Online Appendixes to Financial Stability and Optimal Interest Rate Policy*

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Appendix A. Output Growth and Inflation Expectations in the Great Recession: Evidence from the SPF

In this appendix we report evidence of how professional forecasters' expectations over future output growth and inflation evolved before and during the Great Recession.

Every quarter, participants in the Survey of Professional Forecasters (SPF) report the probability distribution of the growth rate of real average GDP expected over the current and next calendar years. Survey participants are asked to assign probabilities to the events that the growth rate of average real GDP between years 0 and 1 will fall within predetermined ranges.

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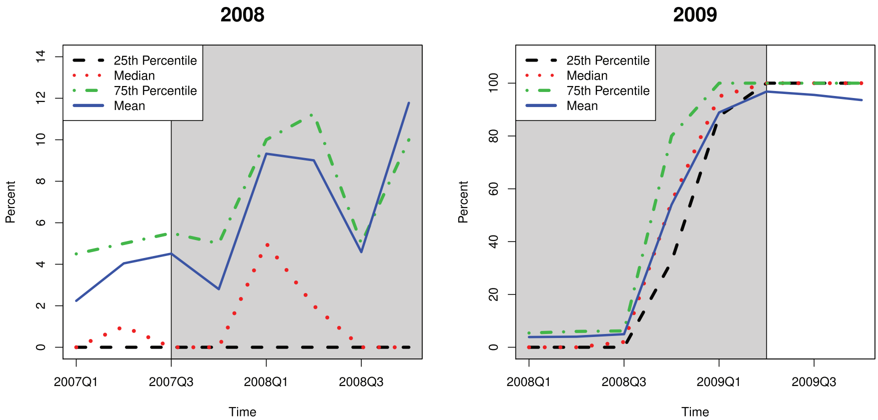
Figure A1. Probability of Negative Forecasted Growth of Average Real GDP in 2008 (left) and 2009 (right)



Data Source: Survey of Professional Forecasters, Research Department, Federal Reserve Bank of Philadelphia.
Notes: See the text for details. The gray area identifies the Great Recession according to NBER dates.

Since 1992:Q1, participants explicitly forecast the likelihood that the growth rate of average real GDP (RGDP) will be lower than 0 percent[1]:

$$PRGDP_{y1}$$
$$= Pr\left[100 \times \ln\left(\frac{\frac{RGDP^{y1}_{Q1}+RGDP^{y1}_{Q2}+RGDP^{y1}_{Q3}+RGDP^{y1}_{Q4}}{4}}{\frac{RGDP^{y0}_{Q1}+RGDP^{y0}_{Q2}+RGDP^{y0}_{Q3}+RGDP^{y0}_{Q4}}{4}}\right) < 0\%\right].$$

We concentrate on the Great Recession episode and study how expectations of professional forecasters behaved before and during the period of financial turmoil that built up to the downturn and to two years of negative growth for average real GDP: 2008 and 2009.

Realized average real GDP fell by –0.29 percent in 2008, and then fell again by –2.81 percent in 2009. The left panel of figure A1 shows

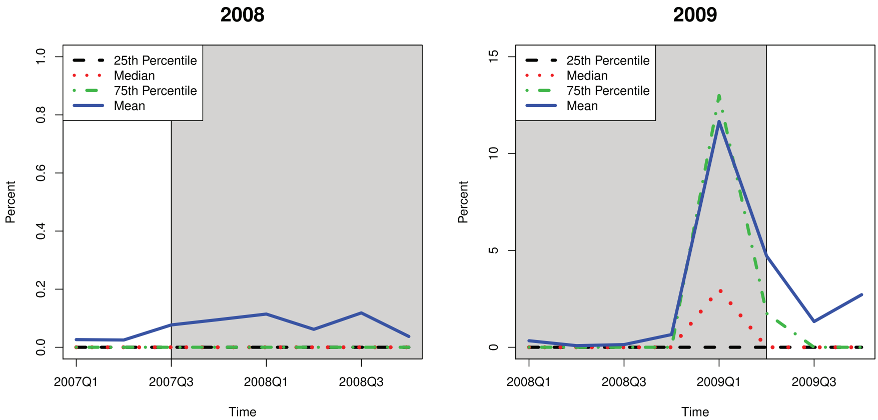
[1]Prior to 1992:Q1, the upper bound of the lowest range in the survey was 2 percent. Moreover, prior to 1981:Q3, participants were surveyed about the probability distribution of nominal (and not real) GDP growth.

