Optimal Monetary Policy and Fiscal Policy Interaction in a Non-Ricardian Economy*

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This paper studies optimal discretionary monetary policy and its interaction with fiscal policy in a New Keynesian model with finitely lived consumers and government debt. Optimal discretionary monetary policy involves debt stabilization to reduce consumption dispersion across cohorts of consumers. The welfare relevance of debt stabilization is proportional to the debt-to-output ratio and inversely related to the household's probability of survival that affects the household's propensity to consume out of financial wealth. Debt-stabilization bias implies that discretionary optimal policy is suboptimal compared with the inflation-targeting rule that fully stabilizes the output gap and the inflation rate while leaving debt to freely fluctuate in response to demand shocks.

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> "It is clear that there are other economic policy instruments which could improve the effectiveness of monetary policy in closing the output gap. In the past, the limited space for the development of fiscal policy has increased the burden on monetary policy."
>
> Mario Draghi[1]

1. Introduction

Researchers have actively pursued the study of optimal monetary policy over the past four decades.[2] One central assumption across several studies is the Ricardian equivalence theorem, which posits that if consumers are infinitely lived and have complete asset market participation, and if taxation is not distortive (i.e., lump-sum taxes), fiscal policy will not affect consumers' decisions. In other words, changes in public debt fail to influence aggregate demand. Within this framework, influential studies by Clarida, Galí, and Gertler (1999) and Woodford (2003) demonstrate that inflation and output gap stabilization are the optimal monetary policy objective. However, it is difficult to reconcile the study of optimal monetary policy in isolation from fiscal policy, given that the remit of several central banks is to operate monetary policy under stable and prudent debt limits set by governments.[3] As indicated by the introducing excerpt of this article, fiscal policy is of primal consideration for the conduct of monetary policy.[4]

The goal of our paper is to investigate the conduct of optimal monetary policy and the interplay between monetary and fiscal policy in a non-Ricardian model. To this purpose, we incorporate the overlapping-generations (OLG) model of Blanchard-Yaari in a tractable New Keynesian framework. The model accommodates consumers of different cohorts who face a constant probability of dying,

[1] Draghi (2015).

[2] See Clarida, Galí, and Gertler (1999) and Woodford (2010) and references therein for a comprehensive review of the literature.

[3] For instance, the U.K. Treasury and the European Commission set the limit of the debt-to-output ratio to 40 percent and below 60 percent, respectively.

[4] See Abbas et al. (2014) for an overview of debt levels across countries for the period 1900–2011.

and movements in debt holdings that generate critical wealth effects that influence consumption. Since nominal price rigidities create a link between real activity and inflation, changes in fiscal policy that affect consumption also generate relevant interplay between fiscal and monetary policy. By setting the consumers' probability of survival equal to 1, the model nests the standard Ricardian framework with infinitely lived consumers. It therefore enables us to draw direct comparisons with related studies. Monetary policy is set optimally under discretion to minimize the welfare loss due to demand shocks while lump-sum taxes change proportionally to debt movements.[5]

This study establishes the following results. First, in non-Ricardian economies, debt stabilization becomes an additional goal for monetary policy over and above the stabilization of inflation and output gap obtained in standard Ricardian economies. The intuition of this result is straightforward. In the model, lifespan is finite and fluctuations in debt holdings fail to be offset by proportional movements in lump-sum taxation, making movements in debt holdings relevant for consumption. The OLG framework produces consumption differentials across different cohorts of consumers that are amplified by issuance of government debt. Because dispersion in consumption is welfare reducing, movements in debt holdings introduce a tradeoff between the stabilization of inflation and the output gap.[6]

Second, optimal discretionary monetary policy is more complex than in Ricardian economies. It requires us to track current and expected inflation and the output gap in conjunction with movements in debt holdings. The relevance of debt stabilization depends on the extent to which consumption reacts to changes in debt holdings that rely on the degree of the debt-to-output ratio and the lifespan of consumers. Life expectancy is critical for the dynamic properties of the model. When the probability of survival matches the average life expectancy of households in advanced economies,

[5]We depart from similar framework in Benigno and Woodford (2003) and Leith and Wren-Lewis (2013) by assuming lump-sum taxation.

[6]Rajan (2015) discusses the relevance of consumption dispersion for welfare and monetary policy. He notes that a serious attempt to link consumption dispersion with monetary policy requires one to dwell on political economy issues, which are ignored by the majority of studies. The inclusion of political economy considerations will certainly be an interesting, albeit contentious, topic for future research.

the dynamic properties of the model are similar to the Ricardian economy. In this case, optimal monetary policy chiefly stabilizes the output gap and inflation, allowing real debt holdings to fluctuate in response to exogenous demand shocks. However, when the probability of survival decreases, changes in real debt holdings retain important welfare effects, and debt stabilization becomes a quantitatively important objective of monetary policy.[7]

Third, this research establishes important interactions between monetary and fiscal policy. The strength of the response of fiscal policy to debt changes is critical for the tradeoff for monetary policy. For example, when the response of taxes to changes in real debt is strong, the real interest rate reacts less to demand shocks and monetary policy primarily focuses on stabilizing output gap and inflation. As the reaction of taxes to debt fluctuations decreases, the real interest rate response to shocks strengthens and monetary policy is more likely to stabilize debt.

Fourth, an inflation-targeting rule is welfare enhancing compared with the optimal discretionary policy. Although debt stabilization in a non-Ricardian economy is important for welfare, the optimal monetary policy under discretion is still characterized by "debt-stabilization bias," and monetary policy may strategically allow the fluctuations of inflation and output gap to reduce movements of real debt holdings, consistent with the findings in Leith and Wren-Lewis (2013) in a model with a representative agent. Any attempt to reduce movements in real debt holdings is welfare reducing compared with an inflation-targeting rule that stabilizes output gap and inflation.

Our paper is related to studies that investigate the tradeoffs of monetary policy in non-Ricardian economies. Bilbiie (2008) develops a non-Ricardian model that embeds limited asset market participation, establishing that optimal monetary policy weakly responds to inflation. Andres, Arce, and Thomas (2013) develop a New Keynesian model with financial frictions in the form of collateral constraints and a monopolistically competitive banking sector. They find that optimal monetary policy faces a non-trivial tradeoff between the stabilization of output gap, inflation, the "consumption

[7]Del Negro, Giannoni, and Patterson (2012) and Nisticó (2016) provide alternative interpretations of the parameter measuring the consumers' probability of survival. We further discuss this issue in section 4.2.

gap" between borrowers and savers, and a "housing gap" that measures the distortion in the distribution of the collateralizable asset between both groups. Nisticó (2016) studies optimal monetary policy and stock price dynamics in a non-Ricardian model that embeds limited asset market participation across agents of different cohorts. He establishes that the objective of the monetary authority also includes financial stability as an additional target to inflation and the output gap. Compared to these studies, we develop a New Keynesian version of the Blanchard-Yaari model that focuses on the effect of government debt, and, in particular, we focus on the interaction between fiscal and monetary policy. By contrast, the aforementioned studies, with the exception of Bilbiie (2008), focus exclusively on monetary policy.

Our paper also is related to studies that investigate the interaction between fiscal and monetary policy. Kirsanova et al. (2007) investigate fiscal policy issues with non-Ricardian consumers based on the Blanchard-Yaari framework. In an open-economy model, they find that simple fiscal rules, which account for differences in inflation and output across countries, are effective in reducing the impact of asymmetric shocks in a monetary union. Chadha and Nolan (2007) also embed the Blanchard-Yaari framework into a New Keynesian model to characterize systematic, simple monetary and fiscal policy over the business cycle. They establish that conducting a stabilization policy requires fiscal policy that accounts for automatic stabilizers and monetary policy that responds strongly to inflation. Leith and von Thadden (2008) find that in non-Ricardian economies, the effect of fiscal policy on the determinacy of the system depends on the level of government debt. Unlike these studies that use optimal exogenous rules, we perform our analysis in the context of a microfounded welfare function, and we focus on the interplay between optimal monetary and fiscal policy. This approach allows us to investigate the extent to which debt holdings and fiscal policy interplay with monetary policy.

Finally, this analysis is related to Benigno and Woodford (2003) and Schmitt-Grohé and Uribe (2004), who study the interplay between monetary and fiscal policies in a representative-agent framework with distortionary taxes and time-inconsistent policies. Using a similar framework, Leith and Wren-Lewis (2013) show that under time consistency the policymaker strategically uses changes in real debt holdings.

The remainder of the paper is structured as follows. Section 2 sets up the model and derives the aggregate equilibrium. Section 3 characterizes the welfare function in a non-Ricardian economy. Section 4 derives the optimal discretionary monetary policy and it obtains the nominal interest rate that implements optimal discretionary policy. This section also investigates the dynamic properties of optimal policy. Section 5 compares the optimal policy against the inflation-targeting rule. Section 6 concludes.

2. Model

The model is a discrete time version of the Blanchard-Yaari OLG framework. It comprises a continuum of consumers indexed by $i \in [0, 1]$ of different ages, s, each of which faces a survival rate of γ; a continuum of imperfectively competitive firms indexed by $j \in [0, 1]$, where firm j produces good j; a government; and a central bank. The model features enough symmetry to allow the analysis to focus on the behavior of a representative consumer for each cohort, making it straightforward to aggregate across different cohorts and firms. In what follows, we describe the activities of each agent and their implications for the evolution of equilibrium prices and quantities.

2.1 Consumers

During each period $t = 0, 1, 2, ...$, a new cohort, s, of unitary size is born while existing consumers face a constant probability of surviving γ, independent of age. Population growth is zero, implying that the population size is constant and equal to $1/(1 - \gamma)$, and there is no bequest motive, implying that newborns do not hold assets at their births, although they own the present discounted value of their labor income (net of taxes and transfers). As in Blanchard (1985), we assume that insurance companies collect financial wealth from deceased consumers and pay a premium to survivors proportional to their financial wealth, such that each consumer receives a return equal to $1/\gamma$ for each unit of financial wealth in each period.

During each period $t = 0, 1, 2, ...$, each consumer i of age s consumes a bundle of goods, $C_{s,t}(i)$, defined by the CES aggregator over the goods manufactured by each firm j,

$$C_{s,t}\left(i\right)=\left[\int_{0}^{1}C_{s,t}\left(i,j\right)^{\frac{\theta-1}{\theta}}dj\right]^{\frac{\theta}{\theta-1}}, \tag{1}$$

where $\theta > 1$ is the elasticity of substitution among different goods varieties, j, and supplies labor, $N_{s,t}\left(i\right)$, to maximize the expected utility function:

$$\Omega_{s,t}\left(i\right)=E_{t}\sum_{t=0}^{\infty}\beta^{t}\gamma^{t}\left\{\log C_{s,t}\left(i\right)+\varphi\log\left[1-N_{s,t}\left(i\right)\right]\right\}, \tag{2}$$

where β is the discount rate, and φ measures the degree to which leisure $(1-N_{s,t}\left(i\right))$ contributes to utility.

Each consumer i of age s enters period t with bonds, $B_{s,t}\left(i\right)$; supplies units of labor, $N_{s,t}\left(i\right)$, at the nominal wage rate, W_t; and earns equity payouts, D_t. The consumer uses income to purchase new bonds, whose expected value is $\gamma F_{t,t+1}^{s}B_{s,t+1}\left(i\right)$, where $F_{t,t+1}^{s}$ denotes the subjective discount factor at time t, to consume $C_{s,t}\left(i\right)$, and to pay lump-sum nominal taxes, Υ_t. Finally, the consumer receives lump-sum transfers, $T_{s,t}$. Hence, the following budget constraint holds:

$$\begin{aligned}&P_{t}C_{s,t}\left(i\right)+\gamma E_{t}\left[F_{t,t+1}^{s}B_{s,t+1}\left(i\right)\right]\\&\quad\leq W_{t}N_{s,t}\left(i\right)+B_{s,t}\left(i\right)+D_{t}-\Upsilon_{t}-T_{s,t},\end{aligned} \tag{3}$$

where $P_t=\left[\int_0^1 P_t\left(j\right)^{1-\theta}dj\right]^{\frac{1}{1-\theta}}$ is the aggregate price index. As we will discuss later, to ensure that the steady state is efficient, we assume that lump-sum transfers are equal to

$$T_{s,t}=S_{t}+V_{s,t}, \tag{4}$$

where S_t is an employment subsidy to firms that offsets distortions generated by monopolistic competition in the goods market, which is set as a proportion $\varkappa$ of the firm's wage bill, such that $S_t=\varkappa W_t N_t(j)$. The second component, $V_{s,t}$, is transferred to consumers to equate the distribution of steady-state debt holdings across generations. This component is a cohort-specific transfer that ensures uniqueness of the steady state and has no effect on the aggregate dynamics of the economy[8]:

$$V_{s,t} = \gamma F^{s}_{t,t+1} V_{s}, \tag{5}$$

with $V_s = (B_s - B)$, where B is the per capita nominal steady-state public debt, and B_s is the steady-state nominal bond holding for the generation of age s in absence of this redistribution scheme.[9] Finally, we impose that the real debt holdings of consumers must satisfy

$$\lim_{k\to\infty} E_t \left\{ F^{s}_{t,t+k} \gamma^{k} B_{s,t+k} \right\} = 0, \tag{6}$$

where $F^{s}_{t,t+k} = \prod_{l=0}^{k} F^{s}_{t,t+l}$. Hence, the consumer chooses $\{C_{s,t}(i), N_{s,t}(i), B_{s,t+1}(i)\}_{t=0}^{\infty}$ to maximize the utility function (2) subject to the budget constraint (3). The first-order conditions for this problem are

$$\frac{1}{C_{s,t}(i)} = \beta E_t \left\{ \frac{1}{C_{s,t+1}(i)} \frac{1}{F^{s}_{t,t+1}} \frac{P_t}{P_{t+1}} \right\} \tag{7}$$

and

$$\frac{W_t}{P_t} = \varphi \frac{C_{s,t}(i)}{1 - N_{s,t}(i)}. \tag{8}$$

Equations (7) and (8) are standard Euler-consumption and labor supply equations, respectively.

