Online Appendixes to Should the ECB Coordinate EMU Fiscal Policies?*

Tatiana Kirsanova,[a] Celsa Machado,[b] and Ana Paula Ribeiro[c]
[a]University of Glasgow
[b]ISCAP
[c]University of Porto

Appendix A. The Model

The model is based on Benigno and Benigno (2003) but with incomplete financial markets as in Benigno (2009). The relatively detailed modeling of the fiscal side follows Woodford (2001) and Leeper and Leith (2016), allowing for variable maturity of government debt.

Specifically, the world economy is populated by a continuum of agents on the interval of $[0;1]$. The population on the segment $[0;n)$ belongs to country H (home), while the rest of the population on $[n;1]$ belongs to country F (foreign). Each economy is populated by households and firms. Households' preferences reflect home bias in consumption. Firms are monopolistically competitive and only use labor to produce differentiated tradable goods. The law of one price holds. Each country has an independent fiscal authority, which finances spending by bonds and distortionary taxes. The government debt is tradable and has geometric maturity structure. We look at implications of complete and incomplete financial markets[1]: in the latter case the portfolio allocation is determined by transaction

*Author contact: Kirsanova: Economics, Adam Smith Business School, Guilbert Scott Building, University of Glasgow, Glasgow G12 8QQ; e-mail: tatiana.kirsanova@glasgow.ac.uk. Machado: ISCAP, Rua Jaime Lopes de Amorim, 4465-111 S. Mamede de Infesta, Portugal; e-mail: celsa@iscap.ipp.pt. Ribeiro: Faculdade de Economia da Universidade do Porto, Gabinete n^o 608, Rua Dr. Roberto Frias S/N 4200 464 Porto, Portugal; e-mail: aribeiro@fep.up.pt.

[1]Baele et al. (2004) argues that in the beginning of the 2000s the public debt market was fairly integrated. However, since the Greece debt restructuring there is a perceived non-zero probability of a sovereign debt default of an individual country.

costs. All profits received by home-country firms and financial intermediaries are rebated to home households. Countries are subject to technology and cost-push shocks.

A.1 Preferences

We assume home bias in consumption. Home households' consumption, C_t, is an aggregate of the continuum of goods $i \in [0, 1]$ produced in the home country and abroad,

$$C_t = \left((1-\gamma)^{\frac{1}{\eta}} C_{Ht}^{\frac{\eta-1}{\eta}} + \gamma^{\frac{1}{\eta}} C_{Ft}^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}},$$

where $\eta > 0$ is the intratemporal elasticity of substitution between home and foreign consumption goods, and $\gamma = (1-n)\alpha$ is the import share. The import share depends on $(1-n)$, which is the relative size of foreign economy, and on α, which is the degree of trade openness. Similarly, for the foreign country C_t^* is an aggregate of the continuum of goods $i \in [0, 1]$ produced in the foreign country and abroad and consumed by foreign households:

$$C_t^* = \left((1-\gamma^*)^{\frac{1}{\eta}} C_{Ft}^{*\frac{\eta-1}{\eta}} + \gamma^{*\frac{1}{\eta}} C_{Ht}^{*\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}},$$

where $\gamma^* = n\alpha$.

Each consumption bundle C_{Ht} and C_{Ft} is composed of imperfectly substitutable varieties with elasticity of substitution $\epsilon > 1$: The optimal allocation within each variety of goods yields household-level relationships

$$c_{Ht}(z) = \frac{1}{n} \left(\frac{p_{Ht}(z)}{P_{Ht}} \right)^{-\epsilon} C_{Ht} \text{ and } c_{Ft}(z) = \frac{1}{1-n} \left(\frac{p_{Ft}(z)}{P_{Ft}} \right)^{-\epsilon} C_{Ft},$$

where z denotes the good's type or variety and $p_{Ht}(z), p_{Ft}(z)$ are prices of individual goods. Throughout the paper we use lowercase letters to denote individual (household) labor, consumption, output, and government spending, while uppercase letters are used to denote country aggregates.

The aggregation at the country level yields

$$C_{Ht} = \left(\left(\frac{1}{n}\right)^{\frac{1}{\epsilon}} \int_0^n c_{Ht}(z)^{\frac{\epsilon-1}{\epsilon}} dz\right)^{\frac{\epsilon}{\epsilon-1}} \quad \text{and}$$

$$C_{Ft} = \left(\left(\frac{1}{1-n}\right)^{\frac{1}{\epsilon}} \int_n^1 c_{Ft}(z)^{\frac{\epsilon-1}{\epsilon}} dz\right)^{\frac{\epsilon}{\epsilon-1}}.$$

Optimal allocation of expenditures between domestic and foreign bundles yields

$$C_{Ht} = (1-\gamma)\left(\frac{P_{Ht}}{P_t}\right)^{-\eta} C_t \text{ and } C_{Ft} = \gamma\left(\frac{P_{Ft}}{P_t}\right)^{-\eta} C_t.$$

P_{Ht}, P_{Ft} are domestic and foreign goods price indexes and

$$P_t = \left((1-\gamma)\, P_{Ht}^{1-\eta} + \gamma P_{Ft}^{1-\eta}\right)^{\frac{1}{1-\eta}},$$

where price indexes are computed as

$$P_{Ht} = \left(\frac{1}{n}\int_0^n p_{Ht}(z)^{1-\epsilon} dz\right)^{\frac{1}{1-\epsilon}} \quad \text{and}$$

$$P_{Ft} = \left(\frac{1}{1-n}\int_n^1 p_{Ft}(z)^{1-\epsilon} dz\right)^{\frac{1}{1-\epsilon}}.$$

Similarly, for the foreign country

$$C^*_{Ht}(z) = \frac{1}{n}\left(\frac{P^*_{Ht}(z)}{P^*_{Ht}}\right)^{-\epsilon} C^*_{Ht} \text{ and } C^*_{Ft}(z) = \frac{1}{1-n}\left(\frac{P^*_{Ft}(z)}{P^*_{Ft}}\right)^{-\epsilon} C^*_{Ft},$$

$$C^*_{Ft} = (1-\gamma^*)\left(\frac{P^*_{Ft}}{P^*_t}\right)^{-\eta} C^*_t \text{ and } C^*_{Ht} = \gamma^*\left(\frac{P^*_{Ht}}{P^*_t}\right)^{-\eta} C^*_t,$$

$$P^*_{Ht} = \left(\frac{1}{n}\int_0^n p^*_{Ht}(z)^{1-\epsilon} dz\right)^{\frac{1}{1-\epsilon}} \quad \text{and}$$

$$P^*_{Ft} = \left(\frac{1}{1-n}\int_n^1 p^*_{Ft}(z)^{1-\epsilon} dz\right)^{\frac{1}{1-\epsilon}},$$

$$P^*_t = \left((1-\gamma^*)\, P^{*1-\eta}_{Ft} + \gamma^* P^{*1-\eta}_{Ht}\right)^{\frac{1}{1-\eta}}.$$

A.2 Law of One Price, the Terms of Trade, and Relative Prices

We assume that the law of one price holds, implying $p_{Ft}(z) = E_t p^*_{Ft}(z), p_{Ht}(z) = E_t p^*_{Ht}(z)$ for all $z \in [0,1]$ where $E_t = [H]/[F]$ is the nominal exchange rate—that is, the price of foreign currency in terms of home currency—and $p^*_{Ft}(z)$ is the price of foreign good z denominated in foreign currency. We define the terms of trade as the relative price of imported goods:

$$S_t = \frac{P_{Ft}}{P_{Ht}}.$$

From here it follows that

$$\frac{P_t}{P_{Ht}} = \left((1-\gamma) + \gamma S_t^{1-\eta}\right)^{\frac{1}{1-\eta}} \text{ and } \frac{P_t^*}{P_{Ht}^*} = \left((1-\gamma^*)\, S_t^{1-\eta} + \gamma^*\right)^{\frac{1}{1-\eta}}.$$

A.3 Households

A.3.1 Home Households

The economy is populated by a continuum of homogeneous households maximizing the expected lifetime utility $w_t^H = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_t, g_t, n_t)$, with period household utility

$$U(c_t, g_t, n_t) \equiv U_t = \frac{c_t^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}}\xi_t + \varpi \frac{g_t^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}}\xi_t - d\frac{n_t^{1+\varsigma}}{1+\varsigma}\delta_t,$$

where $\varsigma \geqslant 0$ measures the labor supply elasticity, $\sigma \geqslant 0$ measures the elasticity of consumption, d is a preference parameter, c_t is consumption, g_t is household consumption of public goods, n_t is labor, β is the discount factor, and ξ_t is a taste shock. Households hold non-contingent bonds issued by governments in home and foreign countries. Each household contributes to the production of all home goods z. Each household is also a firm producing home goods.

We assume that all households in the same country have the same level of initial wealth. As they face the same labor demand and own equal share of all firms, they face identical budget constraints. They all will have identical consumption paths, so we do not use an individual index within each country.

Each household invests in home-currency-denominated home government short- and long-term nominal bonds B_{Ht}^{S} and B_{Ht}^{M} and also in foreign-currency-denominated foreign government long-term nominal bonds B_{Ft}^{M}. The nominal individual intertemporal budget constraint at time t for country H is given by

$$\begin{aligned}(1-\tau_t^w) W_t n_t + \frac{B_{Ht-1}^{S}}{n} + \left(1+\rho_H P_{Ht}^{M}\right) \frac{B_{Ht-1}^{M}}{n} \\ + \left(1+\rho_F P_{Ft}^{M}\right) \frac{B_{Ft-1}^{M}}{n} E_t + P_{Ht} \frac{T_t}{n} \\ \geq P_{Ht}^{S} \frac{B_{Ht}^{S}}{n} + P_{Ht}^{M} \frac{B_{Ht}^{M}}{n} + \frac{P_{Ft}^{M} B_{Ft}^{M} E_t}{n\phi\left(\frac{P_{Ft}^{M} B_{Ft}^{M} E_t}{P_t}\right)} + P_t c_t,\end{aligned}$$

where W_t is the nominal wage, T_t are country-level lump-sum taxes/transfers, and τ_t^w is a country-specific tax on nominal wage income. Here $c_t = \frac{1}{n} C_t$ and $n_t = \frac{1}{n} N_t = \frac{1}{n}\int_0^n n_t(z)\,dz$, where $n_t(z)$ is individual labor supplied to each individual firm-household.

In the main case we assume that financial markets are incomplete, as in Benigno (2009). Households of country H can trade in four nominal bonds. In each country the government issues two forms of bonds. The first is the familiar one-period debt, B_{Ht}^{S}, which has a price equal to the inverse of the gross nominal interest rate, $P_{Ht}^{S} = R_t^{-1} = \frac{1}{1+i_t}$. The second type of bond is actually a portfolio of many bonds which, following Cochrane (2000), pay a declining premium of ρ^j, j periods after being issued, where $0 < \rho < \beta^{-1}$. The duration of the bond is $\frac{1}{1-\beta\rho}$, which means that ρ can be varied to capture changes in the maturity structure of debt. We only need to price a single bond, since any existing bond issued j periods ago is worth ρ^j of new bonds.

Bonds B_{Ht}^{S} and B_{Ht}^{M} are issued by the home government and are denominated in home currency; bonds B_{Ft}^{S} and B_{Ft}^{M} are issued by the foreign government and are denominated in foreign currency.

We assume that only long-term bonds are tradable. Households belonging to country H have to pay an intermediation cost if they want to trade in the foreign bond. These costs are determined by the function $\phi(\cdot)$. Function $\phi(\cdot)$ depends on the real holdings of the foreign assets in the entire economy, and therefore is taken as given by the domestic households. If a home household is in a "borrowing position," it will be charged with a premium on the foreign interest rate, and if the household is in a "lending position," it receives a rate of return lower than the foreign interest rate. We impose the following restrictions on $\phi(\cdot)$: $\phi(x)$ is 1 only if x is in steady state. Furthermore, $\phi(\cdot)$ has to be a differentiable, decreasing function in the neighborhood of steady state, and we denote $\phi'(x) = -\chi$ in steady-state x.

The Lagrangian can be written as

$$\Gamma_t = \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \Bigg\{ U\left(c_t, g_t, n_t\right) + \frac{\Lambda_t}{n} \Bigg(\left(1 + \rho_H P_{Ht}^M\right) B_{Ht-1}^M \\ + \left(1 + \rho_F P_{Ft}^M\right) B_{Ft-1}^M E_t + n\left(1 - \tau_t^w\right) W_t n_t + B_{Ht-1}^S + P_{Ht} T_t \\ - P_{Ht}^S B_{Ht}^S - P_{Ht}^M B_{Ht}^M - \frac{P_{Ft}^M B_{Ft}^M E_t}{\phi\left(\frac{P_{Ft}^M B_{Ft}^M E_t}{P_t}\right)} - n P_t c_t \Bigg) \Bigg\}.$$

And the first-order conditions in the text are taken with respect to c_t, n_t, B_{Ht}^S, B_{Ht}^M, B_{Ft}^M, and Λ_t.

We assume that the net supply of short-term bonds is zero, aggregate all first-order conditions, and arrive at the following system:

$$\frac{W_t}{P_t} = -\frac{U_{n,t}}{\left(1 - \tau_t^w\right) U_{c,t}}, \tag{1}$$

$$\frac{1}{(1 + i_t)} = \beta \mathbb{E}_t \frac{P_t U_{c,t+1}}{P_{t+1} U_{c,t}}, \tag{2}$$

$$P_{Ht}^M = \beta \mathbb{E}_t \frac{U_{c,t+1} P_t}{U_{c,t} P_{t+1}} \left(1 + \rho_H P_{Ht+1}^M\right), \tag{3}$$

$$P_{Ft}^M = \beta \mathbb{E}_t \frac{U_{c,t+1} P_t E_{t+1}}{U_{c,t} P_{t+1} E_t} \phi\left(\frac{P_{Ft}^M B_{Ft}^M E_t}{P_t}\right) \left(1 + \rho_F P_{Ft+1}^M\right), \tag{4}$$

$$P_{Ht}^M B_{Ht}^M + \frac{P_{Ft}^M B_{Ft}^M E_t}{\phi\left(\frac{P_{Ft}^M B_{Ft}^M E_t}{P_t}\right)} = \left(1 + \rho_H P_{Ht}^M\right) B_{Ht-1}^M \\ + \left(1 + \rho_F P_{Ft}^M\right) B_{Ft-1}^M E_t + \left(1 - \tau_t^w\right) W_t N_t \\ + P_{Ht} T_t - P_t C_t, \tag{5}$$

where the budget constraint is written in an aggregated form.

A.3.2 Foreign Households

Similarly, the foreign households maximize the intertemporal utility $w_t^F = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U\left(c_t^*, g_t^*, n_t^*\right)$, where

$$U\left(c_t^*, g_t^*, n_t^*\right) \equiv U_t^* = \frac{c_t^{*1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}}\xi_t^* + \varpi\frac{g_t^{*1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}}\xi_t^* - d\frac{n_t^{*1+\varsigma}}{1+\varsigma}\delta_t^*,$$

and $\psi \geqslant 0$ measures the labor supply elasticity, $\sigma \geqslant 0$ measures the elasticity of consumption, d^* is a preference parameter, c_t^* is consumption, n_t^* is labor, and β is the discount factor in country F. ξ_t^* is a taste shock. The individual household budget constraint in foreign currency is given by

$$\begin{aligned}
&\left(1-\tau_t^{w*}\right) W_t^* n_t^* + \frac{\left(1+\rho_H P_{Ht}^M\right)}{E_t}\frac{B_{Ht-1}^{M*}}{1-n} + \frac{B_{Ft-1}^{S*}}{1-n} \\
&\quad + \left(1+\rho_F P_{Ft}^M\right)\frac{B_{Ft-1}^{M*}}{1-n} + P_{Ft}^*\frac{T_t^*}{1-n} \\
&\geq \frac{P_{Ht}^M B_{Ht}^{M*}}{\left(1-n\right) E_t \phi^*\left(\frac{P_{Ht}^M B_{Ht}^{M*}}{E_t P_t^*}\right)} + P_{Ft}^S\frac{B_{Ft}^{S*}}{1-n} + P_{Ft}^M\frac{B_{Ft}^{M*}}{1-n} + P_t^* c_t^*,
\end{aligned}$$

where W_t^* is the nominal wage, T_t^* are country-level taxes/transfers, and τ_t^{w*} is a country-specific tax on nominal income. Here $c_t^* = \frac{1}{1-n}C_t^*$ and $n_t^* = \frac{1}{1-n}N_t^* = \frac{1}{1-n}\int_n^1 n_t^*\left(z\right) dz$, where $n_t^*\left(z\right)$ is individual labor supplied to each individual firm-household.

The Lagrangian can be written as

$$\begin{aligned}
\Gamma_t = \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \Bigg\{ & U\left(c_t^*, g_t^*, n_t^*\right) + \frac{\Lambda_t^*}{1-n}\Bigg(\left(1+\rho_H P_{Ht}^{M*}\right)\frac{B_{Ht-1}^{M*}}{E_t} \\
& + \left(1+\rho_F P_{Ft}^{M*}\right) B_{Ft-1}^{M*} + \left(1-n\right)\left(1-\tau_t^{w*}\right) W_t^* n_t^* \\
& + B_{Ft-1}^{S*} + P_{Ft}^* T_t^* - P_{Ft}^S B_{Ft}^{S*} - P_{Ft}^M B_{Ft}^{M*} \\
& - \frac{P_{Ht}^M B_{Ht}^{M*}}{E_t \phi^*\left(\frac{P_{Ht}^M B_{Ht}^{M*}}{E_t P_t^*}\right)} - \left(1-n\right) P_t^* c_t^* \Bigg) \Bigg\}.
\end{aligned}$$

And the first-order conditions in the text are taken with respect to c_t^*, n_t^*, B_{Ft}^{S*}, B_{Ft}^{M*}, B_{Ht}^M, and Λ_t^*.

Similarly, assuming zero net supply of short-term bonds and aggregating across all households, we obtain the following system of first-order conditions:

$$\frac{W_t^*}{P_t^*} = -\frac{U_{n,t}^*}{U_{c,t}^*\left(1-\tau_t^{*w}\right)}, \tag{6}$$

$$\frac{1}{\left(1+i_t^*\right)} = \beta\mathbb{E}_t\frac{U_{c,t+1}^* P_t^*}{U_{c,t}^* P_{t+1}^*}, \tag{7}$$

$$P_{Ft}^M = \beta\mathbb{E}_t\frac{U_{c,t+1}^* P_t^*}{U_{c,t}^* P_{t+1}^*}\left(1+\rho_F P_{Ft+1}^M\right), \tag{8}$$

$$P_{Ht}^M = \beta\mathbb{E}_t\frac{U_{c,t+1}^* P_t^* E_t}{U_{c,t}^* P_{t+1}^* E_{t+1}}\left(1+\rho_H P_{Ht+1}^M\right)\phi^*\left(\frac{P_{Ht}^M B_{Ht}^{M*}}{E_t P_t^*}\right), \tag{9}$$

$$P_{Ft}^M B_{Ft}^{M*} = \left(1+\rho_F P_{Ft}^M\right)B_{Ft-1}^{M*} + \frac{\left(1+\rho_H P_{Ht}^M\right)B_{Ht-1}^{M*}}{E_t}$$
$$+\left(1-\tau_t^{w*}\right)W_t^* N_t^* + P_{Ft}^* T_t^* - P_t^* C_t^* - \frac{P_{Ht}^M B_{Ht}^{M*}}{E_t\phi^*\left(\frac{P_{Ht}^M B_{Ht}^{M*}}{E_t P_t^*}\right)}. \tag{10}$$

A.4 Firms

A.4.1 Home Firms

The production side in this model is entirely conventional. An intermediate-good-producing firm in each sector only uses labor as a factor of production. Labor is mobile between firms so wages are equalized across all firms. The firm's optimization problem is standard: a firm i chooses employment and prices to maximize profit

$$\max_{\left\{N_t(i),p_{Hs}^{\times}(i)\right\}_{s=t}^{\infty}} \mathbb{E}_t\sum_{s=t}^{\infty} Q_{t,s}\left(y_s\left(i\right)p_{Hs}\left(i\right) - W_s N_s\left(i\right)\right),$$

where $Q_{t,s}$ is the household's stochastic discount factor subject to the production constraint

$$y_t\left(i\right) = AZ_t N_t\left(i\right),$$

demand constraint

$$y_t(i) = Y_t \left(\frac{p_{Ht}(i)}{P_{Ht}} \right)^{-\varepsilon},$$

and price rigidity

$$p_{Ht}(i) = p_{Ht}^{\times}(i),$$
$$p_{Ht+1}(i) = \begin{cases} p_{Ht}^{\times}(i), \text{ with prob } 1-\theta \\ p_{Ht+1}(i), \text{ with prob } \theta. \end{cases}$$

The profit-maximization problem can be split into two separate problems: choose labor to minimize cost intratemporally and choose prices to maximize future profits. We deal with each of these problems separately.