that the median forecaster in the SPF (purple line) attached probabilities close to 0 percent to the event that average real GDP could fall during the course of 2008, in each quarter he or she was asked to forecast it, over the course of 2007 and 2008. Similarly, the right panel of figure A1 shows that the median forecaster (purple solid line) reported probabilities below 2 percent when asked to forecast the likelihood of negative growth for average real GDP in 2009, at least until the collapse of Lehman Brothers in 2008:Q3. After this point, the median probability of negative growth in 2009 increased from 2 percent in 2008:Q3 up to 55 percent in 2008:Q4, and later converged to 100 percent by the second half of 2009 as more information on the severity of the financial crisis became available. The graphs also report the interquartile range for the same probabilities, as well as mean probabilities, and the NBER-dated recession period is highlighted in gray.

We conduct a similar exercise using the SPF data for the probability distribution of the growth rate of average CPI over the same time frame. We are particularly interested in the forecasters' views on the likelihood of a prolonged deflationary scenario during the Great Recession. The left panel of figure A2 shows that the median forecaster (purple solid line) reported a probability of negative growth of average CPI to be 0 percent for 2008, over the course of the forecasting period (2007 and 2008). Similarly, the right panel of figure A2 shows that the median forecaster kept the expected probability of deflation for 2009 equal to 0 percent until realized CPI inflation recorded a negative entry in 2008:Q4 (–2.3 percent, not shown in the figures). At that point the median forecaster increased the expected likelihood of a deflationary scenario to 3 percent, only to converge back to 0 percent once the temporary effect of the sudden decrease in energy prices of the end of 2008 faded out and realized CPI inflation went back into positive territory.

It is interesting to note that the mean, together with the third quartile (green dash-dotted line) of the distribution of SPF participants included in the graphs, points out that a number of professionals did forecast a higher likelihood of a prolonged drop in real GDP and prices for 2008 and 2009. Nonetheless, the third-quartile forecast of how likely the drop in average real GDP would last through 2009 hovers around 10 percent and only increases rapidly after the collapse of Lehman Brothers. Deflation expectations show a similar

Figure A2. Probability of Negative Forecasted Growth of Average CPI in 2008 (left) and 2009 (right)



Data Source: Survey of Professional Forecasters, Research Department, Federal Reserve Bank of Philadelphia.
Notes: See the text for details. The gray area identifies the Great Recession according to NBER dates.

pattern. We interpret this as evidence of how agents did not anticipate the occurrence and the effects of the financial crisis of 2007–09. Agents' expectations of the likelihood of a prolonged recession adjusted with a lag to the unfolding of the events on financial markets, rather than, for example, responding to the accumulation of financial imbalances over the course of the economic expansion of the 2000s.

Appendix B. Credit Conditions and Crisis Probability

In this appendix we provide further details on our use of Schularick and Taylor (2012)'s data and our adaptation of some of their results.

B.1 The Logit Model

Schularick and Taylor (2012) assume that the probability that a given country, i, will fall into a financial crisis in period t and $t+1$

can be expressed as a logistic function $\gamma_{i,t}$ of a collection of predictors $X_{i,t}$:

$$\gamma_{i,t} = \frac{e^{X_{i,t}}}{1 + e^{X_{i,t}}}.$$

Their baseline specification for $X_{i,t}$ includes a constant, c, country fixed effects, α_i, and five lags of the annual growth rate of loans of domestic banks to domestic households, $B_{i,t}$, deflated by the CPI, $P_{i,t}$:

$$\begin{aligned} X_{i,t} = h_0 + h_i + h_{1,L} \Delta \log \frac{B_{i,t}}{P_{i,t-1}} + h_{2,L} \Delta \log \frac{B_{i,t-1}}{P_{i,t-1}} \\ + h_{3,L} \Delta \log \frac{B_{i,t-2}}{P_{i,t-2}} + h_{4,L} \Delta \log \frac{B_{i,t-3}}{P_{i,t-3}} + h_{5,L} \Delta \log \frac{B_{i,t-4}}{P_{i,t-4}}. \end{aligned}$$

The model is estimated on annual data.