As shown in Woodford (2003), the expected price $E_t \left\{ F^{s}_{t,t+1} \right\}$ of a one-period riskless asset is the reciprocal of the (gross) short-term nominal interest rate, R_t, that is,

$$E_t \left\{ F^{s}_{t,t+1} \right\} = \beta^{k} \frac{C_{s,t}}{C_{s,t+1}} \frac{P_t}{P_{t+1}} = \frac{1}{R_t}. \tag{9}$$

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[8]In principle, the government may use lump-sum taxes to balance the budget in each period or to offset differences in financial wealth across cohorts that are critical to depart from Ricardian equivalence. Since we are interested in the interactions between monetary and fiscal policy in a non-Ricardian economy, we assume that the government is unable to implement these policies and that lump-sum taxes are set according to equation (18). This assumption implies that the government attains a balanced budget and consumers hold the same level of government debt in the steady state only.

[9]Nisticó (2016) uses a similar redistribution scheme in the context of an asset market model.

By solving the Euler equation (7) forward, using the budget constraint (3) and imposing the no-Ponzi scheme condition, we express individual consumption as a linear function of financial wealth, $B_{s,t}(i)$, and human wealth, $H_{s,t}(i)$, defined as an expected stream of future income from wages and dividends net of taxes and transfers:[10]

$$P_t C_{s,t}(i) = \frac{1}{(1+\varphi)\Phi_{s,t}} \left[B_{s,t}(i) - V_s + H_t(i) \right], \tag{10}$$

where

$$H_t(i) = E_t \sum_{k=0}^{\infty} F^{s}_{t,t+k} \gamma^k \left(W_{t+k} + D_{t+k} - \Upsilon_{t+k} - S_{t+k} \right), \tag{11}$$

and $\Phi_{s,t}$ is the inverse of the propensity to consume out of financial and human wealth:

$$\Phi_{s,t} = 1 + \sum_{k=1}^{\infty} (\gamma\beta)^k \prod_{j=1}^{k} E_t \left(\frac{q_{s,t+j}}{q_{s,t+j-1}} \right),$$

with

$$q_{s,t+j} = \frac{\prod_{z=0}^{j-1} R_{t+z}}{P_{t+z} C_{s,t+z}} E_t \left(\frac{P_{t+z} C_{s,t+z}}{\prod_{z=0}^{j-1} R_{t+z}} \right).$$

Equation (11) and equation (9) show that human wealth is the same across cohorts. Therefore, differences in consumption across different cohorts of consumers are driven by the accumulation of financial wealth.

2.2 Firms

During each period $t = 0, 1, 2, ...$, firms hire $N_t(j)$ units of labor from consumers to manufacture $Y_t(j)$ units of a good, j, according to the constant-returns-to-scale production technology described by

[10]The model entails incomplete financial markets and therefore the certainty-equivalence principle fails to hold. However, since the equilibrium of the system is linearized around the non-stochastic steady state, the dynamic model retains certainty-equivalence properties. In the linearized model, the propensity to consume out of financial and human wealth is equal to $(\mathbf{1}-\beta\gamma)$.

$$Y_t(j) = Z_t N_t(j). \tag{12}$$

The aggregate productivity shock, Z_t, evolves according to the law of motion:

$$\ln(Z_t) = \rho_z \ln(Z_{t-1}) + \zeta_{z,t},$$

where $0 < \rho_z < 1$, and $\zeta_{z,t}$ is a stochastic shock with zero mean and standard deviation equal to σ_z. Given the CES form of consumption bundles, integrating the demand for the good j across consumers and assuming that the government allocates spending in the same pattern as consumers implies that the demand for product j is given by

$$Y_t(j) = \left[\frac{P_t(j)}{P_t}\right]^{-\theta} Y_t, \tag{13}$$

where $Y_t = \int_0^1 Y_t(j)dj$.

Following Calvo (1983), we assume that each firm sets a new price, $P_t^*(j)$, with probability $(1-\epsilon)$ during each period $t = 0, 1, 2, ...$ In particular, firm j chooses $\{P_t^*(j)\}_{t=0}^{\infty}$ to maximize its total market value, given by

$$E_t \sum_{t=0}^{\infty} \left(\frac{\epsilon}{R_t}\right)^t \frac{D_t(j)}{P_t}, \tag{14}$$

where

$$D_t(j) = P_t^*(j) Y_t(j) - (1-\varkappa) W_t N_t(j) \tag{15}$$

denotes the firm j profits net of employment subsidy, subject to production technology (12) and the demand function (13). R_t is the gross nominal interest rate. The first-order condition for this problem is

$$P_t^*(j) = \frac{\theta}{\theta-1} \frac{\sum_{t=0}^{\infty} \left(\frac{\epsilon}{R_t}\right)^t (1-\varkappa) \frac{Y_t}{P_t^{1-\theta}} \frac{W_t}{Z_t}}{\sum_{t=0}^{\infty} \left(\frac{\epsilon}{R_t}\right)^t \frac{Y_t}{P_t^{1-\theta}}}. \tag{16}$$

Firms that fail to optimize prices keep them unchanged. As a result, the aggregate price index, P_t, can be rewritten as a weighted average of newly set prices, P_t^*, and those set in the previous period,

P_{t-1}, where the weights are linked to the probability of reoptimizing prices. Therefore, the aggregate price index can be written as

$$P_t = \left[\epsilon \left(P_{t-1}\right)^{1-\theta} + (1-\epsilon)\left(P_t^*\right)^{1-\theta}\right]^{\frac{1}{1-\theta}}.$$

2.3 Government and Central Bank

During each period $t = 0, 1, 2, ...$, the government issues one-period bonds, B_{t+1}, that pay a gross nominal interest rate, R_t, and raises lump-sum taxes, Υ_t, to finance public spending, G_t, and service outstanding debt, B_t. Hence, the government budget constraint is

$$B_{t+1}/R_t + \Upsilon_t = B_t + G_t, \tag{17}$$

where public spending, G_t, evolves according to the law of motion: $\ln(G_t) = \rho_g \ln(G_{t-1}) + \zeta_{g,t}$, with $0 < \rho_g < 1$; the stochastic shock, $\zeta_{g,t}$, has zero mean and standard deviation is equal to σ_g.

As in Leith and von Thadden (2008), fiscal policy is described by a simple rule, which takes the following form:

$$\Upsilon_t = \Upsilon + \phi_1 \left(B_t - B\right), \tag{18}$$

where Υ is the steady-state level of taxes and $\phi_1 > 0$ is the reaction of taxation to outstanding debt. Fiscal policy described in equation (18) implies that the government responds to debt deviations from its steady state.

The central bank is in charge of setting the nominal interest rate, which can be determined optimally to maximize the social welfare. Section 4 characterizes the nominal interest rate that implements optimal discretionary monetary policy.

2.4 Aggregation and Equilibrium

To aggregate across cohorts, we assume that consumers in the same cohort behave identically. Therefore we can drop the index i and consider a representative consumer for each cohort. To study the properties of the aggregate economy, we aggregate across different cohorts and rewrite the model in per capita terms using the rule

$$X_t = (1-\gamma) \sum_{s=0}^{\infty} \gamma^s X_{s,t}, \tag{19}$$

where we take the general variable $X_{s,t}$ referred to as the specific cohort s and rescale it for the number of consumers in the cohort (γ^s), dividing by the size of the population $(1/(1-\gamma))$ to obtain the per capita value of variable X_t. If we apply this rule to equation (10)—recalling that human wealth defined by equation (11) is independent from the cohort and that the redistribution scheme $V_{s,t}$ is equal to zero when aggregated across generations—we obtain the per capita consumption equation:

$$P_t C_t = \frac{1}{(1+\varphi)\Phi}(B_t + H_t). \tag{20}$$

Similarly, the per capita version of the Euler equation (7) is

$$C_t = \frac{1}{\beta R_t} E_t \frac{P_{t+1}}{P_t} \tilde{C}_{t+1}, \tag{21}$$

where $\tilde{C}_{t+1} = \frac{1-\gamma}{\gamma} \sum_{s=1}^{\infty} \gamma^s C_{s,t+1}(i)$ is the per capita consumption in period $t+1$ of consumers alive in period t. Per capita consumption in period $t+1$ can be rewritten as a weighted sum between the per capita consumption of two groups of consumers: those already alive in period t and newborns in period $t+1$ (C_{t+1}^{NB}), where the weights are the size of the two groups:

$$C_{t+1} = \gamma \tilde{C}_{t+1} + (1-\gamma) C_{t+1}^{NB}. \tag{22}$$

Combining equations (22) and (21), we obtain

$$C_t - \frac{1}{R_t \beta} E_t \frac{P_{t+1}}{P_t} C_{t+1} = \frac{(1-\gamma)}{\gamma} \frac{1}{R_t \beta} E_t \frac{P_{t+1}}{P_t} \left(C_{t+1}^{NB} - C_{t+1}\right). \tag{23}$$

Equation (23) differs from the Euler equation for consumption with Ricardian agents for the term proportional to the gap between consumption of newborns in period $t+1$ and the per capita consumption. However, note that deriving equation (10) for newborns (that implies $B_{s,t} = 0$ and $V_s = B$ for newborns) and subtracting the outcome to equation (20) yields

$$E_t\left(C_{t+1}^{NB} - C_{t+1}\right) = E_t \frac{1}{(1+\varphi)\Phi}(b_{t+1} - b), \tag{24}$$

where b_{t+1} and b are the real bond holdings at the beginning of time $t+1$ and at the steady state, respectively. Equation (24) reveals that the difference between the consumption of newborns and per capita consumption is proportional to the deviation of per capita wealth from its equilibrium level.[11] Finally, combining equations (23) and (24), we derive the dynamic equation of per capita consumption:

$$C_t = \frac{1}{R_t \beta} E_t \frac{P_{t+1}}{P_t} \left\{ C_{t+1} + \frac{1-\gamma}{\gamma} \frac{1}{(1+\varphi)\Phi} (b_{t+1} - b) \right\}. \tag{25}$$

Equation (25) shows that when the probability of surviving is high (i.e., $\gamma \to 1$) and when the propensity to consume financial wealth falls (i.e., $1/(1+\varphi)\Phi \to 0$), movements in real bond holdings have a limited effect on aggregate consumption.

Using equation (8), the per capita labor supply is

$$\frac{W_t}{P_t} = \varphi \frac{C_t}{1 - N_t}. \tag{26}$$

Substituting equation (13) into equation (12) and aggregating across firms yields

$$\Delta_t Y_t = Z_t N_t, \tag{27}$$

where $\Delta_t = \int_0^1 \left[P_t(j)/P_t\right]^{-\theta} dj$ is an index of price dispersion. We derive the aggregate resource constraint by combining the aggregate consumer budget constraint (3), the firm profits equation (15), and the government budget constraint (17), which yields

$$Y_t = C_t + G_t. \tag{28}$$

To close the model, we assume that the central bank sets the nominal interest rate under discretion as the result of a linear-quadratic optimization problem, as in Woodford (2003) and Benigno and Woodford (2012).

[11]This property of the model results from the fact that human wealth is the same across cohorts, as shown in equation (11).

2.5 The Linearized Equilibrium

To provide a tractable, analytical solution of the model, we linearize the equilibrium conditions around the stationary steady state.[12] We express each variable in deviation from its value at the efficient equilibrium that occurs when a benevolent central planner sets the allocations. We also derive the allocations in a flexible-price equilibrium to facilitate the interpretation of results.

We determine efficient allocations by a benevolent social planner who maximizes utility (2), subject to the production technology in equation (12) and the resource constraint in equation (28). The first-order conditions are

$$Z_t = \varphi \frac{C_t^e}{1 - N_t^e}, \tag{29}$$

$$Y_t^e = C_t^e + G_t, \tag{30}$$

where the superscript "e" denotes the efficient level of the variable. Equations (29) and (30) show that consumption and the labor supply are equalized across consumers and that debt holdings are irrelevant for the equilibrium.

To achieve the efficient equilibrium defined by equations (29) and (30), we correct for the distortions introduced by monopolistic competition and the differences in debt holdings across generations.

To describe the linearized equilibrium, we let the symbol "$\hat{}$" denote the percentage (logarithmic) deviation of the variable from its steady-state value.[13] To study optimal monetary policy, we rewrite the system in terms of the output gap ($\hat{x}_t$), defined as the deviation of output from its efficient level or as the difference between the output deviations from its steady state under sticky prices ($\hat{y}_t^e$) and efficient allocation ($\hat{y}_t^e$), such that $\hat{x}_t = \hat{y}_t - \hat{y}_t^e$, where[14]

$$\hat{y}_t^e = \hat{z}_t + \frac{1}{(1+\eta)} \hat{g}_t. \tag{31}$$

[12] Appendix 1 reports the derivation of the model.

[13] Appendix 1 shows the derivation of the steady state of the model. Public spending is defined in deviations from the steady state of output (since the steady state of G is zero), such that $\hat{g}_t = (G_t - G) / Y$.

[14] The presence of a subsidy makes the steady-state level of output identical to the efficient level of output ($Y = Y^e$). Therefore, $\hat{x}_t = (Y_t - Y_t^e)/Y = \hat{y}_t - \hat{y}_t^e$.