Aggregate Employment. Cost minimization of firm i is

$$\min_{N_t(i)} (W_t N_t(i))$$

subject to the production constraint

$$y_t(i) = AZ_t N_t(i).$$

Write down the Lagrangian:

$$L = W_t N_t(i) - P_{Ht}\Xi_t (AZ_t N_t(i) - y_t(i))$$

and obtain the expression for the marginal cost

$$\Xi_t = \frac{W_t}{P_{Ht} AZ_t} = mc_t.$$

Aggregation of labor yields (we denote price dispersion $\Delta_t \langle n, \varepsilon \rangle = \frac{1}{n} \int_0^n \left(\frac{p_{Ht}(i)}{P_{Ht}} \right)^{-\varepsilon} di$)

$$N_t = \int_0^n N_t(i)\, di = \int_0^n \frac{y_t(i)}{AZ_t} di = \frac{1}{n} \int_0^n \frac{Y_t}{AZ_t} \left(\frac{p_{Ht}(i)}{P_{Ht}} \right)^{-\varepsilon} di$$
$$= \frac{Y_t}{AZ_t} \Delta_t \langle n, \varepsilon \rangle,$$

so the "aggregated" production function can be written as

$$Y_t = N_t \frac{AZ_t}{\Delta_t}.$$

Price Setting. The following is price setting:

$$\begin{aligned}
&\max_{\{p_{Hs}^{\times}(i)\}_{s=t}^{\infty}} \mathbb{E}_t \sum_{s=t}^{\infty} Q_{t,s} \left(y_s(i) p_{Hs}(i) - W_s N_s(i)\right) \\
&= \mathbb{E}_t \sum_{s=t}^{\infty} Q_{t,s} \left(y_s(i) p_{Hs}(i) - W_s \frac{y_s(i)}{AZ_s}\right) \\
&= \mathbb{E}_t \sum_{s=t}^{\infty} Q_{t,s} \left(y_s(i) p_{Hs}(i) - y_s(i) MC_s\right),
\end{aligned}$$

where

$$MC_s = P_{Ht} mc_t.$$

The constraints are

$$y_t(i) = Y_t \left(\frac{p_{Ht}(i)}{P_{Ht}}\right)^{-\varepsilon_t}$$

$$p_{Ht}(i) = p_{Ht}^{\times}(i)$$

$$p_{Ht+1}(i) = \begin{cases} p_{Ht+1}^{\times}(i), & \text{with prob } 1-\theta \\ p_{Ht+1}(i), & \text{with prob } \theta. \end{cases}$$

The problem for the optimal price setting at time t can, equivalently, be written as

$$\max_{\{p_{Ht}^{\times}(i)\}_{s=t}^{\infty}} \mathbb{E}_t \sum_{s=t}^{\infty} \theta^{s-t} Q_{t,s} \left(y_s(i) p_{Ht}^{\times}(i) - y_s(i) MC_s\right),$$

so we only consider profits of those firms that fix prices at time t. Substitute demand

$$\max_{\{p_t^{\times}(i)\}_{s=t}^{\infty}} \mathbb{E}_t \sum_{s=t}^{\infty} \theta^{s-t} Q_{t,s} P_{Hs} Y_s \left(\left(\frac{p_{Ht}^{\times}(i)}{P_{Hs}}\right)^{1-\varepsilon} - \left(\frac{p_{Ht}^{\times}(i)}{P_{Hs}}\right)^{-\varepsilon} \frac{MC_s}{P_{Hs}}\right).$$

First-order conditions are

$$0 = \mathbb{E}_t \sum_{s=t}^{\infty} \theta^{s-t} Q_{t,s} P_{Hs} y_s (i) \left(\frac{p_{Ht}^{\times}(i)}{P_{Hs}} - \mu mc_s \right),$$

where $\mu = -\frac{\varepsilon}{1-\varepsilon} = \frac{\varepsilon}{\varepsilon-1}$.

We can rewrite these conditions in the form of the three equations:

$$\frac{p_{Ht}^{\times}(i)}{P_{Ht}} = \frac{H_t}{F_t}$$

$$F_t = \mathbb{E}_t \sum_{s=t}^{\infty} (\theta\beta)^{s-t} f_s \left(\frac{P_{Hs}}{P_{Ht}} \right)^{\varepsilon-1}$$

$$H_t = \mathbb{E}_t \sum_{s=t}^{\infty} (\theta\beta)^{s-t} h_s \left(\frac{P_{Hs}}{P_{Ht}} \right)^{\varepsilon}$$

with

$$f_t = U_C (C_t, n_t) \frac{P_{Ht}}{P_t} Y_t = C_t^{-\frac{1}{\sigma}} \xi_t^{-\frac{1}{\sigma}} \frac{P_{Ht}}{P_t} Y_t$$

$$h_t = \mu U_C (C_t, n_t) \frac{P_{Ht}}{P_t} Y_t mc_t$$

$$= \mu C_t^{-\frac{1}{\sigma}} \xi_t^{-\frac{1}{\sigma}} \frac{P_{Ht}}{P_t} Y_t \frac{W_t}{P_{Ht} A Z_t}.$$

We can aggregate over i and remove index i:

$$\frac{p_{Ht}^{\times}}{P_{Ht}} = \frac{H_t}{F_t}.$$

Price in the sector is determined as

$$P_{Ht} = \left[(1-\theta) \left(p_{Ht}^{\times} \right)^{1-\varepsilon} + \theta P_{Ht-1}^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}},$$

from where

$$\Pi_{Ht}^{1-\varepsilon} = \left(\frac{P_{Ht}}{P_{Ht-1}} \right)^{1-\varepsilon} = (1-\theta) \left(\frac{p_{Ht}^{\times}}{P_{Ht}} \frac{P_{Ht}}{P_{Ht-1}} \right)^{1-\varepsilon} + \theta$$

and so

$$\left(\frac{p^{\times}_{Ht}}{P_{Ht}}\right)^{1-\varepsilon} = \frac{\Pi^{1-\varepsilon}_{Ht} - \theta}{(1-\theta)\,\Pi^{1-\varepsilon}_{Ht}} = \frac{1-\theta\Pi^{\varepsilon-1}_{Ht}}{(1-\theta)} = \left(\frac{H_t}{F_t}\right)^{1-\varepsilon}.$$

We obtain the following first-order conditions:

$$H_t = \mu\upsilon_t U_{C,t}\frac{P_{Ht}}{P_t}Y_t\frac{W_t}{P_{Ht}AZ_t} + \theta\beta\mathbb{E}_t\Pi^{\varepsilon}_{Ht+1}H_{t+1}, \tag{11}$$

$$F_t = U_{C,t}\frac{P_{Ht}}{P_t}Y_t + \theta\beta\mathbb{E}_t\Pi^{\varepsilon-1}_{Ht+1}F_{t+1}, \tag{12}$$

$$\left(\frac{H_t}{F_t}\right)^{1-\varepsilon} = \frac{\left(1-\theta\Pi^{\varepsilon-1}_{Ht}\right)}{(1-\theta)}, \tag{13}$$

$$Y_t = N_t\frac{AZ_t}{\Delta_t}, \tag{14}$$

where equations (11)–(13) define optimal price setting and υ_t represents a shock to the desired flexible-price markup $\mu = \frac{\epsilon}{\epsilon-1}$.[2] Equation (14) defines aggregated labor demand.

We derive the evolution of the price dispersion equation. Each period, a measure of $(1-\theta)n$ of firms resets prices and sets $p_{Ht}(i) = p^*_{Ht}$, which is the same for all such firms. A measure of θ does not change prices, and for them $p_{Ht}(i) = p_{Ht-1}(i)$. Therefore,

$$\Delta_t = \frac{1}{n}\int_0^{(1-\theta)n}\left[\frac{p_{Ht}(i)}{P_{Ht}}\right]^{-\varkappa}di + \frac{1}{n}\int_{(1-\theta)n}^{n}\left[\frac{p_{Ht}(i)}{P_{Ht}}\right]^{-\varkappa}di$$

$$= \frac{1}{n}\int_0^{(1-\theta)n}\left[\frac{p^*_{Ht}}{P_{Ht}}\right]^{-\varkappa}di + \frac{1}{n}\int_{(1-\theta)n}^{n}\left[\frac{p_{Ht-1}(i)}{P_{Ht}}\right]^{-\varkappa}di.$$

[2] We could have assumed a variable elasticity ϵ_t. This would result in the identical linearized system but would not allow us to write down non-linear first-order conditions in the compact form (11)–(13). The derivation of linearized first-order conditions would be identical to the one in Leith and Wren-Lewis (2013).

We compute

$$\int_0^{(1-\theta)n} \left[\frac{p_{Ht}^*}{P_{Ht}}\right]^{-\varkappa} di = \left[\frac{p_{Ht}^*}{P_{Ht}}\right]^{-\varkappa} \int_0^{(1-\theta)n} di = (1-\theta)n \left[\frac{p_{Ht}^*}{P_{Ht}}\right]^{-\varkappa}$$

and

$$\begin{aligned} \int_{(1-\theta)n}^{n} \left[\frac{p_{Ht}(i)}{P_{Ht}}\right]^{-\varkappa} di &= \int_{(1-\theta)n}^{n} \left[\frac{p_{Ht-1}(i)}{P_{Ht}}\right]^{-\varkappa} di \\ &= \int_{(1-\theta)n}^{n} \left[\frac{p_{Ht-1}(i)}{P_{Ht}} \frac{P_{Ht-1}}{P_{Ht-1}}\right]^{-\varkappa} di \\ &= \theta n \left(\frac{P_{Ht-1}}{P_{Ht}}\right)^{-\varkappa} \frac{1}{\theta n} \int_{(1-\theta)n}^{n} \left[\frac{p_{Ht-1}(i)}{P_{Ht-1}}\right]^{-\varkappa} di \\ &= \theta n \left(\frac{P_{Ht-1}}{P_{Ht}}\right)^{-\varkappa} \Delta_{t-1}. \end{aligned}$$

Collect two terms together to obtain

$$\Delta_t = (1-\theta)\left[\frac{p_{Ht}^*}{P_{Ht}}\right]^{-\varkappa} + \theta\Delta_{t-1}\Pi_{Ht}^{\varkappa}$$

and with $\varkappa = \epsilon$ we obtain

$$\Delta_t = (1-\theta)\left(\frac{1-\theta\Pi_{Ht}^{\epsilon-1}}{1-\theta}\right)^{\frac{\epsilon}{\epsilon-1}} + \theta\Pi_{Ht}^{\epsilon}\Delta_{t-1}. \tag{15}$$

Aggregate intraperiod nominal profit of monopolistically competitive firms is

$$P_{Ht}\Phi_t = P_{Ht}Y_t - W_tN_t.$$

We assume that the profit is 100 percent taxed by the government and redistributed to households:

$$P_{Ht}\Phi_t = P_{Ht}\bar{T}_t. \tag{16}$$

A.4.2 Flexible Price

$$\max \mathbb{E}_t \sum_{s=t}^{\infty} Q_{t,s} \left(y_s(i) \, p_{Hs}(i) - W_s N_s(i) \right)$$
$$= \mathbb{E}_t \sum_{s=t}^{\infty} Q_{t,s} \left(P_{Hs} Y_s \left(\frac{p_{Hs}(i)}{P_{Hs}} \right)^{1-\varepsilon} - \frac{W_s}{AZ_s} Y_s \left(\frac{p_{Hs}(i)}{P_{Hs}} \right)^{-\varepsilon} \right)$$

The constraints are

$$y_t(i) = Y_t \left(\frac{p_{Ht}(i)}{P_{Ht}} \right)^{-\varepsilon_t}.$$

First-order conditions are

$$0 = (1-\varepsilon) Y_s \left(\frac{p_{Hs}(i)}{P_{Hs}} \right)^{-\varepsilon} + \varepsilon \frac{W_s}{P_{Hs} AZ_s} Y_s \left(\frac{p_{Hs}(i)}{P_{Hs}} \right)^{-\varepsilon-1},$$

where $\mu = -\frac{\varepsilon}{1-\varepsilon} = \frac{\varepsilon}{\varepsilon-1}$.

$$0 = \frac{(1-\varepsilon)}{\varepsilon} \left(\frac{p_{Hs}(i)}{P_{Hs}} \right) + \frac{W_s}{P_{Hs} AZ_s}$$

$$\frac{(\varepsilon-1)}{\varepsilon} = \frac{W_s}{P_{Hs} AZ_s}$$

$$\frac{W_t}{P_t} = -\frac{U_{n,t}}{(1-\tau_t^w) U_{c,t}},$$

$$\frac{P_t}{P_{Ht}} = \left((1-\gamma) + \gamma S_t^{1-\eta} \right)^{\frac{1}{1-\eta}} = \Upsilon_t \text{ and}$$
$$\frac{P_t^*}{P_{Ht}^*} = \left((1-\gamma^*) S_t^{1-\eta} + \gamma^* \right)^{\frac{1}{1-\eta}} = \Gamma_t.$$

$$\frac{1}{\mu} = \frac{\varepsilon-1}{\varepsilon} = -\frac{1}{AZ_t} \frac{U_{n,t}}{(1-\tau_t^w) U_{c,t}} \Upsilon_t$$
$$= \frac{1}{AZ_t} \frac{d n_t^{\varsigma} \delta_t^{-\frac{1}{\sigma}}}{(1-\tau_t^w) c_t^{-\frac{1}{\sigma}} \xi_t^{-\frac{1}{\sigma}}} \Upsilon_t = \frac{\Upsilon_t}{AZ_t} \frac{d n^{-\left(\frac{1}{\sigma}+\varsigma\right)} N_t^{\varsigma}}{(1-\tau_t^w) C_t^{-\frac{1}{\sigma}}} \frac{\delta_t^{-\frac{1}{\sigma}}}{\xi_t^{-\frac{1}{\sigma}}}$$

A.4.3 Foreign Firms

Similarly, foreign firm i chooses employment and prices to maximize profit:

$$\max_{\left\{N_t^*(i),p_s^{*\times}(i)\right\}_{s=t}^{\infty}} \mathbb{E}_t \sum_{s=t}^{\infty} Q_{t,s}^* \left(y_s^*\left(i\right) p_{Fs}^*\left(i\right) - W_s^* N_s^*\left(i\right)\right),$$

subject to constraints

$$y_t^*\left(i\right) = A^* Z_t^* N_t^*\left(i\right),$$

$$y_t^*\left(i\right) = Y_t^* \left(\frac{p_{Ft}^*\left(i\right)}{P_{Ft}^*}\right)^{-\varepsilon},$$

$$p_{Ft}^*\left(i\right) = p_{Ft}^{*\times}\left(i\right)$$

$$p_{Ft+1}^*\left(i\right) = \begin{cases} p_{Ft+1}^{*\times}\left(i\right), \text{ with prob } 1-\theta \\ p_{Ft+1}^*\left(i\right), \text{ with prob } \theta. \end{cases}$$

The first-order conditions are

$$H_t^* = \mu \upsilon_t^* U_{C,t}^* \frac{P_{Ft}^*}{P_t^*} Y_t^* \frac{W_t^*}{P_{Ft}^* A^* Z_t^*} + \theta\beta \mathbb{E}_t \Pi_{Ft+1}^{*\varepsilon} H_{t+1}^*, \tag{17}$$

$$F_t^* = U_{C,t}^* \frac{P_{Ft}^*}{P_t^*} Y_t^* + \theta\beta \mathbb{E}_t \Pi_{Ft+1}^{*\varepsilon-1} F_{t+1}^*, \tag{18}$$

$$\left(\frac{H_t^*}{F_t^*}\right)^{1-\varepsilon} = \frac{\left(1-\theta\Pi_{Ft}^{*\varepsilon-1}\right)}{(1-\theta)}, \tag{19}$$

$$Y_t^* = N_t^* \frac{A^* Z_t^*}{\Delta_t^*}, \tag{20}$$

$$\Delta_t^* = (1-\theta)\left(\frac{1-\theta\Pi_{Ft}^{*\epsilon-1}}{1-\theta}\right)^{\frac{\epsilon}{\epsilon-1}} + \theta\Pi_{Ft}^{*\epsilon}\Delta_{t-1}^*. \tag{21}$$

The profit is

$$\Phi_t^* = Y_t^* - \frac{W_t^*}{P_{Ft}^*} N_t^*. \tag{22}$$

A.4.4 Flexible Price

$$\max \mathbb{E}_t \sum_{s=t}^{\infty} Q_{t,s} \left(y_s^* \left(i \right) p_{Fs}^* \left(i \right) - W_s^* N_s^* \left(i \right) \right)$$

$$= \mathbb{E}_t \sum_{s=t}^{\infty} Q_{t,s} \left(P_{Ft}^* Y_t^* \left(\frac{p_{Ft}^* \left(i \right)}{P_{Ft}^*} \right)^{-\varepsilon_t} - \frac{W_s^*}{AZ_s^*} Y_t^* \left(\frac{p_{Ft}^* \left(i \right)}{P_{Ft}^*} \right)^{-\varepsilon_t} \right)$$

The constraints are

$$y_t^* \left(i \right) = Y_t^* \left(\frac{p_{Ft}^* \left(i \right)}{P_{Ft}^*} \right)^{-\varepsilon_t}.$$

First-order conditions are

$$0 = \left(1 - \varepsilon \right) Y_t^* \left(\frac{p_{Ft}^* \left(i \right)}{P_{Ft}^*} \right)^{-\varepsilon} + \varepsilon \frac{W_s^*}{AZ_s^* P_{Ft}^*} Y_t^* \left(\frac{p_{Ft}^* \left(i \right)}{P_{Ft}^*} \right)^{-\varepsilon - 1},$$

where $\mu = -\frac{\varepsilon}{1-\varepsilon} = \frac{\varepsilon}{\varepsilon - 1}$.

$$0 = \frac{\left(1 - \varepsilon \right)}{\varepsilon} \left(\frac{p_{Ft}^* \left(i \right)}{P_{Ft}^*} \right) + \frac{W_s^*}{AZ_s^* P_{Ft}^*}$$

$$\frac{\left(\varepsilon - 1 \right)}{\varepsilon} = \frac{W_s^*}{AZ_s^* P_{Ft}^*}$$

$$\frac{W_t^*}{P_t^*} = -\frac{U_{n,t}^*}{U_{c,t}^* \left(1 - \tau_t^{*w} \right)},$$

$$\frac{P_t}{P_{Ht}} = \left(\left(1 - \gamma \right) + \gamma S_t^{1-\eta} \right)^{\frac{1}{1-\eta}} = \Upsilon_t \text{ and}$$

$$\frac{P_t^*}{P_{Ht}^*} = \left(\left(1 - \gamma^* \right) S_t^{1-\eta} + \gamma^* \right)^{\frac{1}{1-\eta}} = \Gamma_t.$$

$$\frac{1}{\mu} = \frac{\varepsilon - 1}{\varepsilon} = -\frac{U_{n,t}^*}{AZ_t^* U_{c,t}^* \left(1 - \tau_t^{*w} \right)} \frac{\Gamma_t}{S_t} = \frac{1}{AZ_t^*} \frac{dn_t^{*\varsigma} \delta_t^{*-\frac{1}{\sigma}}}{\left(1 - \tau_t^{*w} \right) c_t^{*-\frac{1}{\sigma}} \xi_t^{*-\frac{1}{\sigma}}} \frac{\Gamma_t}{S_t}$$

$$= \frac{1}{AZ_t^*} \frac{\Gamma_t}{S_t} \frac{d \left(1 - n \right)^{-\left(\frac{1}{\sigma} + \varsigma \right)} N_t^{*\varsigma}}{\left(1 - \tau_t^{*w} \right) C_t^{*-\frac{1}{\sigma}}} \frac{\delta_t^{*-\frac{1}{\sigma}}}{\xi_t^{*-\frac{1}{\sigma}}}$$

A.5 Financial Intermediaries

Financial intermediaries in each country help to sell foreign bonds to residents of their own country. The profit of intermediaries is

$$P_{Ht}\Psi_t = \frac{P_{Ft}^M B_{Ft}^M E_t}{\phi\left(\frac{P_{Ft}^M B_{Ft}^M E_t}{P_t}\right)} - P_{Ft}^M B_{Ft}^M E_t,$$

$$P_{Ft}^*\Psi_t^* = \frac{P_{Ht}^M B_{Ht}^{M*}}{E_t\phi^*\left(\frac{P_{Ht}^M B_{Ht}^{M*}}{E_t P_t^*}\right)} - \frac{P_{Ht}^M B_{Ht}^{M*}}{E_t}.$$

A.6 Governments

The home-country government buys home goods G_{Ht}, taxes labor income τ_t^l, raises lump-sum taxes $\tilde{T}_t$, pays an employment subsidy τ_t^s, expropriates monopoly profit Φ_t and profit of financial intermediaries Ψ_t, pays transfers to households

$$T_t = \bar{T}_t - \tilde{T}_t,$$

and issues nominal debt B_t^M. The value B_t^M of end-of-period public debt then evolves according to the following law of motion:

$$\begin{aligned} &P_{Ht}^M\left(B_{Ht}^M + B_{Ht}^{M*}\right) \\ &\quad = \left(1+\rho_H P_{Ht}^M\right)\left(B_{Ht-1}^M + B_{Ht-1}^{M*}\right) + P_{Ht}G_t + P_{Ht}T_t \\ &\quad\quad - \tau_t^w W_t n_t - P_{Ht}\Phi_t - P_{Ht}\Psi_t, \end{aligned} \tag{23}$$

where we assumed that the aggregate stock of one-period bonds is in zero net supply, $B_t^S = 0$.