In order to reduce the number of lags and state variables in our model, we reestimate a simplified version of Schularick and Taylor's model using the cumulative five-year growth rate of bank loans from time $t - 4$ to t, denoted as L_t, as predictor of a financial crisis in period $t + 1$, instead of the five lags separately.

$$X_{i,t} = h_0 + h_i + h_1 L_t^a, \tag{B1}$$

where

$$L_t^a = \Sigma_{s=0}^4 \Delta \log \frac{B_{i,t-s}}{P_{i,t-s}}.$$

The estimated coefficients for this equation are significant and shown in table B1 (the country fixed effect for the United States is set to zero, for identification purposes).

To adapt the results to our model calibrated to quarterly data, we assume that the annual probability of a crisis $\gamma_{i,t}$ is uniformly distributed over the four quarters within the year, so that the quarterly probability $\gamma_{i,t}^q$ is equal to[2]

$$\gamma_{i,t}^q = \frac{\gamma_{i,t}}{4}.$$

[2]Up to a small approximation error, this is equivalent to solving for the quarterly probability from its definition:

$$(1 - \gamma^a) = (1 - \gamma^q)^4,$$

Table B1. Estimates of the Schularick and Taylor Model for the U.S. Regressor L_t: Five-Year Cumulative Growth Rate

Equation	Variables
h_1	1.880***
	(0.569)
h_0	−3.396***
	(0.544)
Observations	1,253

We define a recursive approximation of L_t as the recursive sum of the quarterly growth rates recorded from time $t-19$ up to quarter t:

$$\Sigma_{s=0}^{19} \Delta \log \frac{B_{t-s}^q}{P_{t-s}} \approx L_t^q = \Delta \log \frac{B_t^q}{P_t} + \frac{19}{20} L_{t-1}^q. \tag{B2}$$

The left panel of figure B1 shows the cumulative annual regressor and its recursive counterpart defined in equation (B2). The right panel of figure B1 shows the quarterly actual and recursive sums defined in equation (B2). The left and right panels of figure B2 show the corresponding fitted probabilities using the logit coefficients in table B1.[3] The series are remarkably similar. As expected, the recursive sums are less volatile than the actual five-year growth rate both for the quarterly and the annual series (the standard deviation of the quarterly actual and recursive sums in figure B1 are 11 percent and 13.5 percent, respectively).

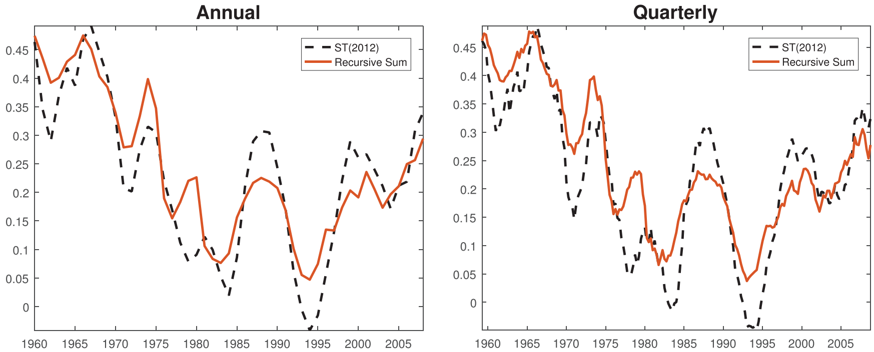
Figure B2 shows the quarterly fitted probability that a crisis arises in period t (hence computed using the quarterly growth rates of bank loans over the past five years of data, up to quarter $t-1$),

by which, the probability of a crisis *not* occurring next year is equal to the probability of the crisis not materializing in any of its four quarters. Taking logs of the above, we can write