The linearized model comprises three endogenous variables $\{\hat{x}_t, \hat{\pi}_t, \hat{b}_t\}$ and two exogenous variables $\{\widehat{z}_t, \widehat{g}_t\}$. The dynamics of endogenous variables are described by the following set of equations:

$$\hat{x}_t = E_t\hat{x}_{t+1} - (\widehat{r}_t - E_t\pi_{t+1} - \hat{\varrho}^e_t) + \frac{(1-\gamma)}{(1+\varphi)\gamma}\frac{\upsilon_B}{\Phi}\hat{b}_{t+1}, \tag{32}$$

$$\hat{\pi}_t = \beta E_t\hat{\pi}_{t+1} + \kappa\hat{x}_t, \tag{33}$$

$$\hat{b}_{t+1} = (\widehat{r}_t - E_t\hat{\pi}_{t+1} - \hat{\varrho}^e_t) + \frac{1}{\beta}(1-\phi_1)\hat{b}_t + f^*_t, \tag{34}$$

where $\kappa = [(1-\beta\epsilon)(1-\epsilon)(1+\eta)]/\epsilon$, $\eta = \bar{N}/(1-\bar{N})$ is the inverse of the Frisch elasticity of labor supply, and υ_B is the steady-state debt-to-output ratio (i.e., $\upsilon_B = B/Y$). The variable $\hat{\varrho}^e_t$ in equations (32) and (34) denotes the efficient natural interest rate:

$$\hat{\varrho}^e_t = \frac{\eta(1-\rho_g)}{1+\eta}\hat{g}_t - (1-\rho_z)\hat{z}_t. \tag{35}$$

The variable f^*_t tracks movements in exogenous shocks:

$$f^*_t = \hat{\varrho}^e_t + \frac{1}{\beta\upsilon_B}\hat{g}_t,$$

and it introduces a tradeoff between the full stabilization of inflation and the output gap in the Euler equation (32) that precludes complete debt stabilization as an objective of monetary policy.

The exogenous variables of technology and government spending, $\widehat{z}_t$ and $\widehat{g}_t$, evolve according to the law of motion: $\widehat{z}_t = \rho_z\widehat{z}_{t-1} + \zeta_{z,t}$ and $\widehat{g}_t = \rho_g\widehat{g}_{t-1} + \zeta_{g,t}$, where $\zeta_{z,t}$ and $\zeta_{z,t}$ are white-noise innovations. Equations (32)–(34) help us interpret tradeoffs for monetary policy. In the Ricardian economy, which assumes that consumers are infinitively lived (i.e., $\gamma = 1$), debt holdings have no real effects (i.e., the last term related to debt holdings in equation (32) drops out), and the optimal policy involves setting the nominal interest rate to track the efficient interest rate. Such policy simultaneously stabilizes inflation and the output gap. However, if the economy is non-Ricardian and consumers are finitely lived (i.e., $\gamma < 1$), debt holdings retain a powerful wealth effect and influence consumption, as shown in equation (32). A policy that sets $\widehat{r}_t = \hat{\varrho}^e_t$ is suboptimal since monetary policy is unable to simultaneously stabilize inflation, the output gap, and debt holdings. In this sense, debt holdings generate an endogenous tradeoff between inflation and output gap stabilization that emerges from any shock occurring in the economy. By contrast, in Ricardian economies, a tradeoff between the output gap and inflation arises in the presence of exogenous cost-push shocks only, as discussed in Woodford (2003).

3. The Welfare Function

This section characterizes the welfare function for the non-Ricardian economy using a linear-quadratic approximation of consumers' utility, as in Woodford (2003) and Benigno and Woodford (2012).[15]

By deriving a microfounded welfare function, we address critical issues pertaining to the use of OLG models for welfare analysis. Calvo (1983) points out that welfare analysis should account for the utility of unborn generations to prevent time inconsistency related to the different impact of policies across cohorts. Similarly, Kirsanova and Wren-Lewis (2012) argue that the welfare function should aggregate consumption across generations, weighted for the utility in each generation. Our analysis internalizes these issues and accounts for the utility of unborn generations by the dispersion of consumption across generations, leading to different welfare objectives.[16]

PROPOSITION 1. *If the steady state is efficient, the aggregate welfare function, Ω_t, can be approximated by (ignoring terms independent from policy and of third or higher order)*

$$\Omega_t \simeq -\frac{1}{2}\sum_{t=0}^{\infty}\beta^t\left[a_x\hat{x}_t^2 + \pi_t^2 + \tilde{a}_b var_s\hat{c}_t\right], \tag{36}$$

where $a_x = \kappa/\theta$ and $\tilde{a}_b = a_x/\eta$.

Proof. See appendix 2.

[15] As outlined in section 2.5, we assume an efficient steady state, achieved by employment subsidies that offset the distortion from monopolistic competition and transfers that equate debt across cohorts in equilibrium.

[16] Nisticó (2016) provides a related analysis in the context of a model with stock-wealth effects.

Proposition 1 shows that in a non-Ricardian economy, the welfare function depends on three terms. The first term is inflation because it creates inefficient price dispersion. The second term is the output gap since nominal price rigidities generate inefficiency in output and labor input fluctuations. These two terms also are present in Ricardian economies. The third term refers to the volatility of consumption across different cohorts of consumers, which encapsulates the distortions associated with non-Ricardian consumers. This additional term becomes part of the welfare function since a convex utility implies that dispersion of consumption across consumers belonging to different cohorts reduces welfare. In a non-Ricardian economy, aggregate utility depends not only on the aggregate level of consumption, as in the Ricardian model, but also on the variability of consumption around the aggregate level.

To further interpret proposition 1, we show that the dispersion of consumption across cohorts is tightly linked to the deviation of debt from its steady state and that movements in bond holdings are relevant for welfare. We re-express the welfare function in equation (36) in terms of deviations in bond holdings from the steady state, which proves convenient for the subsequent analysis and the interpretation of results.

LEMMA 1. *In any period t, the variability of consumption across generations is a function of the dispersion of expected consumption in period $t+1$ and the deviation of the per capita real debt from the steady state:*

$$var_s \hat{c}_t = \frac{1}{\gamma} var_s E_t \hat{c}_{t+1} + (1-\gamma)\left(\frac{\upsilon_B}{\gamma\Phi}\right)^2 \hat{b}_t^2. \tag{37}$$

Proof. See appendix 2.

Lemma 1 identifies a precise mapping between the dispersion of consumption and movements in real bond holdings. In particular, equation (37) shows that the dispersion of consumption relates to the deviation of the average per capita real debt from the steady state. In a non-Ricardian economy populated by finitely lived consumers, movements in bond holdings are not offset by proportional current or future movements in taxes, and government debt exerts real effects on consumption. Using lemma 1, we reinterpret the welfare function, as outlined in proposition 2.

PROPOSITION 2. *If the steady state is efficient, the aggregate welfare function can be expressed as*

$$\Omega_t = \sum_{t=0}^{\infty} \beta^t L_t \approx -\frac{1}{2} \sum_{t=0}^{\infty} \beta^t \left[a_x \hat{x}_t^2 + \hat{\pi}_t^2 + a_b \hat{b}_t^2 \right], \tag{38}$$

where

$$a_b = \frac{(1-\gamma)}{\Phi} \left(\frac{1}{\gamma} \frac{\upsilon_B}{1+\varphi} \right)^2 \tilde{a}_b.$$

Proof. See appendix 2.

Proposition 2 establishes that in a non-Ricardian economy, welfare also depends on the extent to which bond holdings deviate from the equilibrium level since, as discussed above and shown in lemma 1, movements in bond holdings generate dispersion in consumption that is welfare reducing.

By assuming that consumers are infinitely lived (i.e., $\gamma = 1$), the model nests the standard Ricardian economy in which bond holdings have no real effect and therefore no welfare effects. In this instance, a monetary policy that stabilizes inflation also achieves a zero output gap in the absence of cost-push shocks, as shown in Woodford (2003).[17] Instead, when consumers are finitely lived (i.e., $\gamma < 1$), proposition 2 shows that the presence of non-Ricardian consumers generates a tradeoff between inflation, the output gap, and debt stabilization. To gain an understanding of the policy tradeoff faced by the monetary authority, we compare the results to an economy with flexible prices. Under flexible prices, the monetary authority has no influence on the real interest rate, which tracks one-to-one the natural interest rate and consequently fluctuates in response of exogenous shocks. This policy stabilizes the output gap (i.e., $\hat{x}_t = E_t \hat{x}_{t+1}$, from the Euler equation (32)) and allows the real

[17]To see this in the context of the model, by keeping inflation on target, the Phillips curve in equation (33) implies that the output gap also is closed. Therefore no welfare loss occurs.

debt holdings to freely fluctuate in response to exogenous shocks. The monetary authority achieves stabilization of inflation and the output gap by letting debt holdings freely fluctuate in response to exogenous shocks. In contrast, under nominal price rigidities, the monetary authority faces a tradeoff between the stabilization of inflation, the output gap, and debt holdings, and it uses the real interest rate to control debt and improve welfare.

4. Optimal Monetary Policy and Fiscal Interaction

This section focuses on the interaction between optimal monetary and fiscal policy. We first derive the optimal discretionary monetary policy and investigate its dynamic properties in relation to fiscal policy. We then specify and interpret an operational Taylor rule that is consistent with the optimal policy.

4.1 Optimal Discretionary Monetary Policy

The analytical solution for the optimal discretionary policy is complicated by the coexistence of forward- and backward-looking variables. To overcome this issue, we postulate that forward-looking variables evolve in relation to a set of assigned state variables, as in Maskin and Tirole (2001). We assume that the expected output gap ($E_t\hat{x}_{t+1}$) and inflation ($E_t\hat{\pi}_{t+1}$) evolve in relation to the endogenous state variable of debt holdings ($\hat{b}_{t+1}$) and the exogenous state variables of disturbances (f_t^*), according to the equations

$$E_t\hat{x}_{t+1} = m_1\hat{b}_{t+1} + m_2 f_t^*, \tag{39}$$

$$E_t\hat{\pi}_{t+1} = n_1\hat{b}_{t+1} + n_2 f_t^*, \tag{40}$$

where m_1, m_2, n_1, and n_2 are unknown coefficients. Substituting equations (39) and (40) into the Euler equation (32) and the Phillips-curve equation (33), and the law of motion of bond holdings into equation (34), yields

$$\hat{x}_t = -\left(\widehat{r}_t - \hat{\varrho}_t^e\right) + \left(\Psi_B + m_1 + n_1\right)\hat{b}_{t+1} + \left(m_2 + n_2\right) f_t^*, \tag{41}$$

$$\begin{aligned}\hat{\pi}_t = &-\kappa\left(\widehat{r}_t - \hat{\varrho}_t^e\right) + \left[\beta n_1 + \kappa\left(\Psi_B + m_1 + n_1\right)\right]\hat{b}_{t+1} \\ &+ \left(\kappa m_2 + \kappa n_2 + \beta n_2\right) f_t^*,\end{aligned} \tag{42}$$

$$\hat{b}_{t+1} = \left(\frac{\widehat{r}_t - \hat{\varrho}_t^e}{1+n_1}\right) + \frac{1-\phi_1}{\beta\left(1+n_1\right)}\hat{b}_t + \left(\frac{1-n_2}{1+n_1}\right) f_t^*, \tag{43}$$

where $\Psi_B = (1-\gamma)\, \upsilon_B / \gamma\Phi\left(1+\varphi\right)$. Equations (41)–(43) are isomorphic to the original system of equations (32)–(34), but the forward-looking variables, $E_t\hat{x}_{t+1}$ and $E_t\hat{\pi}_{t+1}$, are expressed in terms of the endogenous state variables, $\hat{b}_{t+1}$ and f_t^*.

The optimization problem of the monetary authority solves the following Bellman equation:

$$V\left(b_t, f_t^*\right) = \min_{r_t}\left[-\frac{1}{2}\left(\alpha_x\hat{x}_t^2 + \pi_t^2 + \alpha_b\hat{b}_t^2\right) + \beta E_t V\left(b_{t+1}, f_{t+1}^*\right)\right], \tag{44}$$

subject to constraints given by equations (41)–(43). The nominal interest rate $(\widehat{r}_t)$ is the control variable, and the real debt holding $(\hat{b}_{t+1})$ is the endogenous state variable.[18] The following proposition defines the optimal discretionary monetary policy.

Proposition 3. *Under discretionary monetary policy, the optimal rule is*

$$\begin{aligned}(1-\phi_1)\left[\alpha_x E_t\hat{x}_{t+1} + \kappa E_t\hat{\pi}_{t+1}\right] + \beta\alpha_b\hat{b}_{t+1} \\ = \alpha_x\left(1-\Psi_B-m_1\right)\hat{x}_t + \left[\kappa\left(1-\Psi_B-m_1\right) - \beta n_1\right]\hat{\pi}_t.\end{aligned} \tag{45}$$

Proof. See appendix 2.

Proposition 3 establishes that the condition for optimal monetary policy is more involved than the equivalent policy in the Ricardian framework, which is obtained by assuming infinitely lived consumers (i.e., by setting $\gamma = 1$ in equation (45)):

$$\alpha_x\hat{x}_t + k\widehat{\pi}_t = 0. \tag{46}$$

[18] Inflation and the output gap from equations (41)–(42) can be substituted into equation (44). In this way, the solution of the recursive problem involves the choice of the control variable, $\hat{r}_t$, subject to the law of motion of the real debt holdings. Note that, unlike in the Ricardian economy, the interest rate that stabilizes the output gap cannot be derived from the Euler equation for consumption, which is a binding constraint in our optimization problem.