Lump-sum transfers $\bar{T}_t$ rebate monopolistic profit, $\bar{T}_t = \Phi_t + \Psi_t$, while lump-sum taxes $\tilde{T}_t$ are used to pay the steady-state employment subsidy τ^s, the size of which will be determined later:

$$P_{Ht}\tilde{T}_t = \tau^s W_t N_t,$$

so we define τ_t^l as $\tau_t^w = \tau_t^l - \tau^s$.

For analytical convenience we introduce the home-country real debt $B_{Ht} = B_{Ht}^M / P_{Ht}$, so that (23) becomes

$$P_{Ht}^{M}\left(B_{Ht}+B_{Ht}^{*}\right)=\left(1+\rho_{H}P_{Ht}^{M}\right)\frac{\left(B_{Ht-1}+B_{Ht-1}^{*}\right)}{1+\pi_{Ht}}+G_{t}-\tau_{t}^{l}\frac{W_{t}}{P_{Ht}}N_{t}. \tag{24}$$

Similarly, the foreign-country budget constraint becomes (using $B_{Ft}=B_{Ft}^{M}/P_{Ft}^{*}$)

$$P_{Ft}^{M}\left(B_{Ft}+B_{Ft}^{*}\right)=\left(1+\rho_{F}P_{Ft}^{M}\right)\frac{\left(B_{Ft-1}+B_{Ft-1}^{*}\right)}{1+\pi_{Ft}^{*}}+G_{t}^{*}-\tau_{t}^{l*}\frac{W_{t}^{*}}{P_{Ft}^{*}}N_{t}^{*} \tag{25}$$

and

$$T_{t}^{*}=\bar{T}_{t}^{*}-\tilde{T}_{t}^{*},\qquad \bar{T}_{t}^{*}=\Phi_{t}^{*}+\Psi_{t}^{*},\qquad P_{Ft}^{*}\tilde{T}_{t}^{*}=\tau^{*s}W_{t}^{*}N_{t}^{*}.$$

A.7 Market Clearing and Evolution of the Economy

The two market clearing conditions for countries H and F are

$$Y_{t}P_{Ht}=C_{Ht}P_{Ht}+C_{Ht}^{*}P_{Ht}^{*}E_{t}+G_{t}P_{Ht} \tag{26}$$

$$Y_{t}^{*}P_{Ft}^{*}=C_{Ft}^{*}P_{Ft}^{*}+C_{Ft}\frac{P_{Ft}}{E_{t}}+G_{t}^{*}P_{Ft}^{*}. \tag{27}$$

Aggregating budget constraints, we get two financial constraints, and using market clearing conditions, we obtain the net foreign assets (NFA) equation:

$$\begin{aligned}\gamma^{*}\left(\left(1-\gamma^{*}\right)S_{t}^{1-\eta}+\gamma^{*}\right)^{\frac{\eta}{1-\eta}}C_{t}^{*}-\gamma S_{t}^{1-\eta}\left(\left(1-\gamma\right)+\gamma S_{t}^{1-\eta}\right)^{\frac{\eta}{1-\eta}}C_{t}\\ =\left(P_{Ft}^{M}B_{Ft}-\left(1+\rho_{F}P_{Ft}^{M}\right)\frac{B_{Ft-1}}{1+\pi_{Ft}^{*}}\right)S_{t}\\ -\left(P_{Ht}^{M}B_{Ht}^{*}-\left(1+\rho_{H}P_{Ht}^{M}\right)\frac{B_{Ht-1}^{*}}{1+\pi_{Ht}}\right).\end{aligned} \tag{28}$$

Eight households' first-order conditions (1)–(4) and (6)–(9), ten firms' first-order conditions (11)–(15) and (17)–(21), two market clearing conditions (26)–(27), two government budget constraints,

$$\begin{aligned}P_{Ht}^{M}\left(B_{Ht}+B_{Ht}^{*}\right)=\left(1+\rho_{H}P_{Ht}^{M}\right)\frac{\left(B_{Ht-1}+B_{Ht-1}^{*}\right)}{1+\pi_{Ht}}\\ +G_{t}-\tau_{t}^{l}\frac{W_{t}}{P_{Ht}}N_{t},\end{aligned}$$

$$P_{Ft}^{M}\left(B_{Ft}+B_{Ft}^{*}\right)=\left(1+\rho_{F}P_{Ft}^{M}\right)\frac{\left(B_{Ft-1}+B_{Ft-1}^{*}\right)}{1+\pi_{Ft}^{*}}$$
$$+G_{t}^{*}-\tau_{t}^{l*}\frac{W_{t}^{*}}{P_{Ft}^{*}}N_{t}^{*},$$

one net foreign assets equation (28), and the definition of the nominal exchange rate

$$S_{t}=\frac{E_{t}\ P_{Ft}^{*}}{P_{Ht}}$$

describe the evolution of the economy.

A.8 Private-Sector Equilibrium

To summarize the system as a whole,

$$\frac{W_{t}}{P_{t}}=-\frac{U_{n}\left(c_{t},n_{t}\right)}{U_{c}\left(c_{t},n_{t}\right)\left(1-\tau_{t}^{w}\right)} \tag{29}$$

$$\frac{1}{\left(1+i_{t}\right)}=\beta\mathbb{E}_{t}\frac{U_{c}\left(C_{t+1},n_{t+1}\right)}{U_{c}\left(C_{t},n_{t}\right)}\left(\frac{1-\gamma+\gamma S_{t}^{1-\eta}}{1-\gamma+\gamma S_{t+1}^{1-\eta}}\right)^{\frac{1}{1-\eta}}\frac{1}{1+\pi_{Ht+1}} \tag{30}$$

$$P_{Ht}^{M}=\beta\mathbb{E}_{t}\frac{U_{c}\left(C_{t+1},n_{t+1}\right)}{U_{c}\left(C_{t},n_{t}\right)}\left(\frac{1-\gamma+\gamma S_{t}^{1-\eta}}{1-\gamma+\gamma S_{t+1}^{1-\eta}}\right)^{\frac{1}{1-\eta}}\frac{1}{1+\pi_{Ht+1}}$$
$$\times\left(1+\rho_{H}P_{Ht+1}^{M}\right) \tag{31}$$

$$P_{Ft}^{M}=\beta\mathbb{E}_{t}\frac{U_{c}\left(C_{t+1},n_{t+1}\right)}{U_{c}\left(C_{t},n_{t}\right)}\left(\frac{1-\gamma+\gamma S_{t}^{1-\eta}}{1-\gamma+\gamma S_{t+1}^{1-\eta}}\right)^{\frac{1}{1-\eta}}\frac{S_{t+1}}{S_{t}}\frac{1}{1+\pi_{Ft+1}^{*}}$$
$$\times\left(1+\rho_{F}P_{Ft+1}^{M}\right)\phi\left(P_{Ft}^{M}B_{Ft}S_{t}\left((1-\gamma)+\gamma S_{t}^{1-\eta}\right)^{-\frac{1}{1-\eta}}\right) \tag{32}$$

$$Y_{t}=(1-\gamma)\left((1-\gamma)+\gamma S_{t}^{1-\eta}\right)^{\frac{\eta}{1-\eta}}C_{t}$$
$$+\gamma^{*}\left(\left(1-\gamma^{*}\right)S_{t}^{1-\eta}+\gamma^{*}\right)^{\frac{\eta}{1-\eta}}C_{t}^{*}+G_{t}$$

$$\frac{\left(1-\theta\Pi_{Ht}^{\varepsilon-1}\right)}{(1-\theta)}=\left(\frac{H_t}{F_t}\right)^{1-\varepsilon} \tag{33}$$

$$H_t=\mu v_t U_{C,t}\frac{P_{Ht}}{P_t}Y_t\frac{W_t}{P_{Ht}AZ_t}+\theta\beta\mathbb{E}_t\Pi_{Ht+1}^{\varepsilon}H_{t+1} \tag{34}$$

$$F_t=U_{C,t}\frac{P_{Ht}}{P_t}Y_t+\theta\beta\mathbb{E}_t\Pi_{Ht+1}^{\varepsilon-1}F_{t+1} \tag{35}$$

$$\Delta_t=(1-\theta)\left(\frac{1-\theta\Pi_{Ht}^{\epsilon-1}}{1-\theta}\right)^{\frac{\epsilon}{\epsilon-1}}+\theta\Pi_{Ht}^{\epsilon}\Delta_{t-1} \tag{36}$$

$$Y_t=N_t\frac{AZ_t}{\Delta_t} \tag{37}$$

$$\begin{aligned}P_{Ht}^{M}\left(B_{Ht}+B_{Ht}^{*}\right)=&\left(1+\rho_H P_{Ht}^{M}\right)\frac{\left(B_{Ht-1}+B_{Ht-1}^{*}\right)}{1+\pi_{Ht}}\\&+G_t-\tau_t^{l}\frac{W_t}{P_{Ht}}N_t\end{aligned} \tag{38}$$

$$\frac{W_t^*}{P_{Ft}^*}\frac{P_{Ht}^*}{P_t^*}\frac{P_{Ft}^*}{P_{Ht}^*}=-\frac{U_n\left(c_t^*,n_t^*\right)}{U_c\left(c_t^*,n_t^*\right)\left(1-\tau_t^{*w}\right)} \tag{39}$$

$$\begin{aligned}\frac{1}{\left(1+i_t^*\right)}=&\beta\mathbb{E}_t\frac{U_c\left(c_{t+1}^*,n_{t+1}^*\right)}{U_c\left(c_t^*,n_t^*\right)}\left(\frac{\left(1-\gamma^*\right)S_t^{1-\eta}+\gamma^*}{\left(1-\gamma^*\right)S_{t+1}^{1-\eta}+\gamma^*}\right)^{\frac{1}{1-\eta}}\\&\times\frac{S_{t+1}}{S_t}\frac{1}{1+\pi_{Ft+1}^*}\end{aligned} \tag{40}$$

$$\begin{aligned}P_{Ft}^{M}=&\beta\mathbb{E}_t\frac{U_c\left(c_{t+1}^*,n_{t+1}^*\right)}{U_c\left(c_t^*,n_t^*\right)}\left(\frac{\left(1-\gamma^*\right)S_t^{1-\eta}+\gamma^*}{\left(1-\gamma^*\right)S_{t+1}^{1-\eta}+\gamma^*}\right)^{\frac{1}{1-\eta}}\\&\times\frac{S_{t+1}}{S_t}\frac{1}{1+\pi_{Ft+1}^*}\left(1+\rho_F P_{Ft+1}^{M}\right)\end{aligned} \tag{41}$$

$$\begin{aligned}P_{Ht}^{M}=&\beta\mathbb{E}_t\frac{U_c\left(c_{t+1}^*,n_{t+1}^*\right)}{U_c\left(c_t^*,n_t^*\right)}\left(\frac{\left(1-\gamma^*\right)S_t^{1-\eta}+\gamma^*}{\left(1-\gamma^*\right)S_{t+1}^{1-\eta}+\gamma^*}\right)^{\frac{1}{1-\eta}}\frac{1}{1+\pi_{Ht+1}}\\&\times\left(1+\rho_H P_{Ht+1}^{M}\right)\phi^*\left(P_{Ht}^{M}B_{Ht}^*\left(\left(1-\gamma^*\right)S_t^{1-\eta}+\gamma^*\right)^{-\frac{1}{1-\eta}}\right)\end{aligned} \tag{42}$$

$$Y_t^* = (1-\gamma^*)\, S_t^{-\eta} \left((1-\gamma^*)\, S_t^{1-\eta} + \gamma^* \right)^{\frac{\eta}{1-\eta}} C_t^* \\ + \gamma S_t^{-\eta} \left(1 - \gamma + \gamma S_t^{1-\eta} \right)^{\frac{\eta}{1-\eta}} C_t + G_t^* \tag{43}$$

$$\frac{1-\theta \Pi_{Ft}^{*\varepsilon-1}}{1-\theta} = \left(\frac{H_t^*}{F_t^*} \right)^{1-\varepsilon} \tag{44}$$

$$H_t^* = \mu \upsilon_t^* U_{C,t}^* \frac{P_{Ft}^*}{P_t^*} Y_t^* \frac{W_t^*}{P_{Ft}^* A^* Z_t^*} + \theta \beta \mathbb{E}_t \Pi_{Ft+1}^{*\varepsilon} H_{t+1}^* \tag{45}$$

$$F_t^* = U_{C,t}^* \frac{P_{Ft}^*}{P_t^*} Y_t^* + \theta \beta \mathbb{E}_t \Pi_{Ft+1}^{*\varepsilon-1} F_{t+1}^*, \tag{46}$$

$$\Delta_t^* = (1-\theta) \left(\frac{1-\theta \Pi_{Ft}^{*\epsilon-1}}{1-\theta} \right)^{\frac{\epsilon}{\epsilon-1}} + \theta \Pi_{Ft}^{*\epsilon} \Delta_{t-1}^* \tag{47}$$

$$Y_t^* = N_t^* \frac{A^* Z_t^*}{\Delta_t^*} \tag{48}$$

$$P_{Ft}^M \left(B_{Ft} + B_{Ft}^* \right) = \left(1 + \rho_F P_{Ft}^M \right) \frac{\left(B_{Ft-1} + B_{Ft-1}^* \right)}{1 + \pi_{Ft}^*} \\ + G_t^* - \tau_t^{l*} \frac{W_t^*}{P_{Ft}^*} N_t^* \tag{49}$$

$$0 = \gamma S_t^{1-\eta} \left((1-\gamma) + \gamma S_t^{1-\eta} \right)^{\frac{\eta}{1-\eta}} C_t - \gamma^* \left((1-\gamma^*)\, S_t^{1-\eta} + \gamma^* \right)^{\frac{\eta}{1-\eta}} C_t^* \\ + \left(P_{Ft}^M B_{Ft} - \left(1 + \rho_F P_{Ft}^M \right) \frac{B_{Ft-1}}{1+\pi_{Ft}^*} \right) S_t \\ - \left(P_{Ht}^M B_{Ht}^* - \left(1 + \rho_H P_{Ht}^M \right) \frac{B_{Ht-1}^*}{1+\pi_{Ht}} \right) \tag{50}$$

$$S_t = \frac{E_t \; P_{Ft}^*}{P_{Ht}} \tag{51}$$

$$Q_t = \frac{E_t \; P_t^*}{P_t} = \frac{\left((1-\gamma^*)\, S_t^{1-\eta} + \gamma^* \right)^{\frac{1}{1-\eta}}}{\left((1-\gamma) + \gamma S_t^{1-\eta} \right)^{\frac{1}{1-\eta}}} \tag{52}$$

$$D_t = \frac{P_t}{E_t P_t^*} \frac{c_t^{*-\frac{1}{\sigma}} \xi_t^{*-\frac{1}{\sigma}}}{c_t^{-\frac{1}{\sigma}} \xi_t^{-\frac{1}{\sigma}}} = \frac{1}{Q_t} \frac{c_t^{*-\frac{1}{\sigma}} \xi_t^{*-\frac{1}{\sigma}}}{c_t^{-\frac{1}{\sigma}} \xi_t^{-\frac{1}{\sigma}}}, \tag{53}$$

where we assumed steady-state lump-sum employment subsidy financed by labor taxes

$$\tau_t^w = \tau_t^l - \tau^s$$
$$\tau_t^{*w} = \tau_t^{*l} - \tau^{*s}.$$

The system of twenty-six equations determines twenty-six variables: $C_t, N_t, W_t, Y_t, H_t, F_t, \Delta_t, \pi_{Ht}, P_{Ht}^M, B_{Ht}, B_{Ft}, C_t^*, N_t^*, W_t^*, Y_t^*, H_t^*, F_t^*, \Delta_t^*, \pi_{Ft}^*, P_{Ft}^M$, $B_{Ht}^*, B_{Ft}^*, E_t, S_t, Q_t$, and D_t.

Policy instruments are $i_t, i_t^*, G_t, G_t^*, \tau_t^w, \tau_t^{w*}$ and it remains to describe policy.

A.9 Parameterization

Individual utility and its derivatives are given by

$$U(c_t, g_t, n_t) = \frac{c_t^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} \xi_t^{-\frac{1}{\sigma}} + \varpi \frac{g_t^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} \xi_t^{-\frac{1}{\sigma}} - d \frac{n_t^{1+\varsigma}}{1+\varsigma} \delta_t^{-\frac{1}{\sigma}}$$
$$U_c(c_t, g_t, n_t) = c_t^{-\frac{1}{\sigma}} \xi_t^{-\frac{1}{\sigma}}$$
$$U_n(c_t, g_t, n_t) = -d n_t^{\varsigma} \delta_t^{-\frac{1}{\sigma}}$$
$$U(c_t^*, g_t^*, n_t^*) = \frac{c_t^{*1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} \xi_t^{*-\frac{1}{\sigma}} + \varpi \frac{g_t^{*1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} \xi_t^{*-\frac{1}{\sigma}} - d \frac{n_t^{*1+\varsigma}}{1+\varsigma} \delta_t^{*-\frac{1}{\sigma}}$$
$$U_c(c_t^*, g_t^*, n_t^*) = c_t^{*-\frac{1}{\sigma}} \xi_t^{*-\frac{1}{\sigma}}$$
$$U_n(c_t^*, g_t^*, n_t^*) = -d n_t^{*\varsigma} \delta_t^{*-\frac{1}{\sigma}}.$$

A.10 Steady State and Calibration

The fiscal side of the model is characterized by government-spending-to-output ratios $\Theta_H = G_H/Y$ and $\Theta_F = G_F/Y$, labor income taxes τ^l and τ^{*l}, maturity of debt m_H and m_F, *annualized* steady-state debt-to-output ratios $\delta_H = \beta m_H \frac{(B_H + B_H^*)}{4Y}$ and

$\delta_F = \beta m_F \frac{(B_F+B_F^*)}{4Y^*}$, the share of home-issued debt held by non-residents $\varrho = \frac{B_H^*}{B_H+B_H^*}$, and the share of home-held foreign debt to foreign-held home debt $\omega = \frac{m_H B_F S}{m_F B_H^*}$. Here B_H are home-issued bonds held by home residents, B_H^* are home-issued bonds held by foreign residents, B_F^* are foreign-issued bonds held by foreign residents, and B_F are foreign-issued bonds held by home residents.