$$\gamma_q \approx \frac{\gamma_a}{4}.$$

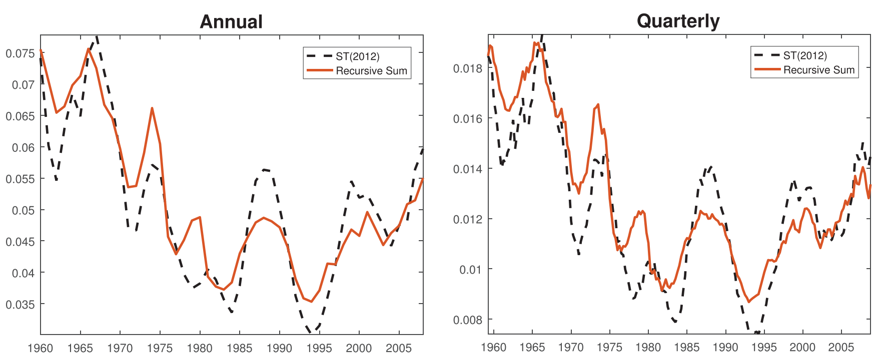
[3]The series are built at both annual and quarterly frequencies for the United States using the total loans and leases and security investments of commercial banks from the Board of Governors of the Federal Reserve System's H.8 release.

Figure B1. Annual (left) and Quarterly (right) Five-Year Growth Rate of Real Banking Loans: Actual vs. Recursive Sum, 1960–2008



Data Source: Total loans and leases and security investments of commercial banks from Assets and Liabilities of Commercial Banks in the United States—H.8 release from the Board of Governors of the Federal Reserve System.

Figure B2. Annual (left) and Quarterly (right) Fitted Crisis Probabilities, 1960–2008



Data Source: Authors' calculations.

from equation (B2). The cyclical properties of the quarterly series are the same as the ones of the annual series.[4]

B.2 Quarterly Bank Loan Growth

We assume that the quarterly growth rate of nominal bank loans is a function of the nominal federal funds rate i_t, of the output gap y_t, and of the inflation rate π_t:

$$\Delta \log B_t = c + \phi_i i_t + \phi_y y_t + \phi_\pi \pi_t + \varepsilon_t^B. \tag{B3}$$

In order to estimate the coefficients of equation (B3), we use data on nominal bank loans for commercial banks from the flow of funds of the United States (as in Schularick and Taylor 2012), the effective federal funds rate, the output gap (defined as the log-difference between GDP and potential GDP as defined by the Congressional Budget Office and available through the Federal Reserve Bank of St. Louis),[5] and the quarterly rate of PCE headline inflation, from 1960:Q1 to 2008:Q1.

Estimating this reduced-form equation (B3) does not allow us to separately identify how shifts in the demand and supply of credit translate into nominal loan growth. Moreover, the direction of causality between the left- and right-hand-side variables can be questioned. To ameliorate a potential simultaneity bias, we use lagged values of the monetary policy rate i_{t-1} as an instrument for its current value, i_t. The output of the first-stage regressions (not reported, but available upon request) shows that the lagged variable enters significantly in the determination of the fitted contemporaneous realization, with positive coefficient close to unity.

Table B2 shows the results of the second-stage regression. That coefficient of the linear relation between nominal bank loans growth on the fitted policy rate, $\hat{\imath}_t$, appears to be statistically insignificant at a 5 percent level. The output gap and inflation enter with a positive

[4]The quarterly observations include intra-annual information. The last observation of 2008 shows a decline in the growth rate due to the inclusion of the negative surprises in the third quarter of 2008. The 2008 value of the annual series in figure B1 instead only contains information up to the end of 2007.

[5]The results are similar when using different definitions of the output gap, i.e., the log-difference between GDP and its one-sided or two-sided HP trends.