Since debt holdings are part of the optimization problem of non-Ricardian consumers, the optimal policy is dynamic and comprises present and expected value of the output gap and inflation, jointly, with the value of debt holding. Therefore, as outlined in the previous section and in proposition 3, the stabilization of output gap and the inflation rate is suboptimal for monetary policy because this policy involves inefficient fluctuations in debt holdings.

On one hand, fluctuations in real debt holdings increase consumption dispersion across cohorts, which in turn reduce aggregate welfare, as established in lemma 1. On the other hand, fluctuations in real debt are important because they also affect the output gap and may have a non-trivial effect on welfare, as shown in equation (32). To disentangle these two effects, we consider the special case of a monetary policy that maximizes the utility of the *average* consumer in contrast to the utility across different cohorts.[19] In this example, consumers are still finitely lived (i.e., $\gamma < 1$), but the monetary authority fails to internalize the effect of consumption dispersion across cohorts. As a result, fluctuations in bond holdings become irrelevant for welfare (which is equivalent to setting $a_b = 0$ in equation (38)) despite movements in debt holdings that continue to influence aggregate consumption. The optimal policy becomes

$$\Psi \left(\alpha_x \hat{x}_t + k\pi_t\right) = \left(1 - \phi_1\right) \left(\alpha_x E_t \hat{x}_{t+1} + k E_t \pi_{t+1}\right), \tag{47}$$

where $\Psi = 1 - \Psi_B$. Since the inequality, $\Psi / \left(1 - \phi_1\right) > 1$, holds for plausible calibrations of the model, the unique solution, $\alpha_x \hat{x}_t + k\widehat{\pi}_t = 0$, to equation (47) rules out explosive paths for output gap and inflation rate. It coincides with the solution in the Ricardian economy, as outlined in equation (46). If consumption dispersion is irrelevant for welfare, optimal policy remains the same as in the Ricardian economy, even though consumers are finitely lived and the law of bonds accumulation in equation (34) holds.

[19]For simplicity, we assume that consumption and worked hours of the representative household are equal to their per capita value. The welfare loss is then derived as described in appendix 2, with the exception that we do not need to aggregate across different cohorts of consumers.

4.2 Dynamic Properties of Optimal Discretionary Policy

To simulate and study the dynamic properties of the model, we find analytical solutions to the four unknown parameters, m_1, m_2, n_1, and n_2, in equations (39) and (40). We define an additional equation that describes the evolution of the endogenous state variable:

$$\hat{b}_{t+1} = w_1 \hat{b}_t + w_2 f_t^*. \tag{48}$$

By using equation (48), we can express the unknown coefficients, m_1, m_2, n_1, and n_2, in equations (39) and (40) in terms of the parameters, w_1 and w_2, in equation (48):

$$m_1 = \frac{\frac{1}{\beta}(1-\phi_1) - \Psi w_1}{1-w_1}, \tag{49}$$

$$m_2 = \rho \tilde{m}_2 = \rho \frac{m_1 w_2 + 1 - \Psi w_2}{1-\rho}, \tag{50}$$

$$n_1 = \frac{\kappa m_1}{1-\beta w_1}, \tag{51}$$

$$n_2 = \rho \tilde{n}_2 = \rho \frac{\beta n_1 w_2 + \kappa \tilde{m}_2}{1-\beta\rho}, \tag{52}$$

$$w_2 = \frac{(\alpha_x \tilde{m}_2 + \kappa \tilde{n}_2)\left[\Psi - m_1 - (1-\phi_1)\rho\right] - \beta n_1 \tilde{n}_2}{(1-\phi_1)\alpha_x m_1 + (1-\phi_1)\kappa n_1 + \beta\alpha_b}. \tag{53}$$

We can solve equations (48)–(53) recursively once we determine the value of w_1.[20] To select a value for w_1 among admissible solutions, we use the minimal state variable (MSV) method described in McCallum (1983, 1999, 2003).[21] To determine w_1, we solve an

[20] To find the unknown coefficient w_1, we solve a fifth-order equation in w_1. This equation is obtained using equations (32) and (34) to substitute inflation and the output gap in equation (45) that describes the optimal discretionary policy. Then we apply equations (39) and (40) to the resulting equation to write the $E_t\hat{x}_{t+1}$ and $E_t\hat{\pi}_{t+1}$ in terms of $\hat{b}_{t+1}$ and f_t^*. An appendix with analytical derivations is available on request.

[21] The method consists in finding a special combination of structural parameters that imply a zero coefficient for at least one backward-looking variable in the system and that solve the unknown coefficients problem for this special case. It is then straightforward to choose the solution of the unknown coefficients problem because it has to return a zero at the coefficient associated with the backward-looking variable. We use this particular solution to find values of the

equation of the fifth order. Therefore the MSV problem lacks an analytical solution, making us resort to numerical simulations. To simulate the model, we calibrate parameters β, θ, γ, ϵ, φ, ρ_z, and ρ_g and allow the parameters υ_B and ϕ_1 to vary across a broad range of plausible values.

The probability of survival, γ, is the critical parameter that controls the size of the non-Ricardian channel in the OLG model. Importantly, this parameter relates to the relevance of real debt fluctuations for consumption, as outlined in equation (32), and therefore controls the overall effect of debt on welfare. In our benchmark calibration, we set this parameter equal to 0.995 to match the average life expectancy of households of fifty years, consistent with Del Negro, Giannoni, and Patterson (2012). Their analysis uses actuarial life tables from the Social Security Administration to construct survival probability for individuals aged twenty years and above, weighting by the population at each age. We also provide results for additional calibrations of this parameter based on alternative interpretations advanced in Del Negro, Giannoni, and Patterson (2012). They interpret the probability of survival as the likelihood that households avoid financial default, suggesting that a plausible value of γ is in the range of 0.987 and 0.94. Similarly, Nisticó (2016) interprets the perpetual youth model as one in which agents move in and out of hand-to-mouth status.[22] Consequently, he proposes a value of γ equal to 0.93.[23] In line with these alternative interpretations, we provide a sensitivity analysis using the alternative values of γ equal to 0.97 and 0.93, respectively. We set the discount rate,

unknown coefficients for any possible combination of the structural parameters of the model. In our model, we focus on the special case of Ricardian consumers by imposing that the probability of survival is equal to 1 (i.e., $\gamma = 1$). This assumption makes the state variable $\hat{b}_{t+1}$ irrelevant for the dynamics of the system (32) and (34), implying that real debt holdings are not part of the minimum set of state variables. Therefore, in this special case, we select the MSV solution by choosing w_1, such that m_1 and n_1 are equal to 0.

[22]For alternative interpretations of the parameter γ, see Del Negro, Giannoni, and Patterson (2012).

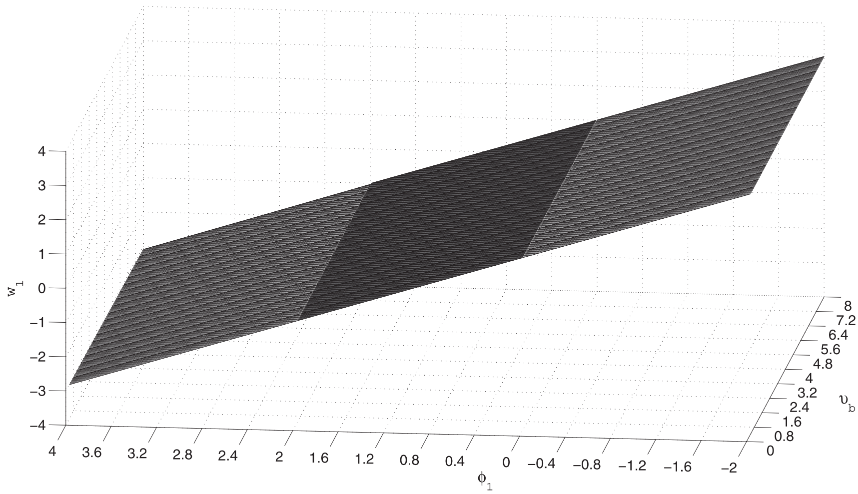
[23]The alternative calibrations for parameter γ proposed in Del Negro, Giannoni, and Patterson (2012) and Nisticó (2016) generate a small value for the parameter that encapsulates the propensity to consume out financial wealth (Φ). These values are smaller than the large estimate in Castelnuovo and Nisticó (2010), obtained using Bayesian methods on aggregate data.

β, equal to 0.9928 to replicate the annual real interest rate of 3 percent in the data. The value of the elasticity of substitution across different goods, θ, is set to equal 8 to match a steady-state markup of approximately 15 percent. The probability of retaining the same price, ϵ, is set equal to 0.75, consistent with an average duration of price changes (i.e., $1/(1-\epsilon)$) of approximately ten months. The degree at which leisure contributes to utility, φ, is set equal to 0.67 to match the inverse of the Frisch elasticity of labor equal to 1.5, in line with micro- and macro-evidence, as described in Keane and Rogerson (2012). We let the parameter, υ_B, vary between 0 and 8, which implies steady-state debt-to-output ratios between 0 and 200 percent. We allow the response of taxes to debt issuance, ϕ_1, to cover a wide range of values between -2 and 4. Finally, the autoregressive coefficients of exogenous shocks to technology and public spending, ρ_z and ρ_g, respectively, are set equal to 0.8, as in Carlstrom, Fuerst, and Paustian (2009).

Figure 1 plots values of the parameter w_1 for a wide range of values of the debt-to-output ratio (υ_B) and the response of taxation to debt (ϕ_1) when $\gamma = 0.955$. The dark grey area shows the values of ϕ_1 and υ_B associated with a stable solution of the system. The model is stable to the extent that real debt is not on an explosive path, which requires the condition $|w_1| < 1$ to hold. The figure shows that the response of fiscal policy to the changes in real debt holdings must be positive and sufficiently strong to ensure debt stability since the coefficient $|w_1|$ becomes greater than 1 for negative values of ϕ_1 and when ϕ_1 is very close to 0. In addition, the response of fiscal policy must be within certain limits since $|w_1|$ becomes greater than 1 for values of ϕ_1 close to 2. In the figure, changes in debt-to-output ratio have limited impact on the value of w_1, showing that the stability of the system is principally related to the response to fiscal policy to debt—the system remains stable for a broad range of values of the debt-to-output ratio. When the probability of survival (γ) decreases, the stability region diminishes but with a limited quantitative effect.

The figure shows that the value of w_1, which enters in the debt equation (48) as the autoregressive term, decreases with ϕ_1 and increases with υ_B. The intuition is straightforward. A strong response of taxes to debt supports the reduction of debt and prevents debt from reaching an explosive path. By contrast, a high value of debt-to-output ratio moves debt further away from the steady state

Figure 1. Model Stability (coefficient w_1)



in response to a shock and therefore obstructs the convergence of debt to the steady state. When ϕ_1 is greater than 1, the coefficient w_1 is negative. In this case, the response of fiscal policy is sufficiently strong to generate a change in taxes that more than offsets movements in real debt holdings in the previous period.

4.2.1 Nominal Interest Rate and Optimal Policy

Several central banks choose the nominal interest rate to be their main policy instrument. Therefore, it is relevant to derive an equation for the nominal interest rate, which implements the equilibrium of optimal discretionary monetary policy.

PROPOSITION 4. *A nominal interest rate rule that implements optimal discretionary monetary policy can be written as*

$$\widehat{r}_t - \hat{\varrho}_t^e = \phi_x E_t \hat{x}_{t+1} + \phi_\pi E_t \hat{\pi}_{t+1} + \phi_b \widehat{b}_{t+1}, \tag{54}$$

where the coefficients of the variables are

$$\phi_x = 1 + \frac{(1-\phi_1)\,\alpha_x}{\left[\alpha_x\,(m_1 - \Psi) + \kappa^2\,(m_1 - \Psi) + \beta n_1 \kappa\right]} \tag{55}$$

$$\phi_\pi = 1 + \frac{\left[\kappa\left(m_1 - \Psi\right) + \beta n_1\right]\beta + \kappa\left(1 - \phi_1\right)}{\left[\alpha_x\left(m_1 - \Psi\right) + \kappa^2\left(m_1 - \Psi\right) + \beta n_1 \kappa\right]} \tag{56}$$

$$\phi_b = \Psi_b + \frac{\beta\alpha_b}{\left[\alpha_x\left(m_1 - \Psi\right) + \kappa^2\left(m_1 - \Psi\right) + \beta n_1 \kappa\right]}. \tag{57}$$

Proof. See appendix 2.

To facilitate interpretation, the optimal discretionary monetary policy rule in equation (54) is expressed in terms of the nominal interest rate gap. We define the gap as the difference between the nominal interest rate under discretion and the efficient equilibrium. Without the cost-push shock and with Ricardian consumers, the optimal discretionary policy implies that monetary policy closes the output gap and stabilizes inflation by setting $\widehat{r}_t$ to track the interest rate under flexible prices, as discussed in Woodford (2003). In our model, equation (54) shows that this policy is no longer optimal since debt issuance introduces consumption dispersion across finitely lived consumers. Although the three policy coefficients, ϕ_x, ϕ_π, and ϕ_b, appear convoluted, they show that an optimal response of the nominal interest rate depends on the steady-state debt-to-output ratio (v_b) and the response of taxes to debt (ϕ_1).