The share of government spending to GDP, Θ_H and Θ_F, is set to 0.20 for each country, in line with the EMU data. In our benchmark calibration we assume that the small home country has a total-debt-to-annual-output ratio of $\delta_H = 1.10$, which is consistent with the employment-weighted average debt level in Portugal, Greece, Ireland, Italy, and Spain. The large foreign country has $\delta_H = 0.6$, consistent with the debt level in the rest of the EMU. The currently observed domestic debt levels are treated as steady-state values rather than initial condition, partly because the current projections (see International Monetary Fund Fiscal Monitor data) suggest that this level of government debt is expected to persist for at least a decade, thus making these values an (implicit) target of policy authorities, as it is expected that all variables are to return to these (steady-state) values in the long run.

As a significant proportion of the government debt is held by non-residents (see tables C2 and C3 in appendix C), we set the benchmark value of the small home-country government debt held by non-residents $\varrho = 0.5$. The IMF survey data suggest that the imbalances in long-term debt holdings imply $\omega = 0.5$, so that the small home country is a net debtor. Finally, we calibrate the adjustment cost parameter $\chi = \chi^* = 0.01$ following Benigno (2009).

Calibration of parameters δ_H, δ_F, m_H, m_F, Θ_H, Θ_F, ϱ, and ω yields the steady-state tax level needed to service debt

$$\frac{\tau^l}{\mu} = \Theta_H + 4\frac{(1-\beta)}{\beta}\delta_H, \qquad \frac{\tau^{*l}}{\mu} = \Theta_F + 4\frac{(1-\beta)}{\beta}\delta_F,$$

and steady-state values of all debt components

$$\frac{B_H}{Y} = 4\,(1-\varrho)\,\frac{\delta_H}{\beta m_H}, \quad \frac{B_H^*}{Y} = 4\varrho\frac{\delta_H}{\beta m_H},$$
$$\frac{B_F}{Y^*} = 4\omega\varrho\frac{Y}{Y^*}\frac{\delta_F}{\beta m_F}, \quad \frac{B_F^*}{Y^*} = 4\left(1-\omega\varrho\frac{Y}{Y^*}\right)\frac{\delta_F}{\beta m_F}.$$

The other variables can be determined from

$$1 = (1-\gamma)\,\Upsilon^{\eta}\frac{c}{y} + \gamma\Gamma^{\eta}\frac{c^*}{y^*}\frac{y^*}{y} + \frac{g}{y}$$
$$1 = (1-\gamma^*)\,S^{-\eta}\Gamma^{\eta}\frac{c^*}{y^*} + \gamma^* S^{-\eta}\Upsilon^{\eta}\frac{c}{y}\frac{y}{y^*} + \frac{g^*}{y^*}$$
$$\frac{c^{-\frac{1}{\sigma}}}{\Upsilon} = dy^{\varsigma}\left(\frac{\delta}{\xi}\right)^{-\frac{1}{\sigma}}$$
$$c^{*-\frac{1}{\sigma}} = \frac{\Gamma}{S}dy^{*\varsigma}\left(\frac{\delta^*}{\xi^*}\right)^{-\frac{1}{\sigma}}$$
$$0 = \gamma S^{1-\eta}\Upsilon^{\eta}\frac{c}{y} - \gamma\Gamma^{\eta}\frac{c^*}{y} + \frac{4\varrho\,(1-\beta)}{\beta}\left[\delta_H - \omega S\delta_F\right],$$

where

$$\Upsilon = \Upsilon(S) = \left((1-\gamma) + \gamma S^{1-\eta}\right)^{\frac{1}{1-\eta}} = \frac{P}{P_H}$$
$$\Gamma = \Gamma(S) = \left((1-\gamma^*)\,S^{1-\eta} + \gamma^*\right)^{\frac{1}{1-\eta}} = \frac{P^*}{P_H^*}$$
$$\Xi = \frac{\Gamma}{S}.$$

Following Benigno (2009), we assume steady-state employment subsidy such that

$$\frac{(1-\tau^{w})}{\mu} = \frac{(1-\tau^{*w})}{\mu} = 1$$

and require the steady-state ratio of taste levels to satisfy

$$\frac{\Gamma}{\Upsilon}\frac{c^{-\frac{1}{\sigma}}}{c^{*-\frac{1}{\sigma}}} = \left(\frac{\xi^*}{\xi}\right)^{-\frac{1}{\sigma}}.$$

The steady state becomes

$$\frac{c}{y} = \frac{S^{-\eta}\,(1-\gamma^*)\,(1-\Theta_H) - \frac{y^*}{y}\gamma\,(1-\Theta_F)}{((1-\gamma^*)\,(1-\gamma) - \gamma\gamma^*)\,S^{-\eta}\Upsilon^{\eta}}$$

$$\frac{c^*}{y^*} = \frac{1-(1-\gamma)\,\Theta_F - (1-\Theta_H)\,\gamma^* S^{-\eta}\frac{y}{y^*}}{S^{-\eta}\Gamma^{\eta}\left((1-\gamma)\,(1-\gamma^*) - \gamma\gamma^*\right)}$$

$$c^{-\frac{1}{\sigma}} = \Upsilon d y^{\varsigma}\left(\frac{\delta}{\xi}\right)^{-\frac{1}{\sigma}}$$

$$\left(\frac{\xi^*}{\xi}\right)^{-\frac{1}{\sigma}} = S\left(\frac{\delta}{\delta^*}\right)^{-\frac{1}{\sigma}}\left(\frac{y}{y^*}\right)^{\varsigma}$$

$$0 = \frac{\left((1-\Theta_H)\,\Gamma^{1-\eta} - (1-\Theta_F)\,\frac{\Upsilon^{1-\eta}}{S^{-\eta}}\frac{y^*}{y}\right)\gamma}{\left((1-\gamma)\,(1-\gamma^*) - \gamma\gamma^*\right)} + (1-\beta)\left[m_H\frac{B_H^*}{Y} - m_F\frac{B_F}{Y}S\right]$$

$$\left(\frac{c}{c^*}\right)^{-\frac{1}{\sigma}} = \frac{\Upsilon}{\Gamma}\left(\frac{\xi^*}{\xi}\right)^{-\frac{1}{\sigma}}.$$

The following are several important cases:

(i) Non-zero debt-to-output ratio $\frac{(B_H+B_H^*)}{Y} \neq 0$. $\frac{(B_F+B_F^*)}{Y^*} \neq 0$. Financial autarky in the steady state: $B_F = B_H^* = 0$:

$$\varrho = 0, \omega < 1.$$

Here NFA in the steady state is zero.

(ii) Non-zero debt-to-output ratio $\frac{(B_H+B_H^*)}{Y} \neq 0$, $\frac{(B_F+B_F^*)}{Y^*} \neq 0$. Zero NFA in the steady state but no financial autarky, $B_F \neq 0$, $B_H^* \neq 0$:

$$\omega = 1, \varrho > 0.$$

(iii) Zero debt-to-output ratio in country H: $0 = \left(\frac{\tau^l}{\mu} - \Theta_H\right) = \frac{(B_H+B_H^*)}{Y}$, $\frac{(B_F+B_F^*)}{Y^*} \neq 0$:

$$\omega\varrho = 0.$$

It implies financial autarky in the steady state: $B_F = B_H^* = 0$.

(iv) Zero debt-to-output ratio in each country: $0 = \left(\frac{\tau^l}{\mu} - \Theta_H\right) = \left(\frac{\tau^{*l}}{\mu} - \Theta_F\right) = \frac{(B_H + B_H^*)}{Y} = \frac{(B_F + B_F^*)}{Y^*}$: ω and ϱ are irrelevant for the calibration of the model. These parameters do not affect equations. This case implies financial autarky with NFA $= 0$ in the steady state.

A.11 Linearized Model

In order to tractably solve the model in the presence of potential strategic interactions between the monetary and fiscal policymakers, we recast the policy problem in a linear-quadratic (LQ) form. In doing so we employ the device of a steady-state employment subsidy which ensures that the deterministic steady-state is efficient. We design this subsidy by contrasting the social planner's allocation with what would be achieved in our decentralized economy in the absence of nominal inertia and with perfect risk sharing, and then choosing a steady-state employment subsidy which equates the two. This ensures that the steady state of our economy is equivalent to the steady state of the flexible-price economy with risk sharing which, in turn, is efficient. This allows us to generate a valid LQ approximation to the underlying policy problem across all the types of policy we consider.[3]

Log-linearization of the system about the deterministic steady state, assuming the fixed exchange rate regime, yields

$$\hat{C}_t = \mathbb{E}_t \hat{C}_{t+1} + \ \sigma\gamma\left(\pi^*_{Ft+1} - \pi_{Ht+1}\right) - \sigma\hat{r}_t \tag{54}$$

$$\hat{C}^*_t = \mathbb{E}_t \hat{C}^*_{t+1} - \sigma\gamma^*\left(\pi^*_{Ft+1} - \pi_{Ht+1}\right) - \sigma\hat{r}^*_t \tag{55}$$

$$\hat{r}_t = \mathbb{E}_t \hat{R}_{t+1} \tag{56}$$

$$\hat{r}^*_t = \mathbb{E}_t \hat{R}^*_{t+1} \tag{57}$$

[3] If we did not adopt an efficient steady state, then the second-order approximation to social welfare would include linear terms which would prevent us calculating a second-order approximation to welfare using a linearized model and would also introduce an inflationary bias to our policy problem.

$$\pi_{Ht} = \lambda \left(\hat{\upsilon}_t + \gamma \hat{S}_t + \varsigma \left(\hat{Y}_t - \hat{Z}_t \right) + \frac{1}{\sigma} \hat{C}_t + \frac{\tau^w}{\mu} \hat{\tau}_t^w - \hat{Z}_t \right) + \beta \mathbb{E}_t \pi_{Ht+1} \tag{58}$$

$$\pi_{Ft}^* = \lambda \left(\hat{\upsilon}_t^* - \gamma^* \hat{S}_t + \varsigma \left(\hat{Y}_t^* - \hat{Z}_t^* \right) + \frac{1}{\sigma} \hat{C}_t^* + \frac{\tau^{*w}}{\mu} \hat{\tau}_t^{*w} - \hat{Z}_t^* \right) + \beta \mathbb{E}_t \pi_{Ft+1}^* \tag{59}$$

$$\hat{Y}_t = \eta \left(\frac{C}{Y} (1-\gamma) \gamma + \frac{C^*}{Y} \gamma^* (1-\gamma^*) \right) \hat{S}_t + (1-\gamma) \frac{C}{Y} \hat{C}_t + \gamma^* \frac{C^*}{Y} \hat{C}_t^* + \frac{G}{Y} \hat{G}_t \tag{60}$$

$$\hat{Y}_t^* = -\frac{Y}{Y^*} \eta \left(\frac{C}{Y} (1-\gamma) \gamma + \frac{C^*}{Y} \gamma^* (1-\gamma^*) \right) \hat{S}_t + (1-\gamma^*) \frac{C^*}{Y^*} \hat{C}_t^* + \gamma \frac{C}{Y^*} \hat{C}_t + \frac{G^*}{Y^*} \hat{G}_t^* \tag{61}$$

$$P_H^M \frac{B_H}{Y} d_{Ht} + \frac{Y^*}{Y} P_H^M \frac{B_H^*}{Y^*} d_{Ht}^* = \frac{1}{\beta} \hat{R}_t \frac{P_H^M B_H + P_H^M B_H^*}{Y} + \frac{1}{\beta} P_H^M \frac{B_H}{Y} d_{Ht-1} + \frac{G}{Y} \hat{G}_t + \frac{1}{\beta} P_H^M \frac{B_H^*}{Y} d_{Ht-1}^* - \frac{N}{Y} w \tau^l \left(\hat{\tau}_t^l + \gamma \hat{S}_t + (\varsigma + 1) \left(\hat{Y}_t - \hat{Z}_t \right) + \frac{1}{\sigma} \hat{C}_t + \frac{\tau^w}{\mu} \hat{\tau}_t^w \right) \tag{62}$$

$$P_F^M \frac{B_F}{Y^*} d_{Ft} + P_F^M \frac{B_F^*}{Y^*} d_{Ft}^* = \frac{1}{\beta} \hat{R}_t^* \frac{P_F^M B_F + P_F^M B_F^*}{Y^*} + \frac{1}{\beta} \frac{Y}{Y^*} P_F^M \frac{B_F}{Y} d_{Ft-1} + \frac{G^*}{Y^*} \hat{G}_t^* + \frac{1}{\beta} P_F^M \frac{B_F^*}{Y^*} d_{Ft-1}^* - \frac{N^*}{Y^*} \tau^{l*} w^* \times \left(\hat{\tau}_t^{l*} - \gamma^* \hat{S}_t + (\varsigma + 1) \left(\hat{Y}_t^* - \hat{Z}_t^* \right) + \frac{1}{\sigma} \hat{C}_t^* + \frac{\tau^{*w}}{\mu} \hat{\tau}_t^{*w} \right) \tag{63}$$

$$\chi^* P_H^M B_H^* \left(d_{Ht}^* - (1-\gamma^*) \hat{S}_t \right) = -\chi P_F^M B_F \left(d_{Ft} + (1-\gamma) \hat{S}_t \right) \tag{64}$$

$$\hat{r}_t - \hat{r}_t^* = \mathbb{E}_t \left(\pi_{Ft+1}^* - \pi_{Ht+1} \right) - \chi^* (1-\gamma^*) \varkappa^* \hat{S}_t + \chi^* P_H^M B_H^* d_{Ht}^* \tag{65}$$

$$0 = \left(\gamma C\left((1-\eta)(1-\gamma)+\gamma S^{1-\eta}\right) - C^{*}\eta\gamma^{*}(1-\gamma^{*})\right)\hat{S}_t$$
$$+\gamma C\hat{C}_t - \gamma^{*}C^{*}\hat{C}_t^{*} + P_F^M B_F\left(d_{Ft}+\hat{S}_t-\frac{1}{\beta}\left(\hat{R}_t^{*}+d_{Ft-1}+\hat{S}_t\right)\right)$$
$$-P_H^M B_H^{*}\left(d_{Ht}^{*}-\frac{1}{\beta}\left(\hat{R}_t+d_{Ht-1}^{*}\right)\right) \tag{66}$$

$$\hat{\imath}_t = \hat{r}_t + \mathbb{E}_t\pi_{Ht+1} \tag{67}$$
$$\hat{\imath}_t^{*} = \hat{r}_t^{*} + \mathbb{E}_t\pi_{Ft+1}^{*} \tag{68}$$
$$\hat{S}_t - \hat{S}_{t-1} + \pi_{Ht} - \pi_{Ft}^{*} = 0. \tag{69}$$

Here and everywhere below, unless stated differently, for each variable X_t with non-zero steady-state value X, we use the notation $\hat{X}_t = \ln(X_t/X)$. Here $\tilde{B}_t = \frac{B}{Y}\hat{B}_t$, $\hat{\imath}_t = \ln\frac{1+i_t}{1+i}$, and $\pi_t = \Pi_t - 1$. Parameter $\lambda = \frac{(1-\theta)(1-\theta\beta)}{\theta}$ is the slope of the Phillips curve. Finally, $\hat{P}_{Ht}^M$, $\hat{P}_{Ft}^M$, $\hat{B}_{Ht}$, $\hat{B}_{Ht}^{*}$, $\hat{P}_{Ft}^M$, $\hat{B}_{Ft}^{*}$ can be obtained from

$$\hat{R}_{t+1} = \beta\rho_H\hat{P}_{Ht+1}^M - \hat{P}_{Ht}^M - \pi_{Ht+1}$$
$$\hat{R}_{t+1}^{*} = \beta\rho_F\hat{P}_{Ft+1}^M - \hat{P}_{Ft}^M - \pi_{Ft+1}^{*}$$
$$d_{Ht} = \hat{B}_{Ht} + \hat{P}_{Ht}^M$$
$$d_{Ht}^{*} = \hat{B}_{Ht}^{*} + \hat{P}_{Ht}^M$$
$$d_{Ft} = \hat{P}_{Ft}^M + \hat{B}_{Ft}$$
$$d_{Ft}^{*} = \hat{B}_{Ft}^{*} + \hat{P}_{Ft}^M.$$

Without loss of generality, we can assume that the central bank controls $\hat{\imath}_t$, and then the portfolio adjustment cost implies that households face different interest rates in two economies with incomplete financial markets. There are also two independent fiscal authorities in countries H and F. Each of them controls government spending ($\hat{G}_t$ and $\hat{G}_t^{*}$) and the income tax rate ($\hat{\tau}_t^l$ and $\hat{\tau}_t^{*l}$).

Appendix B. Quadratic Representation of Social Welfare

To derive the welfare objective for policy analysis, we complete the following steps. First, we consider the social planner's problem.

We then determine the level of the steady-state subsidy required to ensure that the model's initial steady state is socially optimal. Finally, we construct a quadratic approximation to utility in the economy with sticky prices and distortionary taxes. The representation of utility assesses the extent to which the economic variables deviate from the efficient equilibrium levels due to price stickiness and tax distortions present in our model.

B.1 Welfare Components

Worldwide social welfare is written as

$$W = \sum_{t=0}^{\infty} \beta^t \left(\begin{array}{c} n\left(\frac{c_t^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} \xi_t^{-\frac{1}{\sigma}} + \frac{\varpi}{1-\frac{1}{\sigma}} g_t^{1-\frac{1}{\sigma}} \xi_t^{-\frac{1}{\sigma}} - d\frac{n_t^{1+\varsigma}}{1+\varsigma} \delta_t^{-\frac{1}{\sigma}} \right) \\ + (1-n)\left(\frac{c_t^{*1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} \xi_t^{*-\frac{1}{\sigma}} + \varpi \frac{g_t^{*1-\sigma}}{1-\sigma} \xi_t^{*-\frac{1}{\sigma}} - d\frac{n_t^{*1+\varsigma}}{1+\varsigma} \delta_t^{*-\frac{1}{\sigma}} \right) \end{array} \right).$$

Use aggregate variables

$$W = \sum_{t=0}^{\infty} \beta^t \left(\begin{array}{c} n\left(\frac{n^{\frac{1}{\sigma}-1} C_t^{1-\frac{1}{\sigma}} \xi_t^{-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} + \frac{\varpi n^{\frac{1}{\sigma}-1}}{1-\frac{1}{\sigma}} G_t^{1-\frac{1}{\sigma}} \xi_t^{-\frac{1}{\sigma}} - dn^{-(1+\varsigma)} \frac{N_t^{1+\varsigma}}{1+\varsigma} \delta_t^{-\frac{1}{\sigma}} \right) \\ + (1-n)\left(\frac{(1-n)^{\frac{1}{\sigma}-1} C_t^{*1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} \xi_t^{*-\frac{1}{\sigma}} + \frac{\varpi (1-n)^{\frac{1}{\sigma}-1}}{1-\frac{1}{\sigma}} G_t^{*1-\frac{1}{\sigma}} \xi_t^{*-\frac{1}{\sigma}} \right. \\ \left. -d(1-n)^{-(1+\varsigma)} \frac{N_t^{*1+\varsigma}}{1+\varsigma} \delta_t^{*-\frac{1}{\sigma}} \right) \end{array} \right)$$

and substitute out

$$N_t = \frac{Y_t \Delta_t}{AZ_t}$$
$$N_t^* = \frac{Y_t^* \Delta_t^*}{A^* Z_t^*}.$$

