenths of the MVSV model's RMSEs—they are often not statistically significant. Primarily in the cases of France, Italy, and the United States, the MVSV model produces forecasts that are significantly better than projections derived from either a random-walk or UCSV model. Strikingly, the MVSV model rarely fares significantly worse than any of the moving averages or the UCSV model. The MVSV model also generates considerably higher log-predictive scores than the UCSV, suggesting a more accurate predictive density, especially over longer forecast horizons. To quite some extent, this reflects the differences in specification of the stochastic volatilities for the inflation gaps. As described in section 3, the UCSV model embeds the assumption that the log of the inflation-gap shock variance follows a random walk whereas the MVSV model uses an AR(1) specification. At longer forecast horizons, the random-walk assumption for the inflation-gap variance seems to lead to undue extrapolation of temporary changes in volatility—a property that has an adverse bearing on the accuracy of the UCSV model's predictive density.

Comparison of the absolute levels of the RMSEs for the MVSV model across both tables shows that RMSE values are somewhat

[31]The Diebold-Mariano (1995) test is designed to ascertain whether the squared losses generated by two different forecasts are, on average, equal. In light of the overlap in the forecast periods, we computed the standard errors using the Newey-West (1997) robust estimator, with a bandwidth set equal to one plus the forecast horizon. The Diebold-Mariano test can also be used to assess differences in log-predictive scores, as recently shown by Clark and Ravazzolo (2014).

larger for cases in which it is the quarterly rather than annual rate of inflation that is being forecast. This pattern is indicative of the considerable amount of highly transitory—and hence harder to forecast—fluctuations found in quarter-to-quarter variations in prices. These variations figure less heavily in the behavior of four-quarter inflation, a series in which the most violent swings in quarterly inflation are averaged out by construction. Correspondingly, the differences in RMSEs across the different models, which are clearly evident in the quarterly inflation results in table 4, are smaller in size and tend to be less statistically significant when forecasts of annual inflation are considered, as in table 3; a similar pattern holds also for differences in log-predictive scores between MVSV and UCSV model.

We also consider the forecasting performance of the MVSV trend alone, neglecting the horizon-specific information resulting from the VAR component of the model's gap equation (for a given trend-inflation estimate). In this case, forecasts for all horizons are set equal to the models' trend estimate, generated in quasi-real time (and shown in figure 18). For projections of both quarterly and annual inflation—reported in tables 3 and 4—there is typically not a great difference between the (average) forecast errors arising from the MVSV model (from which inflation forecasts are derived from summing the inflation-trend forecast and the inflation-gap forecast) and the errors of inflation projections derived from relying solely on the MVSV-generated inflation trend. This finding is consistent with the notion, espoused by Faust and Wright (2013), that improved forecast accuracy stems from the quality of the estimates of the inflation trend. Applied to the MVSV approach, this notion implies that the model's VAR equation for the inflation gap adds little value beyond its role in shaping the trend estimate itself.

For the United States, the MVSV model tends to outperform either a random-walk or the UCSV model, both in terms of RMSE and predictive density score, and significantly so in most cases. Results are fairly similar when using either the regular CPI or the CPI-U-RS measure, as table 3 shows.

As a final comparison, we consider forecasts from the MVSV-T model that sets the inflation trend equal to each country's inflation goal when applicable. In contrast to the baseline MVSV model, the MVSV-T model does not centers its forecasts on an empirical trend estimate. Instead, it takes the trend as corresponding to each country's official inflation goal (when applicable). Furthermore, once an inflation goal has been introduced for a given country, the MVSV-T model treats the inflation trend as deterministic, thus removing uncertainty about future trend shocks from the predictive density.[32] The forecast performance of the MVSV-T model compared with the MVSV model differs for different countries. In several instances, like those of Australia, Canada, and New Zealand, conditioning on a known inflation goal clearly improves forecasts both in terms of RMSE and predictive density, especially for longer forecast horizons. In other cases, like Switzerland and Ireland, the opposite is true, although the differences are not statistically significant. For several countries—including France, Belgium, and the United States—forecasts derived from the MVSV-T model do not differ greatly from those generated by the baseline version of the MVSV model.[33] If anything, the predictive density of longer-horizon forecasts tends to be improved when generated from the MVSV-T model.

8. Conclusion

Our paper has compared estimates of trend inflation in fourteen advanced economies using two different models. Our preferred model is a multivariate extension of Stock and Watson's (2007) unobserved-components model with stochastic volatility (UCSV) that has been applied to the G7 countries by Cecchetti et al. (2007). Like the UCSV model, our multivariate stochastic volatility model (MVSV) tracks time variation in the variability of shocks to trend inflation and the inflation gap. Inflation-gap estimates from our MVSV model allow for inflation-gap persistence—albeit modeled in a more parsimonious fashion than in Cogley, Primiceri, and Sargent (2010)—while the UCSV model embeds the assumption that gaps are serially uncorrelated. We find that, particularly since the 1980s, the MVSV-based

[32] At a given point in time, future adjustments in the inflation goal are, however, not anticipated by the MVSV-T model.

[33] In the case of the United States, forecasts from the MVSV and the MVSV-T model barely differ, on average, from each other in part also because of the limited number of observations for which the Federal Reserve's longer-term inflation objective, officially introduced in 2012, applies in our sample.

inflation trends are smoother and less variable than their UCSV counterparts, as the underlying filtering procedure implies less influence on the trend estimates of persistent variations in inflation that do not prove to be fully permanent.

A key additional property of the MVSV model is that it conditions on multiple inflation series, on the assumption that they share a common trend, as in the model of Mertens (2011). In contrast to Cogley and Sargent (2005), Kang, Kim, and Morley (2009), and Cogley, Primiceri, and Sargent (2010), our model restricts time variation in inflation-gap parameters only to the evolution of stochastic volatility. This variation is in turn limited to only two sources: drift in the log-variances of shocks to the common trend and separate, but cross-correlated, volatility processes for each inflation gap. Placing a limit in this way on the number of time-varying parameters makes the model more tractable, and it also enables us to handle missing data in some of the inflation series for several countries, while still allowing for the possibility of considerable persistence in the inflation-gap series. This restricted approach also holds out the prospect of greater forecast accuracy. Compared with alternative forecasts—generated either from a simple random-walk model or the UCSV model—our MVSV model typically is associated with a lower average size of forecast errors at various horizons and for most countries. In particular, for the exercise of forecasting four-quarter inflation rates (as distinct from quarter-to-quarter rates), the improvements are quite appreciable. However, with the exception of a few countries, it remains hard to generate inflation forecasts that outperform random-walk forecasts of inflation by a statistically significant amount.

Although our estimates of trend inflation display quite some similarities across countries—notably the shared experiences of persistently elevated values during the 1970s and more reliably anchored inflation expectations over the last two decades—there are also clear cross-country differences in the trend estimates. For example, the extent to which trend inflation underwent a rise, and subsequent fall, over the post-war sample differs notably across countries. In addition, for many countries, distinct, country-specific changes in monetary regime, like the adoption of a formal inflation target, are clearly visible in the evolution of our estimates of trend inflation.

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