Figures 2 and 3 show that the size of parameter ϕ_b is substantially smaller than the size of ϕ_x and ϕ_π, and it is negatively related to the fiscal policy parameter ϕ_1. In contrast, both ϕ_x and ϕ_π increase with the fiscal policy parameter ϕ_1, showing that a high response of taxation to debt increases the reaction of monetary policy to changes in expected inflation and the output gap, since movements in debt holdings are principally stabilized by fiscal policy. When the debt-to-output ratio (v_b) increases, the response of the interest rate to expected inflation and the output gap decreases because the welfare implications of debt stabilization become more relevant. The response of the nominal interest rate to the endogenous variables increases with the shortening of the expected lifespan of consumers.

4.2.2 Impulse Response Functions

To investigate the dynamic properties of the model and study the response of the variables to a public spending shock, we produce impulse response functions. To disentangle the interactions between

Figure 2. Interest Rate Rule Parameters for Different Values of ϕ_1 ($\upsilon_b = 2.4$)

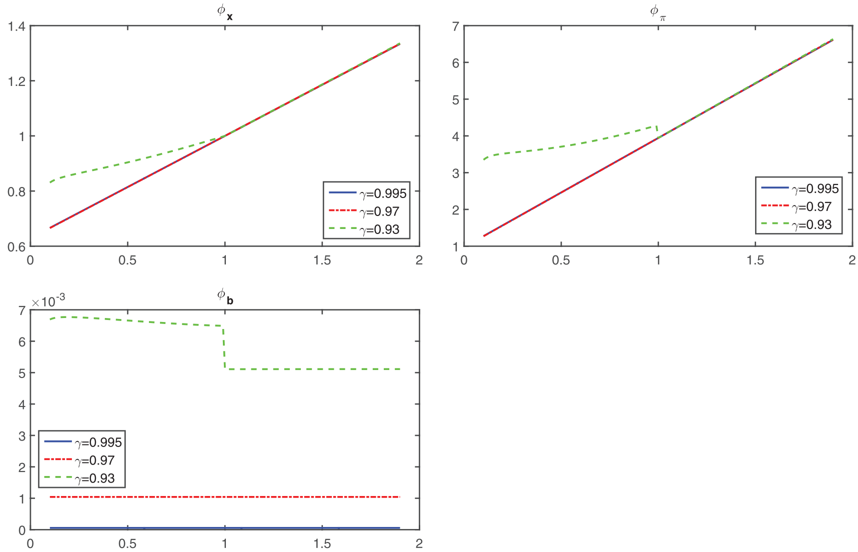
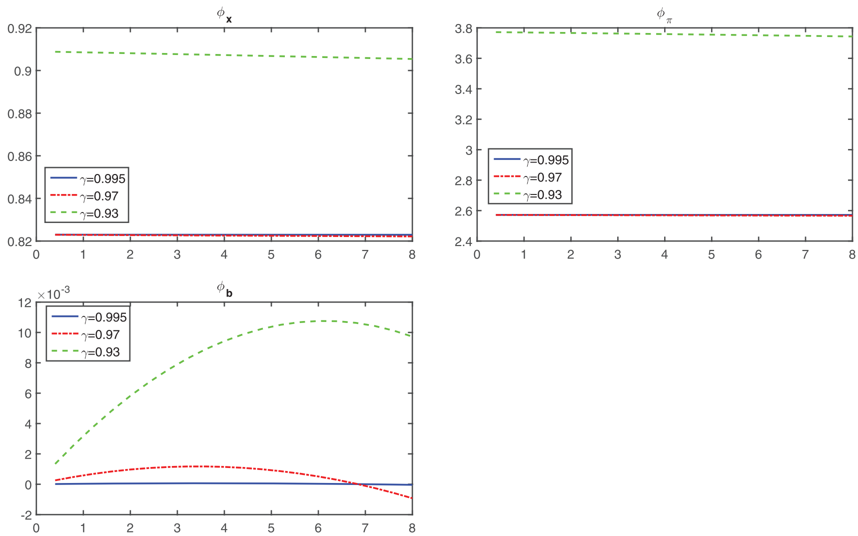


Figure 3. Interest Rate Rule Parameters for Different Values of υ_b ($\phi_1 = 0.5$)



optimal monetary policy and fiscal policy, we plot the variables' responses for different values of the fiscal policy parameter (ϕ_1) and debt-to-output ratios (υ_B).[24] In a Ricardian economy, the central bank reacts to a public spending shock by stabilizing the output gap and the inflation rate, allowing the real debt to fluctuate. However, when consumers are non-Ricardian, debt stabilization becomes an additional target for monetary policy, and the monetary authority now faces a tradeoff across policy objectives. On the one hand, to stabilize inflation and the output gap, the central bank has to raise the real interest rate. On the other hand, the increase in the real interest rate amplifies fluctuations in debt holdings due to debt servicing costs. Figures 4 and 6 show the variables' responses to a positive government spending shock when the probability of surviving is set to $\gamma = 0.93$.[25] Figures 5 and 7 show impulse response functions for $\gamma = 0.995$. The shape of the variables' responses are qualitatively similar for both values of γ, but the reaction to the shock is larger on the output gap and inflation when γ is small. In general, optimal monetary policy entails a limited response in inflation and the output gap and large shifts in debt holdings since they exert a limited wealth effect on the system.

The impulse response functions are indicative of additional interactions between monetary and fiscal policy in a non-Ricardian economy. Figures 4 and 5 show that a strong response of fiscal policy to debt deviations contributes to reduce fluctuations in the economy since it stabilizes real debt holdings.[26] Thus, the interest rate becomes less volatile over the business cycle. However, changes in the fiscal policy parameter (ϕ_1) also affect the tradeoff faced by the central bank, as shown in equation (45). The coefficients ϕ_x, ϕ_π in the interest rate rule increase with the parameter ϕ_1, implying a stronger reaction of the nominal interest rate to fluctuations in the output gap and the inflation rate. These two opposing forces

[24] Figures 9–12 in Appendix 1 show the impulse response functions to the technological shock.

[25] The size of innovation in government public spending is equal to one standard deviation.

[26] Since we focus on values of ϕ_1 that are positive and less than 1, we assume that the three alternative values (0.3, 0.6, and 0.9) are associated with a weak, medium, and strong reaction of lump-sum taxes to debt deviations from the long-run equilibrium.

Figure 4. Impulse Response Functions to a Public Spending Shock ($\gamma = 0.93$; $\upsilon_b = 3.6$)

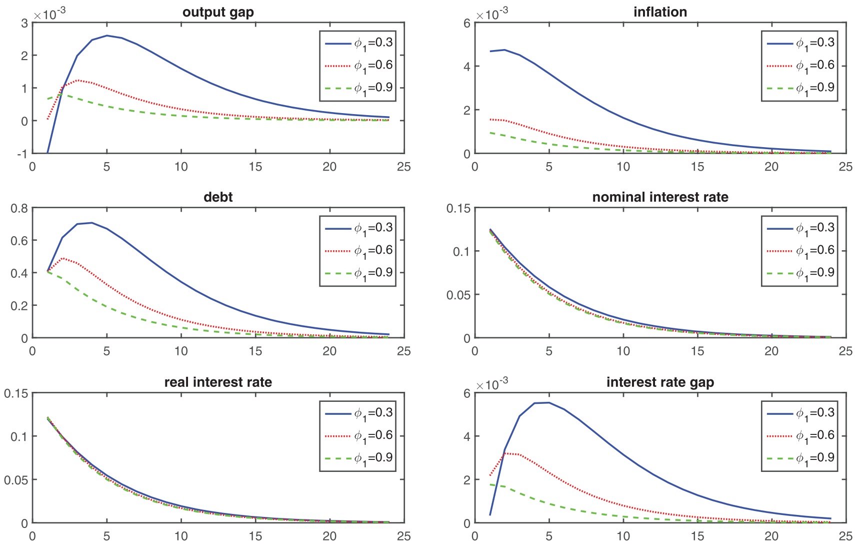


Figure 5. Impulse Response Functions to a Public Spending Shock ($\gamma = 0.995$; $\upsilon_b = 3.6$)

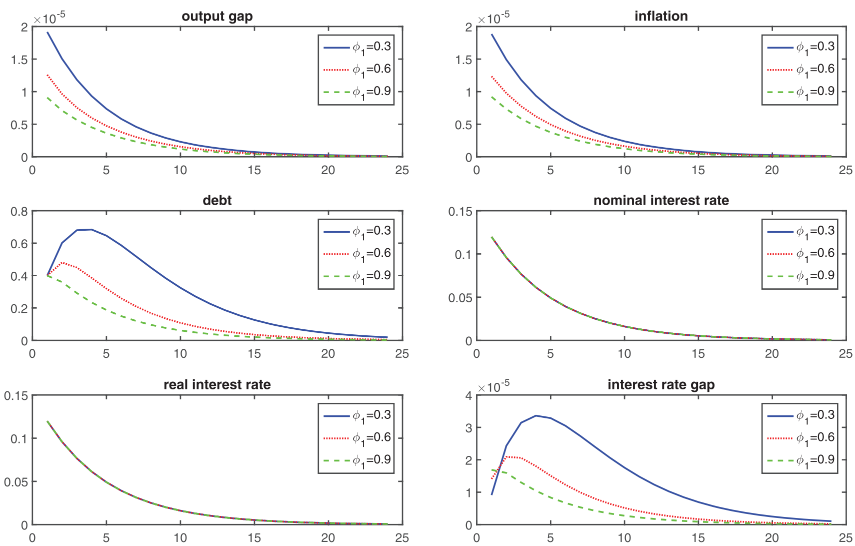
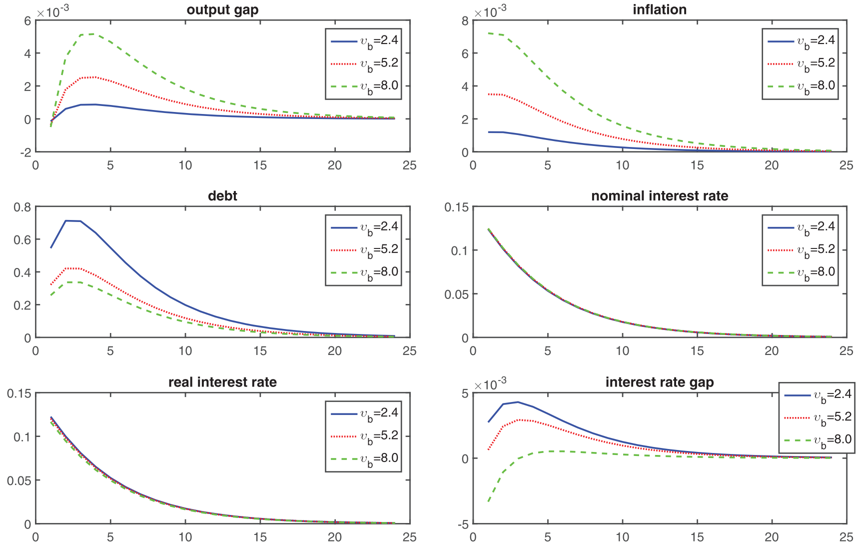


Figure 6. Impulse Response Functions to a Public Spending Shock ($\gamma = 0.93$; $\phi_1 = 0.5$)

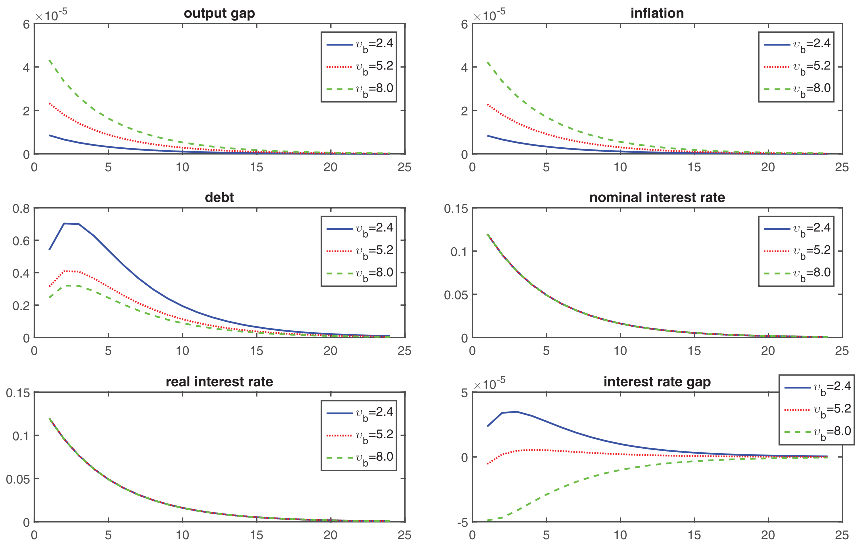


stimulate different responses of the interest rate over the business cycle, and, a priori, the overall effect on the interest rate is unclear. Impulse response functions show that overall the response of the nominal interest rate decreases and the rate primarily tracks the efficient interest rate.[27]

Figures 6 and 7 show that the public spending shock produces sharp movements in the output gap and the inflation rate when the debt-to-output ratio is large. An increase in υ_b magnifies the wealth-effect channel, and fluctuations in real debt holdings have a larger impact on the economy. Although the nominal (real) interest rate primarily tracks the efficient interest rate, it is worth noting that when the debt-to-output ratio is small, the real interest rate is generally larger than the efficient interest rate. However, when the debt-to-output ratio increases, the response of the real interest rate is

[27]The interest rate gap $(\widehat{r}_t - E_t\pi_{t+1} - \hat{\varrho}^e_t)$ shows that, under discretion, the effect of lower variability in the endogenous variables dominates the effect of the increase in the coefficients ϕ_x and ϕ_π, and the real interest rate is less responsive in the presence of a strong response of fiscal policy to movements in debt.