Linearization yields

$$\left(\frac{Y_t \Delta_t}{AZ_t} \right)^{1+\varsigma} \delta_t^{-\frac{1}{\sigma}} = \frac{1}{A^{1+\varsigma}} Y_t^{1+\varsigma} Z_t^{-(1+\varsigma)} \Delta_t^{1+\varsigma}$$
$$= \left(\frac{Y}{A} \right)^{1+\varsigma} \delta^{-\frac{1}{\sigma}} \left(\begin{array}{c} 1 + (1+\varsigma)\hat{\Delta}_t + (1+\varsigma)\hat{Y}_t - (1+\varsigma)^2 \hat{Y}_t \hat{Z}_t \\ +\frac{1}{2}(1+\varsigma)^2 \hat{Y}_t^2 - \frac{1}{\sigma}(1+\varsigma)\hat{Y}_t \hat{\delta}_t \end{array} \right) + tip$$

so that

$$W = \sum_{t=0}^{\infty} \beta^t \times \left(\begin{array}{c} \frac{n^{\frac{1}{\sigma}} C^{1-\frac{1}{\sigma}} \xi^{-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} \left(\begin{array}{c} 1 + \left(1-\frac{1}{\sigma}\right) \hat{C}_t - \left(1-\frac{1}{\sigma}\right) \frac{1}{\sigma} \hat{C}_t \hat{\xi}_t \\ + \frac{1}{2} \left(1-\frac{1}{\sigma}\right)^2 \hat{C}_t^2 \end{array} \right) \\ + \varpi \frac{n^{\frac{1}{\sigma}} G^{1-\frac{1}{\sigma}} \xi^{-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} \left(\begin{array}{c} 1 + \left(1-\frac{1}{\sigma}\right) \hat{G}_t - \frac{1}{\sigma} \left(1-\frac{1}{\sigma}\right) \hat{G}_t \hat{\xi}_t \\ + \frac{1}{2} \left(1-\frac{1}{\sigma}\right)^2 \hat{G}_t^2 \end{array} \right) \\ - \frac{d n^{-\varsigma}}{1+\varsigma} \left(\frac{Y}{A}\right)^{1+\varsigma} \delta^{-\frac{1}{\sigma}} \left(\begin{array}{c} 1 + (1+\varsigma) \hat{\Delta}_t + (1+\varsigma) \hat{Y}_t \\ - (1+\varsigma)^2 \hat{Y}_t \hat{Z}_t + \frac{1}{2} (1+\varsigma)^2 \hat{Y}_t^2 \\ - \frac{1}{\sigma} (1+\varsigma) \hat{Y}_t \hat{\delta}_t \end{array} \right) \\ + \frac{(1-n)^{\frac{1}{\sigma}} C^{*1-\frac{1}{\sigma}} \xi^{*-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} \left(\begin{array}{c} 1 + \left(1-\frac{1}{\sigma}\right) \hat{C}_t^* + \frac{1}{2} \left(1-\frac{1}{\sigma}\right)^2 \hat{C}_t^{*2} \\ - \left(1-\frac{1}{\sigma}\right) \frac{1}{\sigma} \hat{C}_t^* \hat{\xi}_t^* \end{array} \right) \\ + \varpi^* \frac{(1-n)^{\frac{1}{\sigma}} G^{*1-\sigma} \xi^{*-\frac{1}{\sigma}}}{1-\sigma} \left(\begin{array}{c} 1 + \left(1-\frac{1}{\sigma}\right) \hat{G}_t^* + \frac{1}{2} \left(1-\frac{1}{\sigma}\right)^2 \hat{G}_t^{*2} \\ - \frac{1}{\sigma} \left(1-\frac{1}{\sigma}\right) \hat{G}_t^* \hat{\xi}_t^* \end{array} \right) \\ - \frac{d(1-n)^{-\varsigma}}{1+\varsigma} \left(\frac{Y^*}{A^*}\right)^{1+\varsigma} \delta^{*-\frac{1}{\sigma}} \left(\begin{array}{c} 1 + (1+\varsigma) \hat{\Delta}_t^* + (1+\varsigma) \hat{Y}_t^* \\ - (1+\varsigma)^2 \hat{Y}_t^* \hat{Z}_t^* + \frac{1}{2} (1+\varsigma)^2 \hat{Y}_t^{*2} \\ - \frac{1}{\sigma} (1+\varsigma) \hat{Y}_t^* \hat{\delta}_t^* \end{array} \right) \end{array} \right)$$

$$= \sum_{t=0}^{\infty} \beta^t \left(WQ_t + WL_t + WP_t \right),$$

where

$$\begin{aligned} WL_t = {} & n^{\frac{1}{\sigma}} \xi^{-\frac{1}{\sigma}} \left(C^{1-\frac{1}{\sigma}} \hat{C}_t + \varpi G^{1-\frac{1}{\sigma}} \hat{G}_t \right) - d n^{-\varsigma} \left(\frac{Y}{A}\right)^{1+\varsigma} \delta^{-\frac{1}{\sigma}} \hat{Y}_t \\ & + (1-n)^{\frac{1}{\sigma}} \xi^{*-\frac{1}{\sigma}} \left(C^{*1-\frac{1}{\sigma}} \hat{C}_t^* + \varpi^* G^{*1-\sigma} \hat{G}_t^* \right) \\ & - d (1-n)^{-\varsigma} \left(\frac{Y^*}{A^*}\right)^{1+\varsigma} \delta^{*-\frac{1}{\sigma}} \hat{Y}_t^* \end{aligned}$$

$$\begin{aligned} WQ_t = {} & n^{\frac{1}{\sigma}} C^{1-\frac{1}{\sigma}} \xi^{-\frac{1}{\sigma}} \left(\frac{1}{2} \left(1-\frac{1}{\sigma}\right) \hat{C}_t^2 - \frac{1}{\sigma} \hat{C}_t \hat{\xi}_t \right) \\ & + \varpi n^{\frac{1}{\sigma}} G^{1-\frac{1}{\sigma}} \xi^{-\frac{1}{\sigma}} \left(\frac{1}{2} \left(1-\frac{1}{\sigma}\right) \hat{G}_t^2 - \frac{1}{\sigma} \hat{G}_t \hat{\xi}_t \right) \\ & - d n^{-\varsigma} \left(\frac{Y}{A}\right)^{1+\varsigma} \delta^{-\frac{1}{\sigma}} \left(- (1+\varsigma) \hat{Y}_t \hat{Z}_t + \frac{1}{2} (1+\varsigma) \hat{Y}_t^2 - \frac{1}{\sigma} \hat{Y}_t \hat{\delta}_t \right) \\ & + (1-n)^{\frac{1}{\sigma}} C^{*1-\frac{1}{\sigma}} \xi^{*-\frac{1}{\sigma}} \left(\frac{1}{2} \left(1-\frac{1}{\sigma}\right) \hat{C}_t^{*2} - \frac{1}{\sigma} \hat{C}_t^* \hat{\xi}_t^* \right) \\ & + \varpi (1-n)^{\frac{1}{\sigma}} G^{*1-\sigma} \xi^{*-\frac{1}{\sigma}} \left(\frac{1}{2} \left(1-\frac{1}{\sigma}\right) \hat{G}_t^{*2} - \frac{1}{\sigma} \hat{G}_t^* \hat{\xi}_t^* \right) \\ & - d (1-n)^{-\varsigma} \left(\frac{Y^*}{A^*}\right)^{1+\varsigma} \delta^{*-\frac{1}{\sigma}} \Big(- (1+\varsigma) \hat{Y}_t^* \hat{Z}_t^* \\ & + \frac{1}{2} (1+\varsigma) \hat{Y}_t^{*2} - \frac{1}{\sigma} \hat{Y}_t^* \hat{\delta}_t^* \Big) \end{aligned}$$

$$WP_t = -dn^{-\varsigma}\left(\frac{Y}{A}\right)^{1+\varsigma}\xi^{-\frac{1}{\sigma}}\hat{\Delta}_t - d\left(1-n\right)^{-\varsigma}\left(\frac{Y^*}{A^*}\right)^{1+\varsigma}\xi^{*-\frac{1}{\sigma}}\hat{\Delta}_t^*$$

are linear and quadratic components of welfare. We separate price-dispersion components.

Now, the aim is to transform WL_t using second-order approximation of the system of constraints.

We also use second-order approximation to the two market clearing conditions,

$$\begin{aligned} Y_t &= (1-\gamma)\left((1-\gamma)+\gamma S_t^{1-\eta}\right)^{\frac{\eta}{1-\eta}} C_t \\ &\quad + \gamma^*\left((1-\gamma^*) S_t^{1-\eta}+\gamma^*\right)^{\frac{\eta}{1-\eta}} C_t^* + G_t \\ Y_t^* &= (1-\gamma^*) S_t^{-\eta}\left((1-\gamma^*) S_t^{1-\eta}+\gamma^*\right)^{\frac{\eta}{1-\eta}} C_t^* \\ &\quad + \gamma S_t^{-\eta}\left((1-\gamma)+\gamma S_t^{1-\eta}\right)^{\frac{\eta}{1-\eta}} C_t + G_t^*, \end{aligned}$$

which read

$$\begin{aligned} \hat{Y}_t &= HL_t + HQ_t \\ \hat{Y}_t^* &= FL_t + FQ_t, \end{aligned}$$

where

$$\begin{aligned} HL_t &= (1-\gamma)\frac{C}{Y}\Upsilon^\eta\left(\hat{C}_t + \eta\gamma\left(\frac{S}{\Upsilon}\right)^{1-\eta}\hat{S}_t\right) \\ &\quad + \gamma^* S^\eta \Xi^\eta \frac{C^*}{Y^*}\frac{Y^*}{Y}\left(\hat{C}_t^* + \frac{\eta(1-\gamma^*)}{\Xi^{1-\eta}}\hat{S}_t\right) + \frac{G}{Y}\hat{G}_t \\ HQ_t &= (1-\gamma)\frac{C}{Y}\Upsilon^\eta\left(\frac{1}{2}\hat{C}_t^2 + \eta\gamma\left(\frac{S}{\Upsilon}\right)^{1-\eta}\hat{S}_t\hat{C}_t\right. \\ &\quad \left. + \frac{\eta\gamma}{2}\left(\frac{(1-\eta)(1-\gamma)}{\Upsilon^{1-\eta}} + \eta\gamma\left(\frac{S}{\Upsilon}\right)^{1-\eta}\right)\left(\frac{S}{\Upsilon}\right)^{1-\eta}\hat{S}_t^2\right) \\ &\quad + \gamma^* S^\eta \Xi^\eta \frac{C^*}{Y^*}\frac{Y^*}{Y}\left(\frac{1}{2}\hat{C}_t^{*2} + \frac{\eta(1-\gamma^*)}{\Xi^{1-\eta}}\hat{S}_t\hat{C}_t^*\right. \end{aligned}$$

$$+\ \frac{\eta}{2}\frac{(1-\gamma^*)}{\Xi^{1-\eta}}\left(\left(\frac{(1-\eta)\,\gamma^*}{(\Xi S)^{1-\eta}}+\frac{\eta\,(1-\gamma^*)}{\Xi^{1-\eta}}\right)\hat{S}_t^2\right)$$

$$+\ \frac{1}{2}\frac{G}{Y}\hat{G}_t^2-\frac{1}{2}\hat{Y}_t^2$$

$$FL_t=(1-\gamma^*)\,\Xi^\eta\frac{C^*}{Y^*}\left(\hat{C}_t^*-\eta\frac{\gamma^*}{(\Xi S)^{1-\eta}}\hat{S}_t\right)$$

$$+\,\gamma S^{-\eta}\Upsilon^\eta\frac{C}{Y}\frac{Y}{Y^*}\left(\hat{C}_t-\frac{\eta\,(1-\gamma)}{\Upsilon^{1-\eta}}\hat{S}_t\right)+\frac{G^*}{Y^*}\hat{G}_t^*$$

$$FQ_t=(1-\gamma^*)\,\Xi^\eta\frac{C^*}{Y^*}\left(\frac{1}{2}\hat{C}_t^{*2}-\eta\frac{\gamma^*}{(\Xi S)^{1-\eta}}\hat{S}_t\hat{C}_t^*\right.$$

$$\left.+\ \frac{\eta\gamma^*}{2}\left(\frac{(1-\eta)\,(1-\gamma^*)}{\Xi^{1-\eta}}+\frac{\eta\gamma^*}{(\Xi S)^{1-\eta}}\right)\frac{1}{(\Xi S)^{1-\eta}}\hat{S}_t^2\right)$$

$$+\,\gamma S^{-\eta}\Upsilon^\eta\frac{C}{Y}\frac{Y}{Y^*}\left(\frac{1}{2}\hat{C}_t^2-\frac{\eta\,(1-\gamma)}{\Upsilon^{1-\eta}}\hat{S}_t\hat{C}_t\right.$$

$$\left.+\ \frac{\eta}{2}\frac{(1-\gamma)}{\Upsilon^{1-\eta}}\left((1-\eta)\,\gamma\left(\frac{S}{\Upsilon}\right)^{1-\eta}+\frac{\eta\,(1-\gamma)}{\Upsilon^{1-\eta}}\right)\hat{S}_t^2\right)$$

$$+\ \frac{G^*}{Y^*}\frac{1}{2}\hat{G}_t^{*2}-\frac{1}{2}\hat{Y}_t^{*2}.$$

It follows that

$$WL_t=n^{\frac{1}{\sigma}}\xi^{-\frac{1}{\sigma}}\left(C^{1-\frac{1}{\sigma}}\hat{C}_t+\varpi G^{1-\frac{1}{\sigma}}\hat{G}_t\right)$$

$$-\,d\delta^{-\frac{1}{\sigma}}n^{-\varsigma}\left(\frac{Y}{A}\right)^{1+\varsigma}HL_t$$

$$+\,(1-n)^{\frac{1}{\sigma}}\,\xi^{*-\frac{1}{\sigma}}\left(C^{*1-\frac{1}{\sigma}}\hat{C}_t^*+\varpi^*G^{*1-\sigma}\hat{G}_t^*\right)$$

$$-\,d\delta^{*-\frac{1}{\sigma}}\,(1-n)^{-\varsigma}\left(\frac{Y^*}{A^*}\right)^{1+\varsigma}FL_t$$

$$-\,dn^{-\varsigma}\delta^{-\frac{1}{\sigma}}\left(\frac{Y}{A}\right)^{1+\varsigma}HQ_t-d\,(1-n)^{-\varsigma}\,\delta^{*-\frac{1}{\sigma}}\left(\frac{Y^*}{A^*}\right)^{1+\varsigma}FQ_t$$

$$= A_{H,t} + A_{F,t} - dn^{-\varsigma}\delta^{-\frac{1}{\sigma}}\left(\frac{Y}{A}\right)^{1+\varsigma} HQ_t$$
$$- d\left(1-n\right)^{-\varsigma}\delta^{*-\frac{1}{\sigma}}\left(\frac{Y^*}{A^*}\right)^{1+\varsigma} FQ_t,$$

where linear terms

$$A_{H,t} = n^{\frac{1}{\sigma}}\xi^{-\frac{1}{\sigma}}\left(C^{1-\frac{1}{\sigma}}\hat{C}_t + \varpi G^{1-\frac{1}{\sigma}}\hat{G}_t\right)$$
$$- d\delta^{-\frac{1}{\sigma}}n^{-\varsigma}\left(\frac{Y}{A}\right)^{1+\varsigma}\left(\left(1-\gamma\right)\frac{C}{Y}\Upsilon^{\eta}\left(\hat{C}_t + \eta\gamma\left(\frac{S}{\Upsilon}\right)^{1-\eta}\hat{S}_t\right)\right.$$
$$\left. + \gamma^* S^{\eta}\Xi^{\eta}\frac{C^*}{Y^*}\frac{Y^*}{Y}\left(\hat{C}^*_t + \frac{\eta\left(1-\gamma^*\right)}{\Xi^{1-\eta}}\hat{S}_t\right) + \frac{G}{Y}\hat{G}_t\right)$$
$$A_{F,t} = \left(1-n\right)^{\frac{1}{\sigma}}\xi^{*-\frac{1}{\sigma}}\left(C^{*1-\frac{1}{\sigma}}\hat{C}^*_t + \varpi^* G^{*1-\sigma}\hat{G}^*_t\right)$$
$$- d\delta^{*-\frac{1}{\sigma}}\left(1-n\right)^{-\varsigma}\left(\frac{Y^*}{A^*}\right)^{1+\varsigma}$$
$$\times\left(\left(1-\gamma^*\right)\Xi^{\eta}\frac{C^*}{Y^*}\left(\hat{C}^*_t - \eta\frac{\gamma^*}{\left(\Xi S\right)^{1-\eta}}\hat{S}_t\right)\right.$$
$$\left. + \gamma S^{-\eta}\Upsilon^{\eta}\frac{C}{Y}\frac{Y}{Y^*}\left(\hat{C}_t - \frac{\eta\left(1-\gamma\right)}{\Upsilon^{1-\eta}}\hat{S}_t\right) + \frac{G^*}{Y^*}\hat{G}^*_t\right)$$

and their sum

$$A_{H,t} + A_{F,t} = C\left(\xi^{-\frac{1}{\sigma}}c^{-\frac{1}{\sigma}} - \xi^{-\frac{1}{\sigma}}\frac{c^{-\frac{1}{\sigma}}\left(1-\tau^w\right)}{\mu}\left(1-\gamma\right)\Upsilon^{\eta-1}\right.$$
$$\left. - \xi^{*-\frac{1}{\sigma}}\frac{c^{*-\frac{1}{\sigma}}\left(1-\tau^{*w}_t\right)}{\mu}\gamma\frac{S^{-\eta}\Upsilon^{\eta}}{\Xi}\right)\hat{C}_t$$
$$+ C^*\left(\xi^{*-\frac{1}{\sigma}}c^{*-\frac{1}{\sigma}} - \xi^{-\frac{1}{\sigma}}\frac{c^{-\frac{1}{\sigma}}\left(1-\tau^w\right)}{\Upsilon\mu}\gamma^* S^{\eta}\Xi^{\eta}\right.$$
$$\left. - \xi^{*-\frac{1}{\sigma}}\frac{c^{*-\frac{1}{\sigma}}\left(1-\tau^{*w}_t\right)}{\Xi\mu}\left(1-\gamma^*\right)\Xi^{\eta}\right)\hat{C}^*_t$$

$$+ G\xi^{-\frac{1}{\sigma}} \left(\varpi g^{-\frac{1}{\sigma}} - \frac{c^{-\frac{1}{\sigma}} (1-\tau^{w})}{\Upsilon \mu} \right) \hat{G}_t$$

$$+ \begin{pmatrix} \xi^{*-\frac{1}{\sigma}} \frac{c^{*-\frac{1}{\sigma}}(1-\tau^{*w})}{\mu} (1-\gamma^*) \Xi^{\eta-1} \eta \frac{\gamma^*}{(\Xi S)^{1-\eta}} C^* \\ + \xi^{*-\frac{1}{\sigma}} \frac{c^{*-\frac{1}{\sigma}}(1-\tau^{*w})}{\Xi\mu} \gamma S^{-\eta} \Upsilon^{\eta} \frac{\eta(1-\gamma)}{\Upsilon^{1-\eta}} C \\ - \xi^{-\frac{1}{\sigma}} \frac{c^{-\frac{1}{\sigma}}(1-\tau^{w})}{\Upsilon\mu} (1-\gamma) \Upsilon^{\eta} \eta\gamma \left(\frac{S}{\Upsilon}\right)^{1-\eta} C \\ - \xi^{-\frac{1}{\sigma}} \frac{c^{-\frac{1}{\sigma}}(1-\tau^{w})}{\Upsilon\mu} \gamma^* S^{\eta} \Xi^{\eta} \frac{\eta(1-\gamma^*)}{\Xi^{1-\eta}} C^* \end{pmatrix} \hat{S}_t$$

$$+ G^* \left(\xi^{*-\frac{1}{\sigma}} \varpi^* g^{*-\sigma} - \xi^{*-\frac{1}{\sigma}} \frac{c^{*-\frac{1}{\sigma}} (1-\tau_t^{*w})}{\Xi \mu} \right) \hat{G}_t^*.$$

If

$$\frac{S\Xi}{\Upsilon} \xi^{-\frac{1}{\sigma}} c^{-\frac{1}{\sigma}} = \xi^{*-\frac{1}{\sigma}} c^{*-\frac{1}{\sigma}}$$

$$\frac{(1-\tau^{w})}{\mu} = \frac{(1-\tau^{*w})}{\mu} = 1$$

$$\varpi g^{-\frac{1}{\sigma}} = \frac{c^{-\frac{1}{\sigma}}}{\Upsilon}$$

$$\varpi^* g^{*-\sigma} = \frac{c^{*-\frac{1}{\sigma}}}{\Xi},$$

then

$$A_{H,t} + A_{F,t} = 0.$$

This determines the subsidy.