Table B2. Nominal Credit Growth Process

Variables	Δ Log L_t
ϕ_i	−0.26
	(0.14)
ϕ_y	0.18
	(0.04)
ϕ_π	0.43
	(0.18)
c	2.190
	(0.205)
Observations	193
R-squared	0.18

and significant coefficient in the equation for quarterly credit growth: economic expansions and inflationary spells are characterized by a higher growth rate for nominal banking loans. In particular, a positive output gap of 1 percentage point prompts a 0.18 percentage point higher growth rate for bank loans at time t, while a 1 percent increase in inflation leads to an increase in nominal banking loans of 0.43 percent. The calibrated coefficient in equation (20) in the main text for the growth rate of *real* banking loans in the model can be obtained by subtracting π_t from both sides of equation (B3).[6]

We then remain agnostic on the sign and magnitude of the effect of the monetary policy instrument on bank lending growth and set ϕ_i equal to zero. Since the growth rate of nominal loans responds directly to the output and inflation gaps, interest rate policy can affect the degree of financial instability as well, as described in more detail in section 2.3.1 in the main paper.

[6]In a recent paper, Jimenez et al. (2012) carefully identify the exogenous effects of monetary policy and aggregate economic conditions on the demand and supply of banking loans in Spain, using loan-level data. They find that positive changes in the nominal interest rate and negative output growth reduce the likelihood that banks approve loan requests. The effect is larger for banks with poor fundamentals. They use these findings as evidence in support of the bank-lending transmission channel of monetary policy.

Appendix C. The Details of the Optimal Policy

The central bank faces the following optimization problem:

$$W_1 = \min_{i_1, y_1, L_1} u(y_1, \pi_1) + \beta[1 - \gamma_1(L_1)]W_{2,nc} + \beta\gamma_1(L_1)W_{2,c} \quad \text{(C1)}$$

subject to the following constraints defining the private-sector equilibrium conditions:

$$\begin{aligned} y_1 &= -\sigma i_1 + \sigma[(1-\epsilon)\pi_{2,nc} + \epsilon\pi_{2,c}] \\ &\quad + [(1-\epsilon)y_{2,nc} + \epsilon y_{2,c}] && \text{(C2)} \\ \pi_1 &= \kappa y_1 + \beta[(1-\epsilon)\pi_{2,nc} + \epsilon\pi_{2,c}] && \text{(C3)} \\ L_1 &= \rho_L L_0 + \phi_i i_1 + \phi_y y_1 + \phi_\pi \pi_1 + \phi_0 && \text{(C4)} \end{aligned}$$

and where

$$u(y_1, \pi_1) = -\frac{1}{2}(\lambda c_1^2 + \pi_1^2), \quad W_{2,nc} = 0, \quad W_{2,c} = \frac{u(y_{2,c})}{1 - \beta\mu}$$

$$\gamma_1(L_1) = \frac{exp(h_0 + h_1 L_1)}{1 + exp(h_0 + h_1 L_1)} \Rightarrow \gamma_1'(L_1) = \frac{q\, h_1 * exp(h_0 + h_1 L_1)}{(1 + exp(h_0 + h_1 L_1))^2}. \quad \text{(C5)}$$

First-Order Necessary Conditions. Let ω_1, ω_2, and ω_3 be the Lagrange multipliers on the constraints in equations (C2), (C3), and (C4).

$$\frac{\partial}{\partial i_1} = \omega_1\sigma - \omega_3\phi_i = 0 \quad \text{(C6)}$$

$$\Leftrightarrow \omega_1 = \frac{\omega_3\phi_i}{\sigma} \quad \text{(C7)}$$

$$\frac{\partial}{\partial y_1} = \frac{\partial u(y_1, \pi_1)}{\partial y_1} + \omega_1 - \omega_2\kappa - \omega_3\phi_y = 0 \quad \text{(C8)}$$

$$\Leftrightarrow u_{y_1} + \frac{\omega_3\phi_i}{\sigma} - \omega_2\kappa - \omega_3\phi_y = 0$$

$$\Leftrightarrow u_{y_1} + \omega_3\frac{\phi_i - \sigma\phi_y}{\sigma} - \omega_2\kappa = 0$$

$$\Leftrightarrow \omega_2 = \frac{\sigma u_{y_1} + \omega_3(\phi_i - \sigma\phi_y)}{\kappa\sigma} \quad \text{(C9)}$$

$$\frac{\partial}{\partial \pi_1} = \frac{\partial u(y_1, \pi_1)}{\partial \pi_1} + \omega_2 - \omega_3 \phi_\pi = 0 \tag{C10}$$

$$\Leftrightarrow u_{\pi_1} + \frac{\sigma u_{y_1} + \omega_3(\phi_i - \sigma\phi_y)}{\kappa\sigma} - \omega_3\phi_\pi = 0 \tag{C11}$$

$$\frac{\partial}{\partial L_1} = -\beta p'(L_1)W_{2,nc} + \beta p'(L_1)W_{2,c} + \omega_3 = 0 \tag{C12}$$