Figure 7. Impulse Response Functions to a Public Spending Shock ($\gamma = 0.995$; $\phi_1 = 0.5$)



smaller than the response of the efficient rate. This reaction is due to the monetary authority's prevention of an increase in the cost of servicing debt, which is attached to the increase in the real interest rate.

5. Inflation Targeting

In a Ricardian economy with lump-sum taxes and complete asset market participation, the optimal conduct of monetary policy in response to a demand shock is strictly to stabilize inflation since this policy stabilizes the output gap. Woodford (2003) shows that discretionary monetary policy is equivalent to a zero-inflation-targeting policy. Section 4 shows that optimal policy in an economy with non-Ricardian consumers faces a tradeoff in the simultaneous stabilization of inflation and the output gap. In this section, we investigate the extent to which inflation targeting remains the optimal conduct of monetary policy in non-Ricardian economies.

To address this issue, we approximate the changes in the variable f_t^* that tracks exogenous shocks with the AR(1) process:

$f_t^* = \rho f_{t-1}^* + \zeta_t$, where ζ_t is a white-noise process with zero mean and standard deviation, σ_f.[28] We make the standard assumption that monetary policy under inflation targeting seeks to stabilize inflation at the zero target. We evaluate strict inflation targeting against the discretionary policies by comparing unconditional welfare losses.[29]

To derive the unconditional welfare function of each policy, we take the expectations of the welfare function in equation (38) at the beginning of the period (i.e., time t_0):

$$\Omega_t = E_{t_0} \sum_{t=0}^{\infty} \beta^t L_t \approx -\frac{1}{2} \sum_{t=0}^{\infty} \beta^t \left[a_x var\left(\hat{x}_t\right) + var\left(\hat{\pi}_t\right) + a_b var\left(\hat{b}_t\right) \right].$$

The unconditional welfare function under discretion policy (Ω^{dis}) and inflation targeting (Ω^{it}) are, respectively,[30]

$$\Omega^{dis} = \begin{pmatrix} \frac{w_2^2(1+w_1\rho)\left(a_x m_1^2 + n_1^2 + a_b\right)}{\left(1-w_1^2\right)(1-w_1\rho)} + \left(a_x m_2^2 + n_2^2\right) \\ + \frac{2w_2(a_x m_1 m_2 + n_1 n_2)(1+w_1\rho)}{(1-w_1\rho)} \end{pmatrix} \frac{\sigma_f^2}{\left(1-\rho^2\right)\left(1-\beta^2\right)} \tag{58}$$

$$\Omega^{it} = \frac{a_b \beta^2}{\beta^2 \Psi^2 - \left(1-\phi_1\right)^2} \frac{\beta\Psi + \left(1-\phi_1\right)\rho}{\beta\Psi - \left(1-\phi_1\right)\rho} \frac{1}{1-\rho^2} \frac{\sigma_f^2}{1-\beta^2}. \tag{59}$$

Numerical results show that for a wide range of values of parameters, β, θ, γ, ϵ, φ, ρ_z, and ρ_g, welfare is superior under inflation targeting—more so than with optimal discretionary policy. Results are robust to values of the response of taxation to debt (ϕ_1) and the debt-to-output ratio (υ_b), which ensure stability of the system.[31]

[28] In the numerical simulation, the persistence parameter, ρ, is set equal to 0.9. The standard deviation, σ_f, is set equal to 1.

[29] We use the unconditional welfare since results are independent from the initial steady state.

[30] To obtain equation (58), we compute the unconditional variance of the output gap, the inflation rate, and the real debt by applying the unconditional expectation operator to equations (41) and (43) and equation (48). We use the assumption that the variance of the exogenous shock f_t^* is the exogenous source of volatility. To determine the welfare function for inflation targeting in equation (59), we impose the unconditional variance of inflation and the output gap to be zero in the steady state.

[31] An appendix that details the numerical findings is available on request.

Figure 8. Ratio of Unconditional Loss Functions

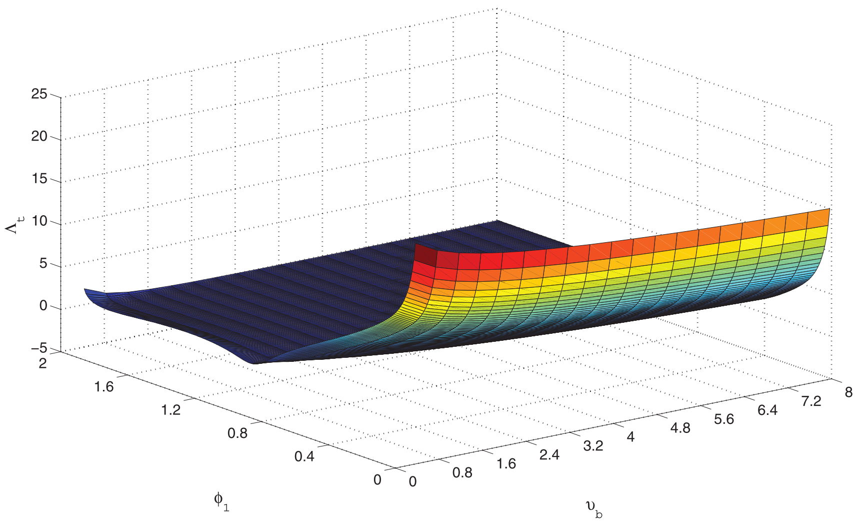


Figure 8 shows the ratios between the unconditional welfare losses under discretion and inflation targeting (i.e., $\Lambda = \log\left(\Omega^{dis}/\Omega^{it}\right)$) when $\gamma = 0.995$. Qualitatively similar results have to be obtained for the alternative calibrations of γ. Welfare losses under inflation targeting are lower than under discretionary monetary policy, as implied by values of Λ larger than 0. However, a strong response of taxes to debt reduces the value of Λ, and as a result, the two unconditional losses become similar for sufficiently large values of ϕ_1. The value of Λ decreases with the debt-to-output ratio, but this effect is quantitatively limited.

The finding is related to the role played by real debt holdings. Under inflation targeting, the monetary authority closes the output gap to achieve the inflation target without considering fluctuations in debt holdings; the unconditional loss is therefore proportional to volatility in real debt holdings. Under discretionary policy, by contrast, the monetary policy responds to changes in debt holdings to strike a balance across the tradeoff in the stabilization of inflation, the output gap, and real debt holdings. A strong response of taxes to debt (i.e., a large value of ϕ_1) attenuates the volatility of debt and therefore its relevance in the optimal discretionary policy, making the welfare losses of the discretionary policy similar to those of strict inflation targeting. When the debt-to-output ratio increases, the relative weights of debt stabilization in the loss function become larger, which increases the welfare loss caused by debt holdings fluctuations.

Why is inflation targeting welfare enhancing compared with the discretionary policy? The monetary authority that acts under a discretionary policy exercises some "debt-stabilization bias" in the attempt to reduce fluctuations in real debt holdings by strategically manipulating private-sector expectations on output gap and inflation, as described by equations (39) and (40).[32] In the non-Ricardian economy, the stabilization of real debt holdings also is motivated by the direct effect of movements in debt in the welfare function. However, our numerical analysis shows that the direct welfare gain from the stabilization of debt is inferior to the loss generated by the debt-stabilization bias.[33]

6. Conclusion

This paper embeds the perpetual youth model of Blanchard-Yaari into a tractable New Keynesian framework to investigate the interactions between monetary and fiscal policies in the presence of demand shocks. Nominal price rigidities create a link between real activity and inflation, while the OLG structure generates non-neutral effects of issuance of government debt. Fiscal policy raises lump-sum taxes proportionally to changes in debt and affects real activity, therefore playing a role in the optimal discretionary monetary policy. We find that the optimal discretionary monetary policy involves debt stabilization to reduce consumption dispersion across cohorts of consumers and requires tracking of current and expected inflation and the output gap. The importance of debt stabilization as an additional objective of monetary policy is proportional to the propensity

[32] See Leith and Wren-Lewis (2013) for a discussion of the issue.

[33] In this respect, our analysis is related to the discussion of rules versus discretion for the conduct of monetary policy. The volumes by Clarida, Galí, and Gertler (1999) and Taylor (2001) contain several contributions to this literature. Recent studies by Galí and Monacelli (2005), Soffritti and Zanetti (2008), Bragoli, Rigon, and Zanetti (2016), and references therein use a similar framework to study the welfare implications of alternative policy rules. Unlike these studies, our analysis is focused on the interactions between fiscal and monetary policy.

of consumers to consume financial wealth, which increases with the debt-to-output ratio and reduces with the household's probability of survival.

Our findings outline important interactions between monetary and fiscal policy. When the response of lump-sum taxes to changes in real debt is strong, monetary policy is primarily focused on stabilizing output gap and inflation. Debt stabilization becomes more relevant as the reaction of taxes to debt fluctuations reduces. Inflation targeting is welfare superior to the optimal discretionary policy, but it is time inconsistent. Under discretionary monetary policy, the monetary authority strategically uses fluctuations in inflation and the output gap to reduce real debt holdings. This "debt-stabilization bias" generates large fluctuations in inflation and the output gap that decrease welfare.

We acknowledge some important limitations and interesting extensions for future research. First, we assume a redistributive scheme that equates the steady-state debt across cohorts of consumers. Although this assumption greatly simplifies the analysis by ensuring the same steady state across different cohorts, it would be interesting to draw more general welfare implications and investigate the extent to which results may differ if the steady-state debt varies across consumers. However, the relaxation of this assumption requires tracking the evolution of steady-state debt across an infinite number of cohorts to derive the equilibrium of the system. Second, the analysis builds on the assumption of lump-sum taxes whereas a realistic tax system comprises distortionary taxes that generate non-Ricardian effects, which may amplify or reduce the real effects of fiscal policy. Finally, we derive the optimal monetary policy under discretion. Although it represents a realistic assumption to describe how monetary policy is implemented, a large volume of research on optimal monetary policy focuses on committed policies. A credible commitment device may contribute to attenuate the debt-stabilization bias that characterizes discretionary policy. We leave the investigation of these issues to future research.

Appendix 1. Steady State

To study the dynamics of the system, we approximate the model around the efficient steady state. In the absence of shocks, the economy converges to a steady state, in which all variables are constant, with $Y_t = Y$, $C_t = C$, $N_t = N$, $b_t = b$, $d_t = d$, $w_t = w$, $R_t = R$, $\Upsilon_t = \Upsilon$, $Z_t = Z$, and $G_t = G$.[34] From the per capita Euler equation (25), we derive the steady state of the gross nominal interest rate equal to $R = 1/\beta$. From the labor supply equation (26), we derive the steady state of the real wage: $w = \varphi C/(1 - N)$. From the production technology (27), we find the steady state of labor input: $N = Y$, where the steady-state level of technology, Z, is normalized to 1. From the optimal price-setting rule (16), we derive $w = (\theta - 1) / [\theta (1 - \varkappa)] Z$. Therefore, once we have set employment subsidy $\varkappa = 1/\theta$ that offsets distortions due to monopolistic competition, we obtain $w = Z = 1$. We assume that the steady-state level of government spending, G, is zero, whereas it is allowed to vary over time around the steady state. This assumption is the same as that of Woodford (2003) and, while simplifying the algebra considerably in the derivation of the welfare function, is innocuous for the main results in the paper and the properties of the dynamic system.[35] Using the aggregate budget constraint (28) together with $N = Y$ and $w = 1$ (the efficient steady state for wages), we derive the steady state of output: $Y = 1/(1 + \varphi)$. We derive the steady-state profit function using equation (14): $d = (1/\theta) Y$. Finally, we use the government budget constraint (17) to derive the steady-state value of real taxation: $\tau = (1 - \beta) b$, where $\tau = \Upsilon/P$. Hence, the steady-state values Y, C, N, d, w, R, and τ depend on the parameters of the model. Note that the level of government debt does not affect the steady state of the variables except for the value of taxation τ since the lump-sum transfers (4) and the redistributive scheme break the link between the debt and the real interest rate. Although the government bond holdings have no effect on the steady state of the economy, we show that different levels of government debt may have an impact in terms of welfare losses due to an exogenous shock.

[34] Note that the steady state of the model is the same across cohorts. To see this, consider that the steady-state consumption equation (10) for a representative agent of age s can be written as $C = (b + h) / [(1 + \varphi) \Sigma]$ since $V_s = 0$ when we aggregate across cohorts, and $B_s = B$ in the steady state since the redistributive scheme compensates agents for any difference across stready-state levels of financial wealth.

[35] An appendix that details the derivation of the welfare functions under more general conditions is available on request.