B.2 Quadratic Representation

$$W = \sum_{t=0}^{\infty} \beta^t \left(WQ_t + WL_t + WP_t \right)$$

Note that

$$\sum_{t=0}^{\infty} \beta^t WP_t = -d \sum_{t=0}^{\infty} \beta^t \left(n^{-\varsigma} \left(\frac{Y}{A} \right)^{1+\varsigma} \xi^{-\frac{1}{\sigma}} \hat{\Delta}_t \right.$$

$$\left. + (1-n)^{-\varsigma} \left(\frac{Y^*}{A^*} \right)^{1+\varsigma} \xi^{*-\frac{1}{\sigma}} \hat{\Delta}_t^* \right)$$

$$= -\frac{1}{2}\frac{\theta\epsilon}{(1-\theta)(1-\beta\theta)} d\sum_{t=0}^{\infty}\beta^t \left(n\left(\frac{y}{A}\right)^{1+\varsigma}\xi^{-\frac{1}{\sigma}}\hat{\Pi}_{Ht}^2\right.$$
$$\left. + (1-n)\left(\frac{y^*}{A^*}\right)^{1+\varsigma}\xi^{*-\frac{1}{\sigma}}\hat{\Pi}_{Ht}^{*2}\right).$$

Therefore, quadratic approximation to the household's utility can be written as

$$W = \sum_{t=0}^{\infty}\beta^t \left(\begin{array}{c} n\xi_t^{-\frac{1}{\sigma}}\left(\frac{c_t^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} + \frac{\varpi}{1-\frac{1}{\sigma}}g_t^{1-\frac{1}{\sigma}} - d\frac{n_t^{1+\varsigma}}{1+\varsigma}\right) \\ +(1-n)\,\xi_t^{*-\frac{1}{\sigma}}\left(\frac{c_t^{*1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} + \varpi\frac{g_t^{*1-\sigma}}{1-\sigma} - d\frac{n_t^{*1+\varsigma}}{1+\varsigma}\right)\end{array}\right)$$
$$= -\frac{1}{2}\xi^{-\frac{1}{\sigma}}\left(\frac{y}{A}\right)^{\varsigma} d\frac{Y}{A}\frac{\epsilon}{\lambda}\sum_{t=0}^{\infty}\beta^t V_t^H$$
$$-\xi^{-\frac{1}{\sigma}}\frac{1}{2}\left(\frac{y}{A}\right)^{\varsigma} d\left(\frac{SY^*}{A}\right)\frac{\epsilon}{\lambda}\sum_{t=0}^{\infty}\beta^t V_t^F + tip + O(2),$$

where where *tip* denotes "terms independent of policy" and $O(2)$ captures terms of order higher than two in the approximation of social welfare, and

$$V_t^H = \hat{\Pi}_{Ht}^2 + \frac{\lambda}{\epsilon}\frac{C}{Y}\left(\frac{1}{\sigma}-1\right)\left(\hat{C}_t + \frac{1}{(1-\sigma)}\hat{\xi}_t\right)^2 + \frac{\lambda}{\epsilon\sigma}\frac{G}{Y}\left(\hat{G}_t + \hat{\xi}_t\right)^2$$
$$+\frac{\varsigma\lambda}{\epsilon}\left(\hat{Y}_t - \left(\frac{(1+\varsigma)}{\varsigma}\hat{Z}_t + \frac{(1+\varsigma)}{\varsigma}\frac{1}{\sigma}\hat{\xi}_t\right)\right)^2$$
$$+\frac{\lambda}{\epsilon}(1-\gamma)\frac{C}{Y}\left(\hat{C}_t + \gamma\eta\hat{S}_t\right)^2 + \frac{\lambda}{\epsilon}\gamma\frac{C}{Y}\left(\hat{C}_t^* + \eta(1-\gamma^*)\hat{S}_t\right)^2$$
$$+\frac{\lambda}{\epsilon}\frac{C}{Y}\eta(1-\eta)\gamma\left((1-\gamma)^2 + (1-\gamma^*)\gamma^*\right)\hat{S}_t^2$$
$$V_t^F = \hat{\Pi}_{Ft}^{*2} + \frac{\lambda(1-\vartheta)}{\epsilon}\left(\frac{1}{\sigma}-1\right)\left(\hat{C}_t^* + \frac{1}{(1-\sigma)}\hat{\xi}_t^*\right)^2$$
$$+\frac{\lambda\vartheta}{\epsilon\sigma}\left(\hat{G}_t^* + \hat{\xi}_t^*\right)^2 + \frac{\varsigma\lambda}{\epsilon}\left(\hat{Y}_t^* - \left(\frac{(1+\varsigma)}{\varsigma}\hat{Z}_t^* + \frac{(1+\varsigma)}{\varsigma}\frac{1}{\sigma}\hat{\xi}_t^*\right)\right)^2$$
$$+\frac{\lambda}{\epsilon}(1-\gamma^*)(1-\vartheta)\left(\hat{C}_t^* - \eta\gamma^*\hat{S}_t\right)^2$$

$$+ \frac{\lambda}{\epsilon}\gamma^* (1-\vartheta)\left(\hat{C}_t - \eta(1-\gamma)\hat{S}_t\right)^2$$
$$+ \frac{\lambda}{\epsilon}(1-\vartheta)\,\eta\,(1-\eta)\,\gamma^*\left((1-\gamma^*)^2 + \gamma(1-\gamma)\right)\hat{S}_t^2,$$

where

$$\lambda = \frac{(1-\theta)(1-\beta\theta)}{\theta}.$$

Parameters are chosen to guarantee $S = 1$ in the steady state. In the main text all taste shocks are assumed to be zero.

Appendix C. Data Tables

Table C1. Selected Statistics for EMU Member Countries

	Employment[1] (% of Total)	Government Expenditures[2] (% of GDP)	Import of Goods and Services[3] (% of GDP)	Net Government Debt[4] (% of GDP)	Average Term to Maturity[4] (Years)	Government Debt Held by Non-residents (% of Debt)	GDP Growth Rate[1] (%)
	2014	**2013**	**2013**	**2013**	**2013**	**2013**	**2013**
AU	2.91	16.1	49.9	48.9	7.5	76.5	0.2
BE	3.19	16.6	81.4	63.6	7.3	49.5	0.3
ES	0.43	17.0	84.6	–3.9	11.7	66.0	1.6
FI	1.72	22.0	39.3	–47.9	6.2	80.7	–1.3
FR	19.08	18.1	30.4	84.7	6.7	56.8	0.7
GE	29.39	11.1	39.8	52.7	6.4	55.7	0.1
GR	*2.69*	*17.4*	*33.2*	*172.1*	*8.2*	*74.7**	*–3.9*
IR	*1.32*	*14.4*	*84.5*	*92.1*	*12.1*	*NA*	*0.2*
IT	*16.25*	*16.9*	*26.5*	*107.5*	*6.4*	*31.8*	*–1.7*
LA	0.61	16.4	62.3	32.2	NA	80.1	4.2
LI	0.91	NA	82.8	16.4	NA	70.4	3.3
NL	5.93	15.6	72.6	32.6	6.7	52.2	–0.7
PT	*3.14*	*17.0*	*38.7*	*119.4*	*5.2*	*66.8*	*–1.6*
SP	*12.42*	*17.1*	*28.1*	*59.5*	*5.5*	*38.9*	*–1.2*

Sources:
[1]OECD, data series ETONC.
[2]OECD, current government expenditures as share of GDP, data series GP132R and B1_GA.
[3]World Bank; see https://data.worldbank.org/indicator/NE.IMP.GNFS.ZS.
[4]IMF Fiscal Monitor October 2013, table 12b.
*Data for 2010. Source: European Central Bank.

Table C2. Portfolio Investment Assets by Economy of Non-resident Issuer (top row), Long-Term Debt Securities, 2011, Mln. U.S. Dollars

	AU	BE	FI	FR	GE	GR	IR	IT	NL	PT	SP
AU		3901	4142	23059	50017	1996	7875	22635	21176	1637	10807
BE	14807		4031	71165	37348	2893	28256	30338	92604	4712	32754
FI	3652	1440		17491	24462	662	2227	5568	13808	1043	5230
FR	69322	77624	11838		182036	13336	64226	227793	215715	29657	164595
GE	84785	30145	22009	245322		12095	92095	164904	212691	21382	151830
GR	769	259	39	2240	1924		1110	1504	1799	245	505
IR	5692	6284	2550	58125	78148	2421		79867	37885	4265	29189
IT	15945	7610	2678	87846	92949	4363	43663		62003	6896	43484
NL	31399	19711	14533	134479	206296	3042	17175	44407		2774	45915
PT	2014	1834	585	9735	6105	1962	26314	12002	9388		11571
SP	5988	7821	1151	37653	26776	3462	19372	59656	35901	12711	

Source: IMF Coordinated Portfolio Investment Survey and authors' calculations.

Table C3. Portfolio Investment Assets by Group of Non-resident Issuer (top row), Long-Term Debt Securities, 2011, Mln. U.S. Dollars

	Small	**Large**
Small		609696
Large	1213859	

Source: IMF Coordinated Portfolio Investment Survey and authors' calculations.

Appendix D. Discretion with Leadership

To illustrate the type of problems we are facing, in this appendix we consider non-cooperative discretionary interactions of three policymakers, labeled A, B, and C, with intraperiod leadership of one of them, but where the other two act simultaneously. Specifically, we assume that policymaker A moves first, then policymakers B and C make decisions simultaneously, and then the private sector reacts. The general principles illustrated here can be used to solve any leadership problem, provided equilibrium exists. The section is based on Oudiz and Sachs (1985), Backus and Driffill (1986), and Currie and Levine (1993), and their approach is used in, e.g., Le Roux and Kirsanova (2013) and here it is a natural extension to a three-agents setting.

We solve the system in a state-space form:

$$\begin{bmatrix} Y_{t+1} \\ H\mathbb{E}_t X_{t+1} \end{bmatrix} = A \begin{bmatrix} Y_t \\ X_t \end{bmatrix} + B \begin{bmatrix} U_t^B \\ U_t^C \end{bmatrix} + DU_t^A + C\varepsilon_{t+1}, \quad (70)$$

where Y_t is an n_1-vector of predetermined state variables, Y_0 is given, and X_t are the effective instruments of the private sector, an n_2-vector of non-predetermined or jump variables ($n = n_1 + n_2$). The policy instruments are represented by U_t^A, U_t^B, and U_t^C. ε_{t+1} is an n_ε-vector of exogenous zero-mean iid shocks. Matrices A, B, C, and D are partitioned conformably with Y_t and X_t as

$$A \equiv \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}; \ B \equiv \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix}$$

$$D \equiv \begin{bmatrix} D_1 \\ D_2 \end{bmatrix}; \ C \equiv \begin{bmatrix} C_1 \\ \mathbf{O} \end{bmatrix}.$$

This system describes the evolution of the economy as observed by policymakers.

D.1 Private Sector's Reaction Function

The forward-looking variables must be linear functions of the state variables,

$$X_t = -N_t Y_t.$$

Take this one step forward, substitute into the upper block of (70), and get

$$\mathbb{E}_t X_{t+1} = -N_{t+1}\left[A_{11}Y_t + A_{12}X_t + B_{11}U_t^B + B_{12}U_t^C + D_1 U_t^A\right]$$

while the lower block of (70) is

$$H\mathbb{E}_t X_{t+1} = A_{21}Y_t + A_{22}X_t + B_{21}U_t^B + B_{22}U_t^C + D_2 U_t^A.$$

Multiplying the former equation by H, setting the result equal to the latter equation, and solving for X_t, we obtain

$$\begin{aligned} X_t = &-\underbrace{(A_{22} + HN_{t+1}A_{12})^{-1}(A_{21} + HN_{t+1}A_{11})}_{J_t}Y_t \\ &- \underbrace{(A_{22} + HN_{t+1}A_{12})^{-1}(B_{21} + HN_{t+1}B_{11})}_{K_t^H}U_t^B \\ &- \underbrace{(A_{22} + HN_{t+1}A_{12})^{-1}(B_{22} + HN_{t+1}B_{12})}_{K_t^F}U_t^C \\ &- \underbrace{(A_{22} + HN_{t+1}A_{12})^{-1}(D_2 + HN_{t+1}D_1)}_{K_t^M}U_t^A \\ = &-J_t Y_t - K_t^A U_t^A - K_t^B U_t^B - K_t^C U_t^C, \end{aligned} \tag{71}$$

where J_t is n_2xn_1, K_t^A is n_2xk_A, K_t^B is n_2xk_B, and K_t^C is n_2xk_C (k_A, k_B, and k_C stand, respectively, for the number of fiscal policy instruments of policymakers A, B, and C, respectively.[4]

[4]It is assumed that $A_{22} + HN_{t+1}A_{12}$ is invertible.

We use (71) in the first n_1 equations in the system (70) to get the reduced-form evolution of the predetermined variables

$$Y_{t+1} = \underbrace{[A_{11} - A_{12}J_t]}_{O_{Y_t}} Y_t + \underbrace{\left[B_{11} - A_{12}K_t^B\right]}_{O_{B_t}} U_t^B + \underbrace{\left[B_{12} - A_{12}K_t^C\right]}_{O_{C_t}} U_t^C + \underbrace{\left[D_1 - B_{12}L_t^A\right]}_{O_{A_t}} U_t^A + C_1\varepsilon_{t+1}$$
$$= O_{Y_t}Y_t + O_{A_t}U_t^A + O_{B_t}U_t^B + O_{C_t}U_t^C + C_1\varepsilon_{t+1}. \quad (72)$$

Being a follower, authority B observes authority A's actions and reacts to them. In a linear-quadratic setup, the optimal solution belongs to the class of linear feedback rules of the form

$$U_t^B = -F_t^B Y_t - L_t^B U_t^A, \quad (73)$$

where F_t^B denotes feedback coefficients on the predetermined state variables and L_t^B is the leadership parameter. The other fiscal authority solves a similar problem and gets

$$U_t^C = -F_t^C Y_t - L_t^C U_t^A. \quad (74)$$

Moving simultaneously, authorities B and C do not respond to each other's actions.

The monetary leadership authority takes into account these fiscal policy reaction functions as well as the private sector's optimal conditions when solving its optimization problem. Thus, the leader can manipulate the follower by changing its policy instrument. The monetary leadership reaction function takes the form of

$$U_t^A = -F_t^A Y_t. \quad (75)$$

D.2 The Followers' Optimization Problem

In the discretionary case, the three policymakers reoptimize every period by taking the process by which private agents form their expectations as given—and where the expectations are consistent with actual policies. Policymakers B and C minimize their loss functions treating policy instrument A parametrically but incorporating the reaction functions of the private sectors. Policymaker B has the following objective function:

$$\frac{1}{2}\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\left(G_t^{B\prime}Q^A G_t^B\right)$$
$$=\frac{1}{2}\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\left(Z_t^{\prime}\mathcal{Q}^B Z_t+2Z_t^{\prime}\mathcal{P}^B U_t+U_t^{\prime}\mathcal{R}^B U_t\right), \tag{76}$$

where G_t^B is the target variables for authority B while Q^B is the corresponding matrix of weights. The target variables can be rewritten in terms of the predetermined and non-predetermined state variables collected on vector Z_t, in terms of the policy instruments collected in $U_t = [U_t^{A\prime}, U_t^{B\prime}, U_t^{C\prime}]^{\prime}$ and their cross-terms.

Authority B optimizes every period, taking into account that she will be able to reoptimize next period. The model is linear quadratic, thus the solution in $t+1$ gives a period return which is quadratic in the state variables, $W_{t+1}^B \equiv Y_{t+1}^{'} S_B^{t+1} Y_{t+1} + w_{t+1}^B$, where S_B^{t+1} is a positive semidefinite matrix and w_{t+1}^B is a scalar independent of Y_{t+1}.

Hence, the value function of authority B in t will then satisfy the Bellman equation:

$$W_t^B=\min_{U_t^B}\frac{1}{2}\left[\left(Z_t^{\prime}\mathcal{Q}^B Z_t+Z_t^{\prime}\mathcal{P}^B U_t+U_t^{\prime}\mathcal{P}^{B\prime}Z_t+U_t^{\prime}\mathcal{R}^B U_t\right)\right.$$
$$\left.+\ \beta\mathbb{E}_t\left(W_{t+1}^B\right)\right] \tag{77}$$
$$s.t.\ \mathbb{E}_t X_{t+1}=-N_{t+1}\mathbb{E}_t Y_{t+1},$$
$$W_{t+1}^B\equiv Y_{t+1}^{'}S_B^{t+1}Y_{t+1}+w_{t+1}^B,$$

and Y_t given.

Therefore we can substitute equations (71) and (72) into (77) to obtain an equivalent minimization problem[5]:

$$2\widetilde{W}_t^B\equiv\min_{U_t^H}\left\{Y_t^{\prime}\left[Q_B^S+\beta O_{Y_t}^{\prime}S_B^{t+1}O_{Y_t}\right]Y_t+U_t^{B^{\prime}}\left[\mathcal{U}_B^{S,B^{\prime}}+\beta O_{B_t}^{\prime}S_B^{t+1}O_{Y_t}\right]Y_t\right.$$
$$+\,Y_t^{\prime}\left[\mathcal{U}_B^{S,B}+\beta O_{Y_t}^{\prime}S_B^{t+1}O_{B_t}\right]U_t^B+U_t^{C^{\prime}}\left[\mathcal{U}_C^{S,B^{\prime}}+\beta O_{C_t}^{\prime}S_B^{t+1}O_{Y_t}\right]Y_t$$

[5] We have made use of the fact that w_{t+1}^B is independent of Y_{t+1} and $\mathbb{E}_t\varepsilon_{t+1}=0$.

$$
\begin{aligned}
&+ Y_t' \left[\mathcal{U}_C^{S,B} + \beta O_{Y_t}' S_B^{t+1} O_{C_t}\right] U_t^C + U_t^{A'} \left[\mathcal{U}_A^{S,B'} + \beta O_{A_t}' S_B^{t+1} O_{Y_t}\right] Y_t \\
&+ Y_t' \left[\mathcal{U}_A^{S,B} + \beta O_{Y_t}' S_B^{t+1} O_{A_t}\right] U_t^A + U_t^{B'} \left[\mathcal{R}_B^{S,B} + \beta O_{B_t}' S_B^{t+1} O_{B_t}\right] U_t^B \\
&+ U_t^{C'} \left[\mathcal{R}_C^{S,B} + \beta O_{C_t}' S_B^{t+1} O_{C_t}\right] U_t^C + U_t^{A'} \left[\mathcal{R}_A^{S,B} + \beta O_{A_t}' S_B^{t+1} O_{A_t}\right] U_t^A \\
&+ U_t^{B'} \left[\mathcal{P}_{BC}^{S,B} + \beta O_{H_t}' S_B^{t+1} O_{C_t}\right] U_t^C + U_t^{C'} \left[\mathcal{P}_{BC}^{S,B'} + \beta O_{C_t}' S_B^{t+1} O_{B_t}\right] U_t^B \\
&+ U_t^{B'} \left[\mathcal{P}_{BA}^{S,B} + \beta O_{H_t}' S_B^{t+1} O_{A_t}\right] U_t^A + U_t^{A'} \left[\mathcal{P}_{BA}^{S,B'} + \beta O_{A_t}' S_B^{t+1} O_{B_t}\right] U_t^B \\
&+ U_t^{C'} \left[\mathcal{P}_{CA}^{S,B} + \beta O_{C_t}' S_B^{t+1} O_{A_t}\right] U_t^A + U_t^{A'} \left[\mathcal{P}_{CA}^{S,B'} + \beta O_{A_t}' S_B^{t+1} O_{C_t}\right] U_t^C \Big\}, \qquad (78)
\end{aligned}
$$

where

$$
\begin{aligned}
Q_B^S &= \mathcal{Q}_{11}^B - J_t' \mathcal{Q}_{21}^B - \mathcal{Q}_{12}^B J_t + J_t' \mathcal{Q}_{22}^B J_t, \\
\mathcal{U}_B^{S,B} &= J_t' \mathcal{Q}_{22}^B K_t^B - \mathcal{Q}_{12}^B K_t^B + \mathcal{P}_{12}^B - J_t' \mathcal{P}_{22}^B \\
\mathcal{U}_F^{S,B} &= J_t' \mathcal{Q}_{22}^B K_t^F - \mathcal{Q}_{12}^B K_t^C + \mathcal{P}_{13}^B - J_t' \mathcal{P}_{23}^B, \\
\mathcal{U}_A^{S,B} &= J_t' \mathcal{Q}_{22}^B K_t^A - \mathcal{Q}_{12}^B K_t^A + \mathcal{P}_{11}^B - J_t' \mathcal{P}_{21}^B \\
\mathcal{R}_B^{S,B} &= K_t^{B\prime} \mathcal{Q}_{22}^B K_t^B - K_t^{B\prime} \mathcal{P}_{22}^B - \mathcal{P}_{22}^{B\prime} K_t^B + \mathcal{R}_{22}^B, \\
\mathcal{R}_C^{S,B} &= K_t^{C\prime} \mathcal{Q}_{22}^B K_t^C - K_t^{C\prime} \mathcal{P}_{23}^B - \mathcal{P}_{23}^{B\prime} K_t^C + \mathcal{R}_{33}^B \\
\mathcal{R}_A^{S,B} &= K_t^{A\prime} \mathcal{Q}_{22}^B K_t^A - K_t^{A\prime} \mathcal{P}_{21}^B - \mathcal{P}_{21}^{B\prime} K_t^A + \mathcal{R}_{11}^B, \\
\mathcal{P}_{BC}^{S,B} &= K_t^{B\prime} \mathcal{Q}_{22}^B K_t^C - K_t^{B\prime} \mathcal{P}_{23}^B - \mathcal{P}_{22}^{B\prime} K_t^C + \mathcal{R}_{23}^B \\
\mathcal{P}_{BA}^{S,B} &= K_t^{B\prime} \mathcal{Q}_{22}^B K_t^A - K_t^{B\prime} \mathcal{P}_{21}^B - \mathcal{P}_{22}^{B\prime} K_t^A + \mathcal{R}_{21}^B, \\
\mathcal{P}_{CA}^{S,B} &= K_t^{C\prime} \mathcal{Q}_{22}^B K_t^A - K_t^{C\prime} \mathcal{P}_{21}^B - \mathcal{P}_{23}^{B\prime} K_t^A + \mathcal{R}_{31}^B.
\end{aligned}
$$