Solving for $\boldsymbol{y_1, i_1, L_1, \pi_1, \omega_3}$. We obtain that

$$\begin{aligned} y_1 = & -\sigma i_1 + \sigma[(1-\epsilon)\pi_{2,nc} + \epsilon\pi_{2,c}] \\ & + \ \alpha_c y_0 + [(1-\epsilon)y_{2,nc} + \epsilon y_{2,c}] \end{aligned} \tag{C13}$$

$$\pi_1 = \ \kappa y_1 + \beta[(1-\epsilon)\pi_{2,nc} + \epsilon\pi_{2,c}] \tag{C14}$$

$$L_1 = \ \rho_L L_0 + \phi_i i_1 + \phi_y y_1 + \phi_\pi \pi_1 + \phi_0 \tag{C15}$$

$$\frac{\partial}{\partial L_1} = -\beta p\prime(L_1)W_{2,nc} + \beta p\prime(L_1)W_{2,c} + \omega_3 = 0 \tag{C16}$$

$$\frac{\partial}{\partial \pi_1} = u_{\pi_1} + \frac{\sigma u_{y_1} + \omega_3(\phi_i - \sigma\phi_y)}{\kappa\sigma} - \omega_3\phi_\pi = 0 \tag{C17}$$

$$\Leftrightarrow \pi_1 = \frac{\sigma u_{y_1} + \omega_3(\phi_i - \sigma\phi_y)}{\kappa\sigma} - \omega_3\phi_\pi \tag{C18}$$

and hence

$$y_1 = -\sigma i_1 + \sigma[(1-\epsilon)\pi_{2,nc} + \epsilon\pi_{2,c}] + [(1-\epsilon)y_{2,nc} + \epsilon y_{2,c}] \tag{C19}$$

$$\frac{\sigma u_{y_1} + \omega_3(\phi_i - \sigma\phi_y)}{\kappa\sigma} - \omega_3\phi_\pi = \kappa y_1 + \beta[(1-\epsilon)\pi_{2,nc} + \epsilon\pi_{2,c}] \tag{C20}$$

$$L_1 = \rho_L L_0 + \phi_i i_1 + \phi_y y_1 + \phi_\pi\left(\frac{\sigma u_{y_1} + \omega_3(\phi_i - \sigma\phi_y)}{\kappa\sigma} - \omega_3\phi_\pi\right) + \phi_0 \tag{C21}$$

$$-\beta p'(L_1)W_{2,nc} + \beta p'(L_1)W_{2,c} + \omega_3 = 0. \tag{C22}$$

Appendix D. The Details of the Optimal Policy under Uncertainty

D.1 The Bayesian Policymaker

We solve the optimization problem of the Bayesian policymaker numerically. For each L_0, we evaluate the welfare loss on 1,001 grid points of the interest rate on the interval $[x - 0.1/400, x + 0.1/400]$, where x is the optimal policy rate in the absence of uncertainty, and choose the policy rate that minimizes the welfare loss.

D.2 The Robust Policymaker

We solve the optimization problem of the robust policymaker numerically. For each L_0, we evaluate the welfare loss on 1,001 grid points of the interest rate on the interval $[x - 0.1/400, x + 0.1/400]$, where x is the optimal policy rate in the absence of uncertainty, and choose the policy rate that minimizes the welfare loss. For a given interest rate, we need to solve the optimization problem of the hypothetical evil agent inside the head of the policymaker. We do so again numerically by evaluating the objective function of the evil agent. In particular, when only one parameter is uncertain, we compute the objective function on twenty-one grid points on the interval $[\theta_{min}, \theta_{max}]$ and choose the parameter value that maximizes the welfare loss. When two parameters are uncertain, we compute the objective function on 21-by-21 grid points on the interval $[(\theta_{1,min}, \theta_{2,min}), (\theta_{1,max}, \theta_{2,max})]$ where θ_1 and θ_2 are two parameters under consideration, and choose the combination of parameter values that maximizes the welfare loss.