Figure 9. Impulse Response Functions to a Technological Shock ($\gamma = 0.93$; $\upsilon_b = 3.6$)

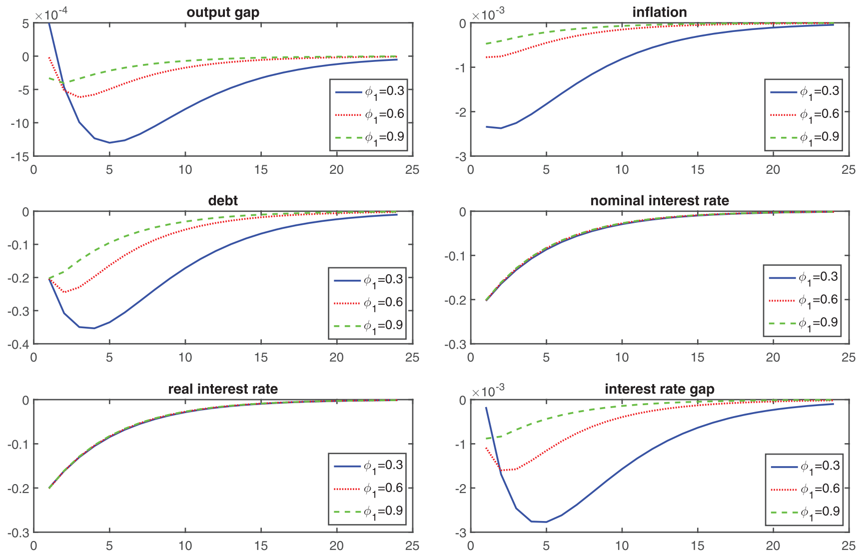


Figure 10. Impulse Response Functions to a Technological Shock ($\gamma = 0.995$; $\upsilon_b = 3.6$)

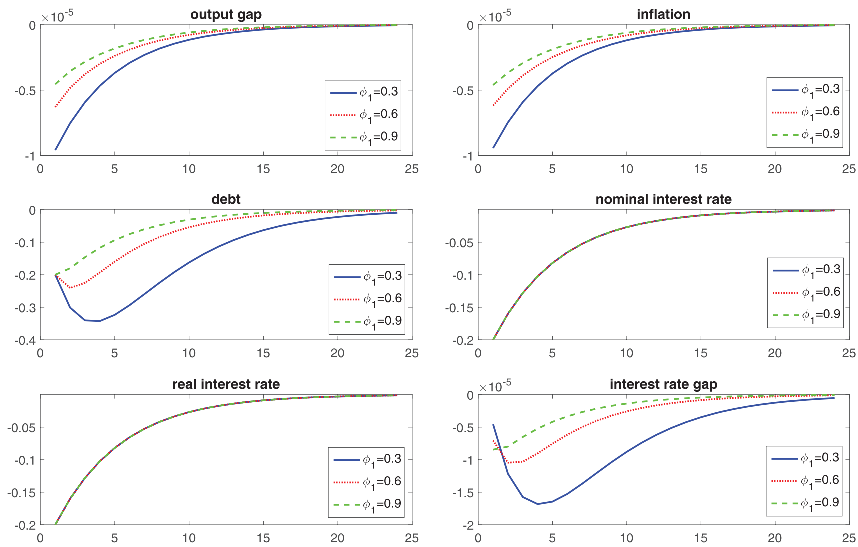


Figure 11. Impulse Response Functions to a Technological Shock ($\gamma = 0.93$; $\phi_1 = 0.5$)

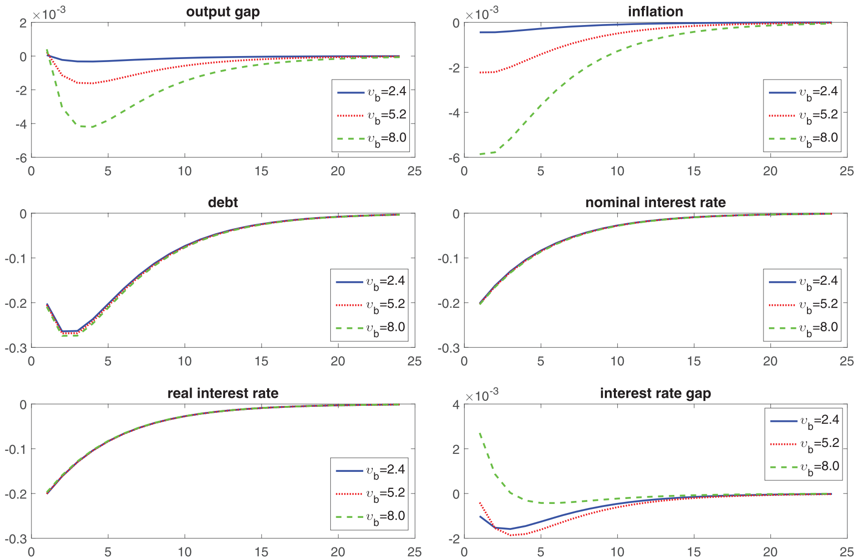
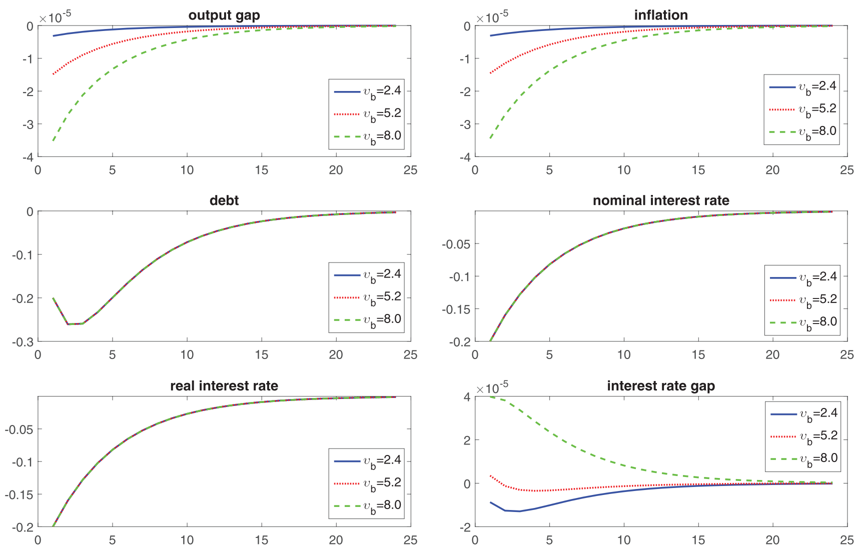


Figure 12. Impulse Response Functions to a Technological Shock ($\gamma = 0.995$; $\phi_1 = 0.5$)



Appendix 2. Proofs of Propositions and Lemmas

This appendix provides the proofs to propositions and lemmas in the paper.

Proof of Proposition 1 (Aggregate Welfare Function, Ω_t)

Consumer i born in cohort s has the following utility function:

$$\Omega_{s.t}(i) = E_t \sum_{t=0}^{\infty} \beta^t \left[U\left(C_{s,t}(i)\right) - V\left(N_{s,t}(i)\right)\right], \tag{60}$$

where utility is assumed to be separable in consumption and leisure, in which

$$U\left(C_{s,t}\right) = \ln C_t, \tag{61}$$

and

$$V\left(N_{s,t}\right) = \varphi \ln\left(1 - N_{s,t}\right). \tag{62}$$

We drop index i since all agents belonging to the same cohort consume the same basket of goods and provide the same amount of labor.

The second-order Taylor expansion of consumption utility function (61) around the steady state implies

$$U\left(C_{s,t}\right) \approx U\left(\overline{C}\right) + U_c\left(C_{t,s} - C\right) + \frac{1}{2}U_{cc}\left(C_{s,t} - C\right)^2 + tip + o\left(\|\xi\|^3\right),$$

where $\|\xi\|$ is a bound on the amplitude of the exogenous disturbances, and the term $o\left(\|\xi\|^3\right)$ indicates that we neglect terms of third or higher order in deviations from their steady-state values. Moreover, expanding $C_{s,t}$ around its steady state yields

$$\frac{C_{s,t}}{C} \approx 1 + \hat{c}_{s,t} + \frac{1}{2}\hat{c}_{s,t}^2 + o\left(\|\xi\|^3\right).$$

Therefore,

$$U\left(C_{s,t}\right) \approx U\left(C\right) + U_c C\hat{c}_{s,t} + \frac{1}{2}U_c C\hat{c}_{s,t}^2 + \frac{1}{2}U_{cc}^2 C^2\hat{c}_{s,t}^2 + o\left(\|\xi\|^3\right).$$

Using

$$U_c = \frac{1}{C}$$
$$U_{cc} = -\frac{1}{C^2},$$

we obtain

$$U\left(C_{s,t}\right) \approx U\left(C\right) + U_c C \hat{c}_{s,t} + tip + o\left(\|\xi\|^3\right), \tag{63}$$

where *tip* collects policy-independent terms (such as constants and functions of exogenous disturbance). The representative-agent model (63) represents the second-order Taylor expansion of consumption utility. However, in the overlapping-generations framework, we need to compute the per capita version of such equation:

$$\begin{aligned} U\left(C_t\right) &= \left(1-\gamma\right)\sum_{s=0}^{\infty}\gamma^s U\left(C_{s,t}\right) \\ &\approx U\left(C\right) + U_c C\left[E_s \hat{c}_{s,t}\right] + tip + o\left(\|\xi\|^3\right). \end{aligned} \tag{64}$$

The second-order log-linearization of equation (19) applied to consumption yields

$$\hat{c}_t = E_s \hat{c}_{s,t} + \frac{1}{2} var_s \hat{c}_t, \tag{65}$$

where $var_s \hat{c}_t$ is the dispersion of consumption across cohorts. Plugging (65) into (64) yields

$$U\left(C_t\right) \approx U_c C\left[\hat{c}_t - \frac{1}{2} var_s \hat{c}_t\right] + tip + o\left(\|\xi\|^3\right). \tag{66}$$

The next step is to obtain an approximation for the disutility of work. The second-order Taylor expansion of (62) yields

$$V\left(N_{t,s}\right) \approx V\left(N\right) + V_n\left(N_{t,s} - N\right) + \frac{1}{2} V_{nn}\left(N_{t,s} - N\right)^2 + o\left(\|\xi\|^3\right),$$

where $N_{t,s}$ is the labor effort of cohort s. In the steady state, we have

$$\frac{V_{nn} N}{V_n} = \frac{N}{1-N} = \eta.$$

Therefore,

$$V\left(N_{s,t}\right) \approx V_n N\left[\widehat{n}_{s,t} + \frac{1+\eta}{2}\widehat{n}_{s,t}^2\right] + tip + o\left(\|\xi\|^3\right). \tag{67}$$

Aggregation across cohorts of the second-order Taylor-approximated labor utility function (67) yields

$$(1-\gamma)\sum_{s=0}^{\infty}\gamma^s V\left(N_{s,t}\right) \approx V_n\overline{N}\left[E_s\widehat{n}_{s,t} + \frac{1+\eta}{2}E_s\left(\widehat{n}_{s,t}^2\right)\right] + tip + o\left(\|\xi\|^3\right).$$

The second-order log-linearization of (19) applied to labor is

$$\widehat{n}_t = E_s\widehat{n}_{s,t} + \frac{1}{2}var_s\hat{n}_t$$

and the definition for the dispersion of labor across cohorts $(var_s\hat{n}_t)$ is

$$var_s\widehat{n}_t = \widehat{n}_t^2 - \left(E_s\hat{n}_{s,t}\right)^2,$$

allowing us to write the approximated utility of leisure as

$$V\left(N_t\right) \approx V_n N\left[\hat{n}_t + \frac{\eta}{2}var_s\hat{n}_t + \frac{1+\eta}{2}\hat{n}_t^2\right] + tip + o\left(\|\xi\|^3\right). \tag{68}$$

To get rid of $\hat{n}_t$, we use the second-order log-linear approximation of equation (27):

$$\hat{n}_t + \frac{1}{2}\hat{n}_t^2 = \hat{y}_t + \frac{1}{2}\hat{y}_t^2 + \widehat{z}_t + \frac{1}{2}\widehat{z}_t^2 + \hat{y}_t\widehat{z}_t + \hat{\chi}_t, \tag{69}$$

where $\hat{\chi}_t$ denotes the log-price dispersion across monopolistic firms. Substituting (69) into (68) yields

$$V\left(N_t\right) \approx V_n N\left[\hat{y}_t + \frac{1+\eta}{2}\hat{y}_t^2 - (1+\eta)\,\hat{y}_t\widehat{z}_t + \hat{\chi}_t + \frac{\eta}{2}var_s\widehat{n}_{s,t}\right] + tip + o\left(\|\xi\|^3\right). \tag{70}$$

Since we assume that the distortions due to monopolistic competition have been removed by an employment subsidy financed by lump-sum taxes, the steady-state labor/leisure tradeoff equation (26) implies that

$$\varphi V_n N = U_c C.$$

We therefore can write equation (70) as

$$V\left(N_t\right) \approx U_c C\left[\hat{y}_t + \frac{1+\eta}{2}\hat{y}_t^2 - (1+\eta)\,\hat{y}_t\widehat{z}_t + \hat{\chi}_t + \frac{\eta}{2}var_s\widehat{n}_{s,t}\right] + tip + o\left(\|\xi\|^3\right). \tag{71}$$

By combining equation (66) and (71) and using the identity

$$\hat{c}_t + \hat{g}_t = \hat{y}_t,$$

we are able to write the second-order approximation of equation (60) as

$$\Omega_t \approx -U_c\overline{C}\left[\frac{1+\eta}{2}\hat{y}_t^2 - (1+\eta)\,\hat{y}_t\widehat{z}_t + \hat{g}_t + \hat{\chi}_t + \frac{\eta}{2}var_s\hat{n}_t + \frac{1}{2}var_s\hat{c}_t\right] + tip + o\left(\|\xi\|^3\right).$$

Moreover, using the definition of efficient output (31), we arrive at

$$\Omega_t \approx -U_c C\left[\frac{1+\eta}{2}\left(\hat{y}_t - \hat{y}_t^e\right)^2 + \hat{\chi}_t + \frac{\eta}{2}var_s\widehat{N}_t + \frac{1}{2}var_s\hat{c}_t\right] + tip + o\left(\|\xi\|^3\right). \tag{72}$$