Hence, the problem faced by authority B has been transformed to a standard linear-quadratic regulator problem without forward-looking variables but with time-varying parameters. The first-order condition is

$$
\begin{aligned}
0 = &\left[\mathcal{U}_B^{S,B'} + \beta O_{B_t}' S_B^{t+1} O_{Y_t}\right] Y_t + \left[\mathcal{R}_B^{S,B} + \beta O_{B_t}' S_B^{t+1} O_{B_t}\right] U_t^B \\
&+ \left[\mathcal{P}_{BC}^{S,B} + \beta O_{B_t}' S_B^{t+1} O_{C_t}\right] U_t^C + \left[\mathcal{P}_{BA}^{S,B} + \beta O_{B_t}' S_B^{t+1} O_{A_t}\right] U_t^A.
\end{aligned}
$$

Since $U_t^B = -F_t^B Y_t - L_t^B U_t^A$ and $U_t^C = -F_t^C Y_t - L_t^C U_t^A$, the first-order condition can be solved for the feedback coefficients of the reaction function of authority B:

$$F_t^B \equiv \left[\mathcal{R}_B^{S,B} + \beta O'_{B_t} S_B^{t+1} O_{B_t}\right]^{-1} \begin{Bmatrix} \left[\mathcal{U}_B^{S,B'} + \beta O'_{B_t} S_B^{t+1} O_{Y_t}\right] \\ -\left[\mathcal{P}_{BC}^{S,B} + \beta O'_{B_t} S_B^{t+1} O_{C_t}\right] F_t^C \end{Bmatrix} \tag{79}$$

$$L_t^B \equiv \left[\mathcal{R}_B^{S,B} + \beta O'_{B_t} S_B^{t+1} O_{B_t}\right]^{-1} \begin{Bmatrix} \left[\mathcal{P}_{BM}^{S,B} + \beta O'_{B_t} S_B^{t+1} O_{M_t}\right] \\ -\left[\mathcal{P}_{BC}^{S,B} + \beta O'_{B_t} S_B^{t+1} O_{C_t}\right] L_t^C. \end{Bmatrix} \tag{80}$$

Substituting the decision rules (73), (74), and (75) into (78), we obtain the recursive equations for

$$S_B^t \equiv T_{0,t}^B + \beta T_t^{B\prime} S_B^{t+1} T_t^B \tag{81}$$

$$\begin{aligned} T_{0,t}^B = Q_B^S & - \mathcal{U}_B^{S,B}\left(F_t^B - L_t^B F_t^A\right) - \left(F_t^B - L_t^B F_t^A\right)' \mathcal{U}_B^{S,B'} \\ & - \mathcal{U}_C^{S,B}\left(F_t^C - L_t^C F_t^A\right) - \left(F_t^C - L_t^C F_t^A\right)' \mathcal{U}_C^{S,B'} - \mathcal{U}_A^{S,B} F_t^A \\ & - F_t^{A'} \mathcal{U}_A^{S,B'} + \left(F_t^B - L_t^B F_t^A\right)' \mathcal{R}_B^{S,B}\left(F_t^B - L_t^B F_t^A\right) \\ & + \left(F_t^C - L_t^C F_t^A\right)' \mathcal{R}_C^{S,B}\left(F_t^C - L_t^C F_t^A\right) + F_t^{A\prime} \mathcal{R}_A^{S,B} F_t^A \\ & + \left(F_t^B - L_t^B F_t^A\right)' \mathcal{P}_{BC}^{S,B}\left(F_t^C - L_t^C F_t^A\right) \\ & + \left(F_t^C - L_t^C F_t^A\right)' \mathcal{P}_{BC}^{S,B'}\left(F_t^B - L_t^B F_t^A\right) \\ & + \left(F_t^B - L_t^B F_t^A\right)' \mathcal{P}_{BA}^{S,B} F_t^A + F_t^{A'\prime} \mathcal{P}_{BA}^{S,B'}\left(F_t^B - L_t^B F_t^A\right) \\ & + \left(F_t^C - L_t^C F_t^A\right)' \mathcal{P}_{CA}^{S,B} F_t^A + F_t^{A'\prime} \mathcal{P}_{CA}^{S,B'}\left(F_t^C - L_t^C F_t^A\right) \end{aligned}$$

and

$$T_t^B = O_{Y_t} - O_{B_t}\left(F_t^B - L_t^B F_t^A\right) - O_{C_t}\left(F_t^C - L_t^C F_t^A\right) - O_{A_t} F_t^A.$$

Similar formulas can be derived for authority C.

D.3 The Leader's Optimization Problem

This part of the problem is the standard optimization problem described by Oudiz and Sachs (1985) and Backus and Driffill (1986) where the system under control evolves as

$$\begin{bmatrix} Y_{t+1} \\ H\mathbb{E}_t X_{t+1} \end{bmatrix} = \begin{bmatrix} A_{11} - B_{11}F_t^B - B_{12}F_t^C & A_{12} \\ A_{21} - B_{21}F_t^B - B_{22}F_t^C & A_{22} \end{bmatrix} \begin{bmatrix} Y_t \\ X_t \end{bmatrix} + \begin{bmatrix} D_{11} - B_{11}L_t^B - B_{12}L_t^C \\ D_{21} - B_{21}L^A - B_{22}L^B \end{bmatrix} U_t^A + C\varepsilon_{t+1}. \quad (82)$$

Authority A's loss function is

$$\frac{1}{2}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(G_t^{A'} Q^A G_t^A \right).$$

But, since the leadership integrates the followers' reaction functions—$U_t^B = -F_t^B Y_t - L_t^B U_t^A$ and $U_t^F = -F_t^F Y_t - L_t^F U_t^A$—into its optimization problem, the leadership's loss function has to be rewritten in terms of the relevant variables for the leadership authority. Since

$$\begin{bmatrix} Y_t \\ X_t \\ U_t^A \\ U_t^B \\ U_t^C \end{bmatrix} = \underbrace{\begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \\ -F_t^B & 0 & -L_t^B \\ -F_t^C & 0 & -L_t^C \end{bmatrix}}_{\mathcal{C}} \begin{bmatrix} Y_t \\ X_t \\ U_t^A \end{bmatrix},$$

we can set $G_t^{A'} Q^A G_t^A = \begin{bmatrix} Y_t' & X_t' & U_t^{A\prime} \end{bmatrix} \widetilde{\mathcal{K}}^A \begin{bmatrix} Y_t \\ X_t \\ U_t^A \end{bmatrix}$, where $\widetilde{\mathcal{K}}^A = \mathcal{C}' \underbrace{C^{A\prime} Q^A C^A}_{\mathcal{K}^A} \mathcal{C}$ and $\widetilde{\mathcal{K}}^A$ have to be partitioned conformably with $\begin{pmatrix} Y_t' & X_t' & U_t^{A\prime} \end{pmatrix}'$.

D.4 The Iterative Procedure

We start with initial approximation for policy rules, with $F_{(0)}^B, F_{(0)}^C, F_{(0)}^A, L_{(0)}^B$, and $L_{(0)}^C$, and solve the follower's problem, using equations (79)–(81) for authority B and equivalent equations for authority C. We will improve $F_{(1)}^B$ and $L_{(1)}^B$, as well as $F_{(1)}^C$ and $L_{(1)}^C$, but not $F_{(0)}^A$. We then take into account the policy reaction functions of B and C and compute new matrices in equation (82), update the target variable, and solve the problem for A. This will give us new optimal reaction $F^A_{(1)}$. Then we again solve the problem for the authorities B and C to update $F^B_{(2)}$, $L^B_{(2)}$, $F^B_{(2)}$, and $L^B_{(2)}$ and so on.

Appendix E. Financial Integration

The results on policy coordination under international risk sharing are given in tables E1 and E2. They are different from those for incomplete financial markets on two accounts.

First, under technology shocks the losses to all policymakers are smaller under international risk sharing than under the incomplete financial markets, while the opposite happens under the cost-push shocks. This is consistent with Auray and Eyquem (2014), who showed that incomplete markets (autarky) may produce lower welfare costs than complete markets, primarily because of inflation volatility costs.

To understand this result, consider an asymmetric cost-push shock, negative in country H and positive in country F, which is plotted in figure 1 in the main text. Complete financial markets allow for international borrowing and risk sharing among households of the two countries. Real marginal utilities from consumption are equated across countries, periods, and states. Thus, since the real exchange rate increases due to the presence of home bias in consumption, relative consumption also increases, but by a relatively small amount.

Incomplete financial markets do not allow that H households lend (F households borrow) abroad as much as they would under complete financial markets, and thus relative consumption increases by much more. This helps to stabilize producer inflation rates and the terms of trade. In effect, larger consumption at country H reduces the marginal utility of consumption and lowers labor supply, exerting an upward pressure on wages and mitigating deflation at country H. Hence, compared to complete markets, producer inflation rates and terms of trade are less volatile, while consumptions are more volatile and relative consumption is inefficiently high.

Therefore, under international risk sharing and relative to the incomplete financial markets, tax rates and government spending need to move by more to stabilize more volatile producer inflation

Table E1. Policy Equilibriums under International Risk Sharing in a Symmetric Monetary Union

A. Equilibrium under Technology Shock					
		Foreign Country			
		L_N		F_N	
Home Country	L_N	**(0.522,0.522)**		(0.521,0.544)	
	F_N	(0.544,0.521)		(0.522,0.522)	
B. Equilibrium under Cost-Push Shock					
		Foreign Country			
Home Country	L_N	(1.070,1.070)		**(0.997,1.064)**	
	F_N	**(1.064,0.997)**		(1.070,1.070)	
C. Equilibrium under Technology Shock					
		Foreign Country			
		L_N	L_U	F_N	F_U
Home Country	L_N	**(0.522,0.522)**	(0.489,0.544)	(0.521,0.544)	(0.484,0.577)
	L_U	(0.544,0.489)	(0.510,0.510)	(0.534,0.498)	(0.510,0.510)
	F_N	(0.544,0.521)	(0.498,0.534)	(0.522,0.522)	(0.516,0.544)
	F_U	(0.577,0.484)	(0.510,0.510)	(0.544,0.516)	(0.510,0.510)
D. Equilibrium under Cost-Push Shock					
		Foreign Country			
		L_N	L_U	F_N	F_U
Home Country	L_N	(1.070,1.070)	(1.073,1.052)	(0.997,1.064)	(0.978,1.054)
	L_U	(1.052,1.073)	**(1.029,1.029)**	(0.990,1.070)	(1.029,1.029)
	F_N	(1.064,0.997)	(1.070,0.990)	(1.070,1.070)	(1.073,1.052)
	F_U	(1.054,0.978)	(1.029,1.029)	(1.052,1.073)	(1.029,1.029)

Note: Losses to (home, foreign); losses in Nash equilibriums are shown in bold fonts.

rates and terms of trade. The higher volatility of inflation, terms of trade, and government spending outweighs the smaller volatility of consumption and output under cost-push shocks.

Second, under technology shocks there are no qualitative differences with the case of incomplete financial markets, but there are differences under cost-push shocks.

Table E2. Policy Equilibriums under International Risk Sharing in an Asymmetric Monetary Union

A. Equilibrium under Technology Shock

		Large Foreign Country	
		L_N	F_N
Small Home Country	L_N	**(0.701,0.413)**	(0.701,0.416)
	F_N	(0.721,0.413)	(0.701,0.413)

B. Equilibrium under Cost-Push Shock

		Foreign Country	
		L_N	F_N
Small Home Country	L_N	(1.297,0.944)	**(1.267,0.944)**
	F_N	**(1.286,0.888)**	(1.297,0.944)

C. Equilibrium under Technology Shock

		Large Foreign Country			
		L_N	L_U	F_N	F_U
Small Home Country	L_N	**(0.701,0.413)**	(0.635,0.429)	(0.701,0.417)	(0.628,0.447)
	L_U	(0.710,0.407)	(0.649,0.422)	(0.705,0.409)	(0.649,0.422)
	F_N	(0.721,0.413)	(0.644,0.426)	(0.701,0.413)	(0.635,0.429)
	F_U	(0.730,0.407)	(0.649,0.422)	(0.710,0.407)	(0.649,0.422)

D. Equilibrium under Cost-Push Shock

		Large Foreign Country			
		L_N	L_U	F_N	F_U
Small Home Country	L_N	(1.297,0.944)	(1.295,0.933)	(1.267,0.944)	(1.213,0.934)
	L_U	(1.315,0.955)	(1.297,0.919)	(1.281,0.956)	(1.297,0.919)
	F_N	**(1.286,0.888)**	(1.288,0.888)	(1.297,0.944)	(1.295,0.933)
	F_U	(1.315,0.897)	(1.297,0.919)	(1.315,0.955)	(1.297,0.919)

Under cost-push shocks, equilibrium (F_N, L_N) is the unique Nash equilibrium under complete financial markets in a country-size asymmetric monetary union, while it is only one of the three Nash equilibriums if markets are incomplete. Moreover, as we discussed above, it only exists under incomplete financial markets if there are external imbalances and the large foreign country is a net foreign creditor, creating bigger "asymmetry" and the desire for the smaller country to choose national objectives.

More generally, incentives to adopt union-wide objectives are smaller the larger the asymmetries (different country size and/or different steady-state external imbalances) within the monetary union. Since the risk sharing leads to larger demand shifts across countries, it further amplifies existing asymmetries under incomplete financial markets. Country-size asymmetry reveals to be high enough to rule out any equilibriums where at least one national authority adopts the union-wide objectives. Thus, (F_N, L_N) is the only surviving equilibrium in the fiscal sequential leadership where the large country leads.

Appendix F. Heterogeneous Monetary Union

F.1 Country-Size Effects

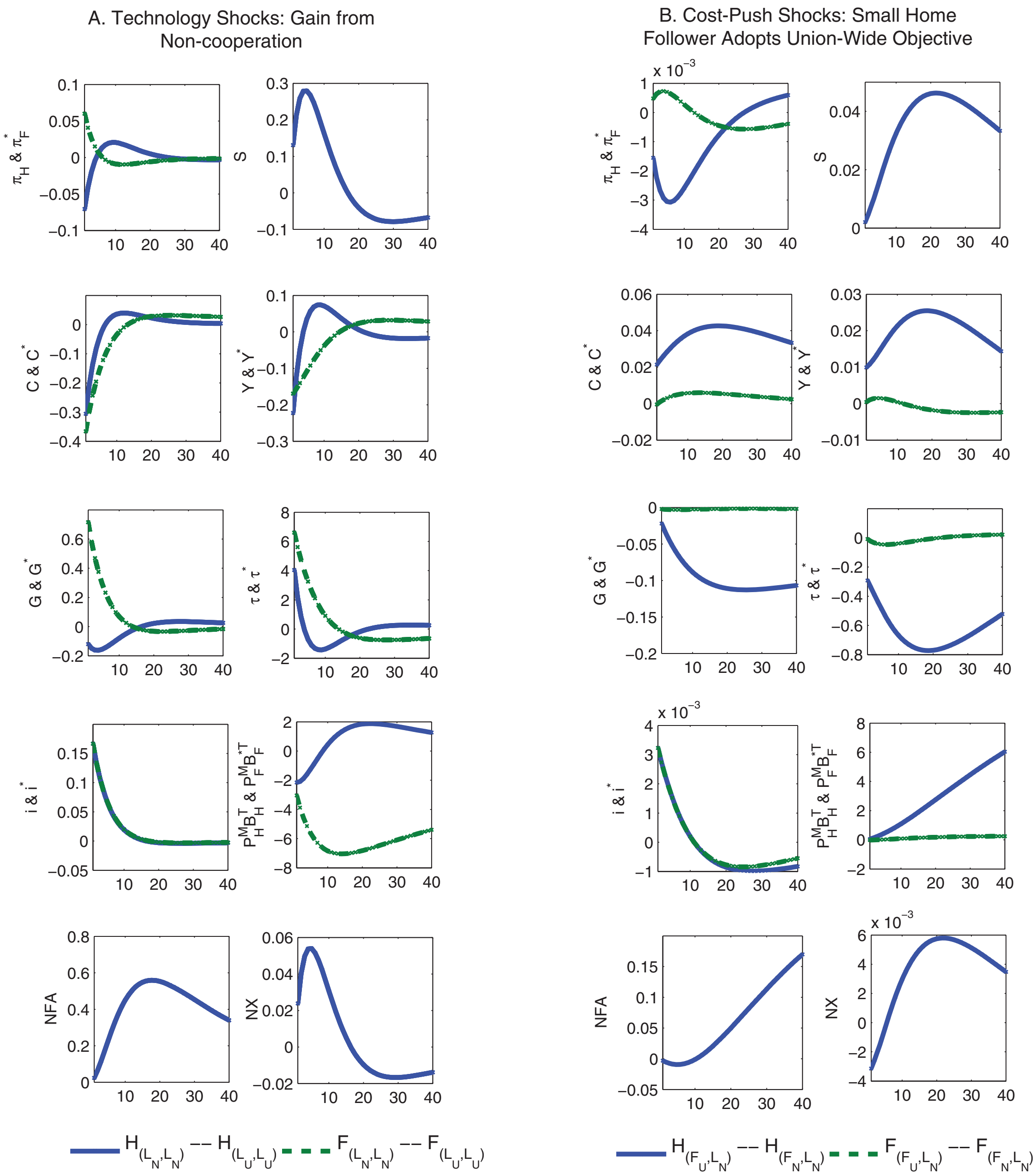
F.1.1 Technology Shocks

In the case of technology shocks, the country-size effects on the equilibrium outcome in table 2 in the main text are limited. The outcome is determined by the individual ranking of outcomes in non-cooperative regimes [HF]M, HFM, and FHM. Regardless of size, the leader gains and the follower loses in the sequential regimes HFM and FHM relative to their payoffs in [HF]M.

The large country, however, gains more than under cooperation. This result is the consequence of the size asymmetry, as in the case of symmetric countries the cooperative outcome was the best for each fiscal authority. The larger country has a stronger effect on monetary policy and is able to reduce the volatility of its fiscal instruments, in particular of costly spending; see panel A in figure F1.