Appendix E. More Sensitivity Analyses

E.1 Annual Calibration

There is empirical evidence that credit cycles and business cycles evolve over different time frequencies, with credit cycles showing a higher degree of persistence than business cycles (see, for example, Borio 2012 and Aikman, Haldane, and Nelson 2015). To account for the potential effect of prolonged spells of higher nominal interest rates in reducing financial instability, we solve our model calibrated at annual frequencies.

Table E1 shows the parameter values for the annual calibration of the model. The discount factor β is annualized to be consistent with a 2 percent real interest rate, as in the quarterly version described in section 2.3 of the main paper. The other parameters pertaining to the standard New Keynesian model are unchanged from the quarterly calibration, under the assumption that the policy rate and inflation are now annualized—e.g., with $\sigma = 1$, a 1 percent increase in the real interest rate translates into a 1 percent widening of the output gap on an annual basis. The annual inflation target, π^*, is assumed to be 2 percent, and hence our choice of the long-run equilibrium policy rate, i^*, of 4 percent implies an equilibrium real short-term rate of 2 percent in a model without financial instability. The weight $\lambda = 1$ in the central bank's period loss function implies equal concern for annualized inflation gaps and output gaps. The calibration of the probability of a financial crisis, γ_1, is consistent with the annual estimates of the adapted Schularick and Taylor (2012)'s model described in appendix B, while the probability of a crisis as it is perceived by the private sector is four times the value in the quarterly calibration, $\epsilon = 0.2/100$. The evolution of the credit conditions index, L, has an annualized persistence of 0.8 (consistent with the quarterly value of 0.95). The elasticities of annual credit growth with respect to the output gap and inflation are the same as in the quarterly calibration. (In the annual version of the model, the output gap is the present discounted value of future annual real rate gaps, while the inflation rate is itself expressed in annual terms. In other words, both the left- and the right-hand-side variables of the equation are annualized, with no changes required to the equation coefficients.) Finally, the persistence of the crisis is set at 0.66, so that the average spell lasts two years.

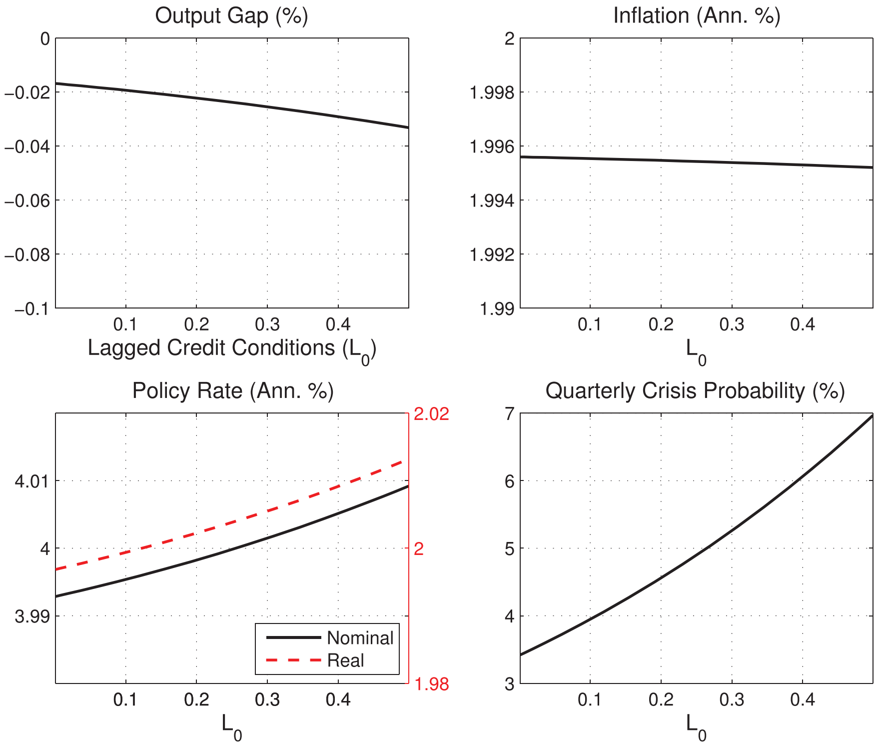
Results for the annual calibration of the model are summarized in figure E1 and are largely consistent with those of the quarterly model. Maintaining a tighter monetary policy stance for one year has a modest effect on the crisis probability, while the cost of higher rates for longer in terms of output and inflation gaps in normal times is larger than in the quarterly calibration.