As shown by Galí and Monacelli (2008),

$$\hat{\chi}_t = +\frac{\theta}{2}var_j\hat{p}_t\left(h\right). \tag{73}$$

Moreover, following Woodford (2003, chapter 6), we arrive at this definition:

$$\sum_{t=0}^{\infty}\beta^t\Delta_t = \frac{\varepsilon}{\left(1-\varepsilon\beta\right)\left(1-\varepsilon\right)}\sum_{t=0}^{\infty}\beta^t\pi_t^2 + tip, \tag{74}$$

where

$$\Delta_t = var_h \hat{p}_t (h).$$

Inserting (73) and (74) into (72) yields

$$\Omega_t \approx -U_c C \left[\frac{1+\eta}{2} \hat{x}_t^2 + \frac{\epsilon}{(1-\epsilon\beta)(1-\epsilon)} \frac{\theta}{2} \hat{\pi}_t^2 + \frac{\eta}{2} var_s \widehat{n}_t + \frac{1}{2} var_s \hat{c}_t \right] + tip + o\left(\|\xi\|^3\right). \tag{75}$$

Finally, from the log-linearized version of (8), we note that

$$var_s \widehat{n}_t = \frac{var_s \hat{c}_t}{\eta^2},$$

which inserted into (75) yields

$$\Omega_t \approx -\left[a_x \hat{x}_t^2 + \hat{\pi}_t^2 + \tilde{a}_b var_s \hat{c}_t\right] + tip + o\left(\|\xi\|^3\right),$$

where

$$\begin{aligned} \hat{x}_t &= \hat{y}_t - \hat{y}_t^e, \\ a_x &= (1+\eta) \frac{(1-\epsilon\beta)(1-\epsilon)}{\theta\epsilon} = \frac{\kappa}{\theta}, \\ \tilde{a}_b &= \frac{1+\eta}{\eta} \frac{(1-\epsilon\beta)(1-\epsilon)}{\theta\epsilon} = \frac{a_x}{\eta}. \end{aligned}$$

Proof of Proposition 2 (Welfare Function and Debt Holdings)

First we note that from the log-linear version of equation (10), the dispersion of consumption across generations is proportional to the dispersion of debt:

$$var_s \hat{c}_t = \frac{1}{(1+\varphi)^2 \Phi^2} var_s \hat{b}_t.$$

Since both $var_s \hat{b}_t$ and $var_s \hat{b}_{t+1}$ are predetermined at time t, we can follow Woodford (2003) in solving equation (37) backward to obtain

$$var_s \hat{c}_t = \gamma var_s \hat{c}_{-1} + (1-\gamma) \sum_{f=0}^{t} \gamma^{t-f} \left(\frac{\upsilon_B}{\gamma\Phi}\right)^2 \hat{b}_t^2, \tag{76}$$

where $var_s c_{-1}$ could be any (small) degree of consumption dispersion in the period before the first period for which a new policy is contemplated that is independent from policy. Taking the discounted value of (76) over the entire period at $t \geq 0$, we obtain

$$\sum_{t=0}^{\infty} \beta^t var_s \widehat{c}_t = \frac{1-\gamma}{1-\beta\gamma} \left(\frac{1}{\gamma\Phi} \frac{\upsilon_B}{1+\varphi} \right)^2 \sum_{t=0}^{\infty} \beta^t \hat{b}_{t+1}^2,$$

which substituted in equation (36) leads to the welfare function (38).

Proof of Proposition 3 (Optimal Discretionary Monetary Policy)

The problem of an infinitively lived central banker is to choose a path for $\hat{r}_t - \varrho_t^e$, to solve the Bellman equation:

$$V\left(b_t, f_t^*\right) = \min_{r_t - \varrho_t^*} \left[-\frac{1}{2} \left(\alpha_x \hat{x}_t^2 + \pi_t^2 + \alpha_b \hat{b}_t^2 \right) + \beta E_t V\left(b_{t+1}, f_{t+1}^*\right) \right],$$

subject to the law of motion for debt

$$\hat{b}_{t+1} = \left(\frac{\widehat{r}_t - \varrho_t^*}{1+n_1} \right) + \frac{1-\phi_1}{\beta\left(1+n_1\right)} \hat{b}_t + \left(\frac{1-n_2}{1+n_1} \right) f_t^*,$$

where

$$E_t \hat{x}_{t+1} = m\left(\hat{b}_{t+1}, f_t^*\right) = m_1 \hat{b}_{t+1} + m_2 f_t^*, \tag{77}$$

$$E_t \hat{\pi}_{t+1} = n\left(\hat{b}_{t+1}, f_t^*\right) = n_1 \hat{b}_{t+1} + n_2 f_t^*, \tag{78}$$

$$\hat{b}_{t+1} = \left(\widehat{r}_t - E_t \hat{\pi}_{t+1} - \varrho_t^*\right) + \frac{1}{\beta}\left(1-\phi_1\right)\hat{b}_t + f_t^*.$$

The first-order conditions with respect to the control variable $(\hat{r}_t - \varrho_t^e)$ and the state variable $\hat{b}_t$ are, respectively,

$$E_t \frac{\partial V\left(b_{t+1}, f_{t+1}^*\right)}{\partial b_{t+1}} = +\alpha_x \hat{x}_t \left\{ \frac{\Psi_b + m_1 - 1}{\beta} \right\} + \hat{\pi}_t \left\{ \frac{\beta n_1 + \kappa\left(\Psi_b + m_1 - 1\right)}{\beta} \right\} + \alpha_b \hat{b}_{t+1} \tag{79}$$

and

$$E_t \frac{\partial V\left(b_{t+1}, f_{t+1}^*\right)}{\partial b_{t+1}} = \frac{(1+n_1)}{(1-\phi_1)} \frac{\partial V\left(b_t, f_t^*\right)}{\partial \hat{b}_{t+1}} + \alpha_b \hat{b}_{t+1}$$
$$+ \alpha_x \left(\frac{\Psi_b + m_1 + n_1}{\beta}\right) \hat{x}_t$$
$$+ \left[\frac{\beta n_1 + \kappa\left(\Psi_b + m_1 + n_1\right)}{\beta}\right] \hat{\pi}_t. \tag{80}$$

Combining equations (79) and (80) yields

$$\frac{\partial E_t V\left(b_t, f_t^*\right)}{\partial b_t} = -\frac{1-\phi_1}{\beta} \alpha_x \hat{x}_t - \kappa \frac{1-\phi_1}{\beta} \hat{\pi}_t. \tag{81}$$

Forwarding equation (81) one period ahead and substituting in (79), we obtain the optimal condition in proposition 3:

$$-\frac{1-\phi_1}{\beta} \alpha_x E_t \hat{x}_{t+1} - \kappa \frac{1-\phi_1}{\beta} E_t \hat{\pi}_{t+1} - \alpha_b \hat{b}_{t+1}$$
$$= \alpha_x \left\{\frac{\Psi_b + m_1 - 1}{\beta}\right\} \hat{x}_t + \frac{\beta n_1 + \kappa\left(\Psi_b + m_1 - 1\right)}{\beta} \hat{\pi}_t.$$

Proof of Proposition 4 (Nominal Interest Rate that Implements Discretionary Optimal Policy)

Using the Phillips curve (78) to substitute for $\hat{\pi}_t$ in the optimal condition (45) and then using the IS equation (77) to get rid of $\hat{x}_t$, we obtain

$$\hat{r}_t - \varrho_t^* = \phi_x E_t \hat{x}_{t+1} + \phi_\pi E_t \pi_{t+1} + \phi_b \hat{b}_{t+1},$$

where the parameters ϕ_x, ϕ_π, and ϕ_b are defined as part of the main text.

Proof of Lemma 1 (Deviation of per Capita Real Debt)

Taking the log-linearized version of equations (7) and (25), subtracting one from the other and computing the dispersion across generation yields

$$var_s \hat{c}_t = \frac{1}{\gamma} var_s E_t \hat{c}_{t+1} + \frac{1-\gamma}{\gamma} \left(\hat{c}_{t+1}^{NB} - \hat{c}_{t+1} \right)^2 ,$$

which, after applying the log-linearized version of equation (24), reduces to the recursive expression for the dispersion of consumption in equation (37).

References

Abbas, S. M. A., L. Blattner, M. D. Broeck, A. A. Elganainy, and M. Hu. 2014. "Sovereign Debt Composition in Advanced Economies: A Historical Perspective." IMF Working Paper No. 14/162.

Andres, J., O. Arce, and C. Thomas. 2013. "Banking Competition, Collateral Constraints, and Optimal Monetary Policy." *Journal of Money, Credit and Banking* 45 (s2): 87–125.

Benigno, P., and M. Woodford. 2003. "Optimal Monetary and Fiscal Policy: A Linear Quadratic Approach." NBER Working Paper No. 9905.

———. 2012. "Linear-Quadratic Approximation of Optimal Policy Problems." *Journal of Economic Theory* 147 (1): 1–42.

Bilbiie, F. O. 2008. "Limited Asset Markets Participation, Monetary Policy and (Inverted) Aggregate Demand Logic." *Journal of Economic Theory* 140 (1): 162–96.

Blanchard, O. J. 1985. "Debt, Deficits, and Finite Horizons." *Journal of Political Economy* 93 (2): 223–47.

Bragoli, D., M. Rigon, and F. Zanetti. 2016. "Optimal Inflation Weights in the Euro Area." *International Journal of Central Banking* 12 (2, June): 357–83.

Calvo, G. A. 1983. "Staggered Prices in a Utility-Maximizing Framework." *Journal of Monetary Economics* 12 (3): 383–98.

Carlstrom, C. T., T. S. Fuerst, and M. Paustian. 2009. "Inflation Persistence, Monetary Policy, and the Great Moderation." *Journal of Money, Credit and Banking* 41 (4): 767–86.

Castelnuovo, E., and S. Nisticó. 2010. "Stock Market Conditions and Monetary Policy in a DSGE Model for the U.S." *Journal of Economic Dynamics and Control* 34 (9): 1700–1731.

Chada, J. S., and C. Nolan. 2007. "Optimal Simple Rules for the Conduct of Monetary and Fiscal Policy." *Journal of Macroeconomics* 29 (4): 665–89.

Clarida, R., J. Galí, and M. Gertler. 1999. "The Science of Monetary Policy: A New Keynesian Perspective." *Journal of Economic Literature* 37 (4): 1661–1707.

Del Negro, M., M. Giannoni, and C. Patterson. 2012. "The Forward Guidance Puzzle." Staff Report No. 574, Federal Reserve Bank of New York.

Draghi, M. 2015. "Monetary Policy and Structural Reform in the Euro Area." Speech given at event hosted by Prometeia, held in Bologna, December 14.

Galí, J., and T. Monacelli. 2005. "Monetary Policy and Exchange Rate Volatility in a Small Open Economy." *Review of Economic Studies* 72 (3): 707–34.

———. 2008. "Optimal Monetary and Fiscal Policy in a Currency Union." *Journal of International Economics* 76 (1): 116–32.

Keane, M., and R. Rogerson. 2012. "Micro and Macro Labor Supply Elasticities: A Reassessment of Conventional Wisdom." *Journal of Economic Literature* 50 (2): 464–76.

Kirsanova, T., M. Satchi, D. Vines, and S. Wren-Lewis. 2007. "Optimal Fiscal Policy Rules in a Monetary Union." *Journal of Money, Credit and Banking* 39 (7): 1759–84.

Kirsanova, T., and S. Wren-Lewis. 2012. "Optimal Fiscal Feedback on Debt in an Economy with Nominal Rigidities." *Economic Journal* 122 (559): 238–64.

Leith, C., and L. von Thadden. 2008. "Monetary and Fiscal Policy Interactions in a New Keynesian Model with Capital Accumulation and Non-Ricardian Consumers." *Journal of Economic Theory* 140 (1): 279–313.

Leith, C., and S. Wren-Lewis. 2013. "Fiscal Sustainability in a New Keynesian Model." *Journal of Money, Credit and Banking* 45 (8): 1477–1516.

Maskin, E., and J. Tirole. 2001. "Markov Perfect Equilibrium: I. Observable Actions." *Journal of Economic Theory* 100 (2): 191–219.

McCallum, B. T. 1983. "On Non-uniqueness in Rational Expectations Models: An Attempt at Perspective." *Journal of Monetary Economics* 11 (2): 139–68.

———. 1999. “Role of the Minimal State Variable Criterion in Rational Expectations Models.” *International Tax and Public Finance* 6 (4): 621–39.

———. 2003. “Multiple-Solution Indeterminacies in Monetary Policy Analysis.” *Journal of Monetary Economics* 50 (5): 1153–75.

Nisticó, 8. 2016. “Optimal Monetary Policy and Financial Stability in a Non-Ricardian Economy.” *Journal of the European Economic Association* 14 (5): 1225–52.

Rajan, R. G. 2015. “The Political Economy of Inflation Dynamics.” In *Inflation Dynamics and Monetary Policy*, 497–502. Proceedings of the Economic Policy Symposium sponsored by the Federal Reserve Bank of Kansas City, held in Jackson Hole, Wyoming, August 27–29.

Schmitt-Grohé, S., and M. Uribe. 2004. “Optimal Fiscal and Monetary Policy under Sticky Prices.” *Journal of Economic Theory* 114 (2): 198–230.

Soffritti, M., and F. Zanetti. 2008. “The Advantage of Tying One’s Hands: Revisited.” *International Journal of Finance and Economics* 13 (2): 135–49.

Taylor, J. B., ed. 2001. *Monetary Policy Rules*. National Bureau of Economic Research Studies in Business Cycles Series. University of Chicago Press.

Woodford, M. 2003. *Interest and Prices: Foundations of a Theory of Monetary Policy.* Princeton University Press.

———. 2010. “Optimal Monetary Stabilization Policy.” In *Handbook of Monetary Economics,* Vol. 3, ed. B. M. Friedman and M. Woodford, 723–828 (chapter 14). Elsevier.