Both countries gain because of less negative terms of trade gap, and lose due to higher inflation volatility. Since the interest rate needs to react by more than cooperation, output and consumption are better stabilized for the large country, while the reverse occurs for the small country. As a consequence, the total loss for the small home

Figure F1. Country-Size Effects, Impulse Responses to Asymmetric Shocks, Monetary Union with $n = 0.3$, $\varrho = 0$, $\omega = 1$, $\delta_H = \delta_F = 0.6$



country is higher than under cooperation, while [HF]M is preferred to cooperation for the large foreign country.[6]

F.1.2 Cost-Push Shocks

If there are no international holdings of debt in the steady state, $\varrho = 0$, then under cost-push shocks the country-size effects result in only two Nash equilibriums in the extended 4x4 game, (L_U, L_U) and (L_N, F_U).

Neither (F_N, L_N) nor (F_U, L_N) is a Nash equilibrium. In contrast to the country-size symmetric union discussed above, if the intraperiod follower country is small, then it does not want to adopt the union-wide objective unilaterally. On the one hand, when the small H country adopts union-wide objectives, it now attaches a large relative weight to the objectives of the large F country. On the other hand, since cross-border effects from H are too small, F does not change its policy reaction meaningfully. As country H fiscal policy produces a small negative cross-border effect and F country has a large weight in the union, H will react by much less than if it cares only for its own (national) objectives. Since H is too small to get enough stabilization feedback from monetary policy, its weaker fiscal policy reaction produces larger welfare stabilization costs than otherwise. Panel B of figure F1 shows that, except for government spending, welfare-relevant variables at H display higher volatility under (F_U, L_N) than under (F_N, L_N). As a result, (F_U, L_N) is not a Nash equilibrium. Furthermore, while the small country H, being a follower, prefers keeping national objectives, the large country F, being a leader, prefers to adopt the union-wide welfare, aligning with the monetary policymaker, and so (F_N, L_N) is also not a Nash equilibrium.

F.2 International Exposure, Imbalances, and Debt Levels

The previous analysis assumed steady-state financial autarky, with zero external debt $\varrho = 0$. Greater international exposure of both countries with positive steady-state external debt ($\varrho > 0$) and equal

[6]Here and below in all simulations we also assume that there are no international holdings of debt in the steady state, $\varrho = 0$. We relax this assumption later in the text.

steady-state external debts ($\omega = 1$) can produce stabilization welfare gains under both technology and cost-push shocks.

With a positive steady-state external debt, price levels and the terms of trade have first-order effects on real external debt and, thus, on intermediation costs. Under either cost-push or technology shocks, consumption becomes more volatile while inflation and terms of trade become more stabilized than in the case of steady-state financial autarky. In the face of an asymmetric technology shock (positive in H and negative in F), H becomes the net lender and real external debt is now worse stabilized through the price-level effect. Consequently, interest rate at H decreases by more than under steady-state financial autarky (see panel A in figure F2).In turn, in the face of an asymmetric cost-push shock (negative in H and positive in F), H becomes the net borrower and real external debt is now better stabilized through the price-level effect. Consequently, the interest rate at H increases by less than under steady-state financial autarky (see panel B in figure F2). In both cases, consumption increases by more at H, depleting stabilization under cost-push shocks (larger deviation from the efficient level of consumption) while improving stabilization under technology shocks. However, overall welfare improves under both shocks, since higher external debt exposures promotes a better stabilization of inflation.

Suppose that, in addition to $\varrho > 0$, we reduce $\omega < 1$ and make the home country the net external debtor. In our example of symmetric monetary union with $n = 0.5$, but with $\varrho = 0.5$ and $\omega = 0.5$, the steady-state consumption and spending at home are lower than they are in the foreign country, because the net external debtor country has to generate current account surplus to pay for debt service. The union is not symmetric any more, with an effectively "smaller" home country, and an asymmetric shock now produces aggregate effects dominated by the effects of the shocks at the "larger" F country. In fact, country H bears larger stabilization costs as monetary policy reacts to averages so that it helps to stabilize F inflation while destabilizing H inflation. Terms of trade further increase, reducing the negative terms of trade gap under technology shocks, while increasing positive terms of trade gap under cost-push shocks; see figure F3. This results in better stabilization under technology shocks but worse stabilization under cost-push shocks for the net lender F country.

Figure F2. Effect of Greater International Exposure in Regime [HF]M, Monetary Union with $n = 0.3$ and $\omega = 1$, $\delta_H = \delta_F = 0.6$

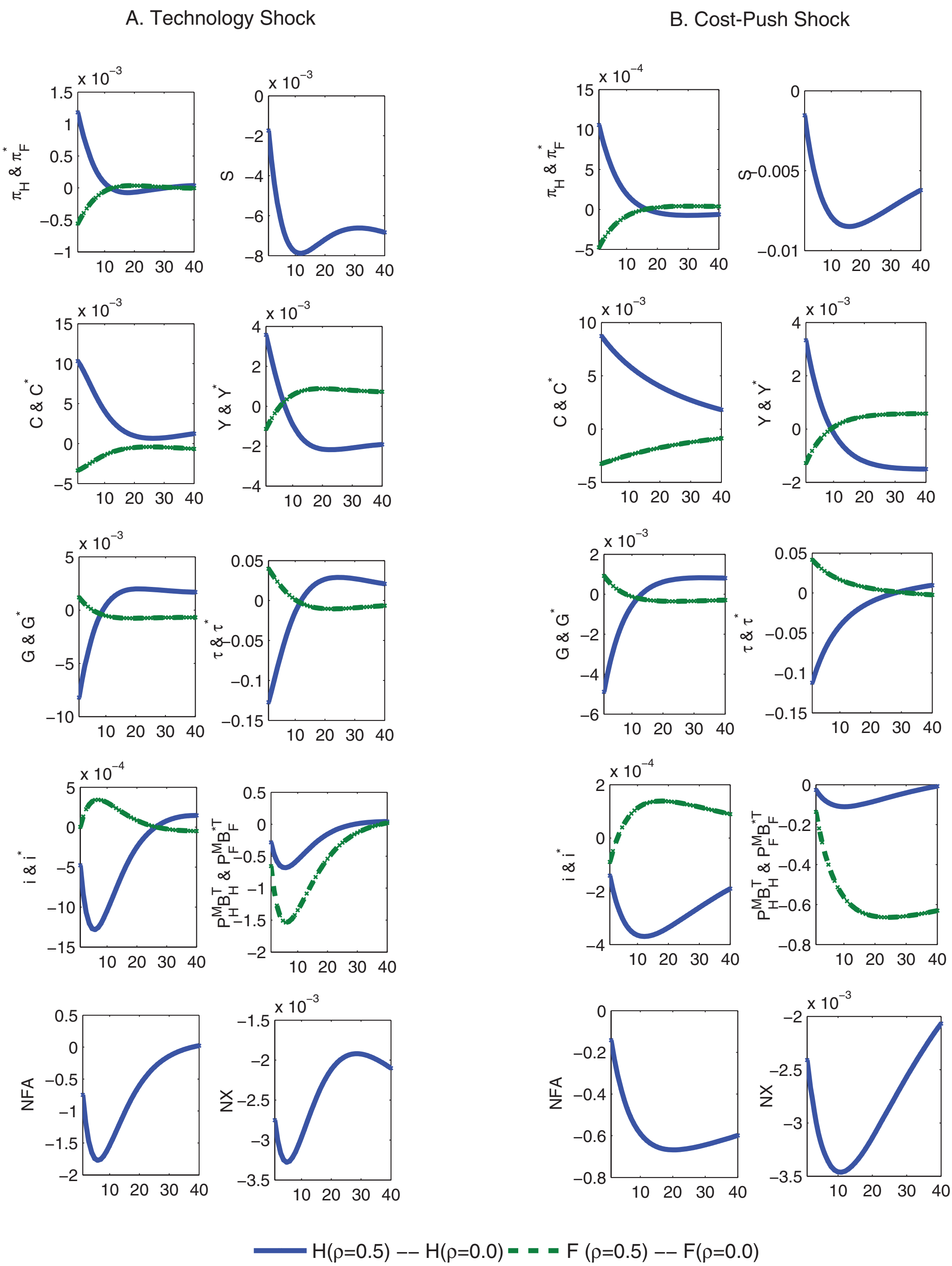


Figure F3. Effect of Greater Imbalances in Regime [HF]M, Monetary Union with $n = 1/2$ and $\varrho = 0.5$, $\delta_H = \delta_F = 0.6$

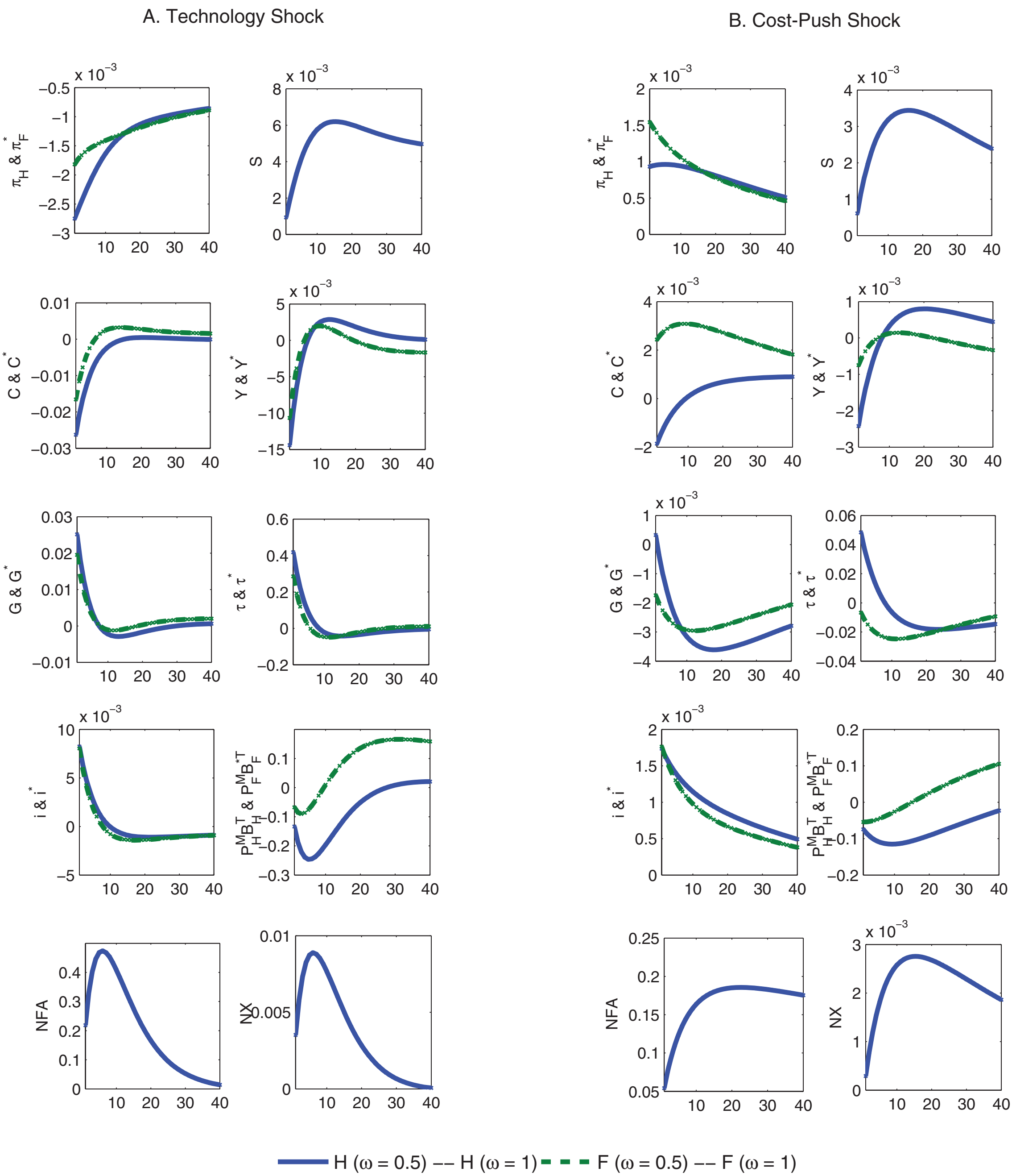


Table F1. Emergence of Equilibrium (F_N, L_N) with Positive Net External Debt of the Home Country

A. Equilibriums under Cost-Push Shock, with $\varrho = 0.5; \omega = 1; \delta_H = \delta_F = 0.6$					
		Large Foreign Country			
		L_N	L_U	F_N	F_U
Small Home Country	L_N	(0.983,0.922)	(0.981,0.906)	(0.969,0.922)	(0.937,0.905)
	L_U	(0.980,0.926)	**(0.962,0.889)**	(0.965,0.926)	(0.963,0.889)
	F_N	(0.977,0.873)	(0.978,0.872)	(0.983,0.923)	(0.981,0.906)
	F_U	(0.981,0.873)	(0.963,0.889)	(0.980,0.926)	(0.963,0.889)
B. Equilibriums under Cost-Push Shock, with $\varrho = 0.5; \omega = 0.5; \delta_H = \delta_F = 0.6$					
		Large Foreign Country			
		L_N	L_U	F_N	F_U
Small Home Country	L_N	(0.992,0.928)	(0.990,0.912)	(0.979,0.928)	**(0.949,0.911)**
	L_U	(0.988,0.931)	**(0.969,0.893)**	(0.972,0.931)	(0.969,0.893)
	F_N	**(0.988,0.876)**	(0.988,0.876)	(0.992,0.928)	(0.990,0.913)
	F_U	(0.990,0.874)	(0.969,0.893)	(0.988,0.931)	(0.969,0.893)
C. Equilibriums under Cost-Push Shock, $\varrho = 0.5; \omega = 0.5; \delta_H = 1.2; \delta_F = 0.6$					
		Large Foreign Country			
		L_N	L_U	F_N	F_U
Small Home Country	L_N	(1.051,0.945)	(1.049,0.933)	(1.037,0.944)	**(1.006,0.931)**
	L_U	(1.051,0.946)	**(1.033,0.915)**	(1.034,0.947)	(1.033,0.915)
	F_N	**(1.047,0.899)**	(1.048,0.901)	(1.051,0.945)	(1.049,0.933)
	F_U	(1.054,0.897)	(1.033,0.915)	(1.051,0.946)	(1.033,0.915)

Under cost-push shocks, this additional asymmetry results in the third Nash equilibrium (F_N, L_N), which existed in the 2x2 game but disappeared in the 4x4 game in the country-size asymmetric monetary union, reappearing; see panels A and B in table F1. As a net debtor, the "smaller" H country has now fewer incentives to adopt the union-wide objectives; see the relative ranking of outcomes in the first column in panel B, table F1. As discussed above, when a small H country is a follower, it achieves a worse stabilization outcome under union-wide objectives than under national objectives. Indeed, as cross-border effects of its policy are small, and the H country attaches a large relative weight to the objectives of the large F country under union-wide metrics, H reacts much less than it does under its national objectives, and optimal actions of the larger country have destabilizing effects on H. These asymmetries become sufficiently large when the small-size country is also a net debtor country, H country optimally chooses the national objective, and (F_N, L_N) emerges as a Nash equilibrium.

Finally, a unilateral increase of steady-state debt-to-output ratio in small country H yields it higher stabilization losses for both types of shocks, while large country F gains under technology shocks and loses under cost-push shocks; see panel C in table F1. However, the relative change is numerically small, without implications for the ranking of regimes and for stability of policy equilibriums.

Appendix G. Coordinating Role of a Central Bank

Countries' budget constraints can be written as

$$P_{Ht}^{M}\left(B_{Ht}+B_{Ht}^{*}\right)=\left(1+\rho_{H}P_{Ht}^{M}\right)\frac{\left(B_{Ht-1}+B_{Ht-1}^{*}\right)}{1+\pi_{Ht}} + G_t-\tau_t^{l}\frac{W_t}{P_{Ht}}N_t+\left[T_t^{CB}-I_t^{CB}\right]$$

$$P_{Ft}^{M}\left(B_{Ft}+B_{Ft}^{*}\right)=\left(1+\rho_{F}P_{Ft}^{M}\right)\frac{\left(B_{Ft-1}+B_{Ft-1}^{*}\right)}{1+\pi_{Ft}^{*}} + G_t^{*}-\tau_t^{l*}\frac{W_t^{*}}{P_{Ft}^{*}}N_t^{*}+\left[T_t^{CB*}-I_t^{CB*}\right].$$

Here,

- B_{Ht} are home-issued bonds held by home residents,
- B_{Ht}^{*} are home-issued bonds held by foreign residents,
- B_{Ft} are foreign-issued bonds held by foreign residents, and
- B_{Ft}^{*} are foreign-issued bonds held by home residents.

Two terms in square brackets are zero in the benchmark model. Now T_t^{CB} and T_t^{CB*} are extra taxes (transfers) to the fund. Variables I_t^{CB} and I_t^{CB*} are the net borrowing from the fund by H and F governments. However, they are not independent, as

$$I_t^{CB} + I_t^{CB*} = T_t^{CB} + T_t^{CB*}. \tag{83}$$

Equation (83) assumes that all funds are loaned to/borrowed from countries H and F.

Let

$$L_t = T_t^{CB} - I_t^{CB}.$$

We can substitute out I_t^{CB*} to get governments' budget constraints:

$$\begin{aligned} P_{Ht}^M \left(B_{Ht} + B_{Ht}^*\right) &= \left(1 + \rho_H P_{Ht}^M\right) \frac{\left(B_{Ht-1} + B_{Ht-1}^*\right)}{1 + \pi_{Ht}} \\ &\quad + G_t - \tau_t^l \frac{W_t}{P_{Ht}} N_t + L_t \\ P_{Ft}^M \left(B_{Ft} + B_{Ft}^*\right) &= \left(1 + \rho_F P_{Ft}^M\right) \frac{\left(B_{Ft-1} + B_{Ft-1}^*\right)}{1 + \pi_{Ft}^*} \\ &\quad + G_t^* - \tau_t^{l*} \frac{W_t^*}{P_{Ft}^*} N_t^* - L_t. \end{aligned}$$

In other words, the net lending of one country is net borrowing of another.

Table G1 reports results if a small penalty (0.01) on volatility of L_t is imposed for cost-push shocks only.

The net lending L_t can be an additional instrument of the central bank. There can be constraints on its use, like the intertemporal budget constraint. There can be access criteria for fiscal authorities, based on fiscal conditions. This will affect the price of the central bank bonds. The detailed and more realistic modeling of such policy instrument is left for future research.

Table G1. Choice of Leadership, Central Bank Has Two Instruments

A. Choice of Leadership					
		A1. Technology Shocks, $\%C\times10^2$		*A2. Cost-Push Shocks*, $\%C\times10^3$	
		Foreign Country		Foreign Country	
		L_N	F_N	L_N	F_N
Home Country	L_N	**(0.547,0.547)**	(0.544,0.568)	(0.11662,0.11662)	**(0.11370,0.11660)**
	F_N	(0.568,0.544)	(0.547,0.547)	**(0.11660,0.11370)**	(0.11662,0.11662)
B. Choice of Leadership and the Type of Objective					
B1. Technology Shocks, $\%C\times10^2$					
		Foreign Country			
		L_N	L_U	F_N	F_U
Home Country	L_N	**(0.547,0.547)**	(0.516,0.570)	(0.544,0.568)	(0.510,0.602)
	L_U	(0.570,0.516)	(0.538,0.538)	(0.559,0.525)	(0.538,0.538)
	F_N	(0.568,0.544)	(0.525,0.559)	(0.547,0.547)	(0.516,0.570)
	F_U	(0.602,0.510)	(0.538,0.538)	(0.570,0.516)	(0.538,0.538)
B2. Cost-Push Shocks, $\%C\times10^3$					
		Foreign Country			
		L_N	L_U	F_N	F_U
Home Country	L_N	(0.11662,0.11662)	(0.11638,0.11621)	(0.11370,0.11660)	(0.11247,0.11622)
	L_U	(0.11621,0.11638)	**(0.11497,0.11497)**	(0.11416,0.11650)	(0.11497,0.11497)
	F_N	(0.11660,0.11370)	(0.11650,0.11416)	(0.11662,0.11662)	(0.11638,0.11621)
	F_U	(0.11622,0.11247)	(0.11497,0.11497)	(0.11621,0.11638)	(0.11497,0.11497)

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