Figure E1 shows the policy functions for the output gap, inflation, the nominal and real policy rate, and the crisis probability in period 1 as they depend on the level of the state variable L_0, along the horizontal axis. Since an increase in the policy rate reduces the

Table E1. Parameter Values: Annual Calibration

Param.	Description	Value	Note
	Standard Parameters		
β	Discount Factor	0.980	Standard
σ	Interest Rate Sensitivity of Output	1.0	Standard
κ	Slope of the Phillips Curve	0.024	Standard
λ	Weight on Output Stabilization	1	Equal Weights on y and the Annualized π
i^*	Long-Run Natural Rate of Interest	0.04	4% (Annual)
π^*	Long-Run Inflation Target	0.02	2% (Annual)
	Parameters for the Equation Governing the Crisis Probability		
h_0	Constant Term	–3.396	
h_1	Coefficient on L	1.88	
	Parameters for the Equation Governing the Financial Conditions		
ρ_L	Coefficient on the Lagged L	4/5	
ϕ_0	Intercept	$(1 - \rho_L) * 0.2$	
ϕ_y	Coefficient on Output Gap	1.14	See Appendix B
ϕ_π	Coefficient on Inflation Gap	–0.57	(0.43 – 1); See Appendix B
	Parameters Related to the Second Period		
$y_{2,nc}$	Output Gap in the Non-crisis State	0	
$\pi_{2,nc}$	Inflation Gap in the Non-crisis State	0	
$W_{2,nc}$	Loss in the Non-Crisis State	0	
$y_{2,c}$	Output Gap in the Crisis State	–0.1	"Great Recession"
$\pi_{2,c}$	Inflation Gap in the Crisis State	–0.02	"Great Recession"
μ	Persistence of the Crisis State	0.66	Two Years
$W_{2,c}$	Loss in the Crisis State	$\dfrac{u(y_{2,c}, \pi_{2,c})}{1 - \beta\mu}$	
	Auxiliary Parameters		
ϵ	Perceived Crisis Probability	0.2/100	Arbitrarily Small

Figure E1. Credit Growth and Optimal Policy: Annual Calibration



Data Source: Authors' calculations.
Note: This figure shows the optimal policy as a function of the initial level of the credit condition variable, L_0.

crisis probability by more when credit growth is high, the optimal policy rate increases with lagged credit conditions. When $L_0 = 0$—roughly the minimum of this variable observed in the United States over the past five decades—the central bank finds it optimal to *decrease* the policy rate just by about 1 basis point below 4 percent (the optimal rate that would prevail in a model without financial instability). For low levels of L_0, the central bank will optimally lean *with* the wind, albeit modestly, to avoid incurring prolonged output and inflation gap losses with minimal benefits in terms of reduced likelihood of a financial crisis, as suggested by Svensson (2016). Only for higher levels of L_0 will the central bank find it optimal to increase the policy rate above 4 percent. However, when $L_0 = 0.5$, the peak observed in the United States in postwar data (see figure B1), the optimal increase in the policy rate is merely 1 basis point, compared with an already small adjustment of 7 basis points in the quarterly model. Thus, even under conditions similar to those prevailing immediately prior to the onset of the financial crisis, the optimal increase to the short-term interest rate in response to potential financial stability risks would have been minimal.

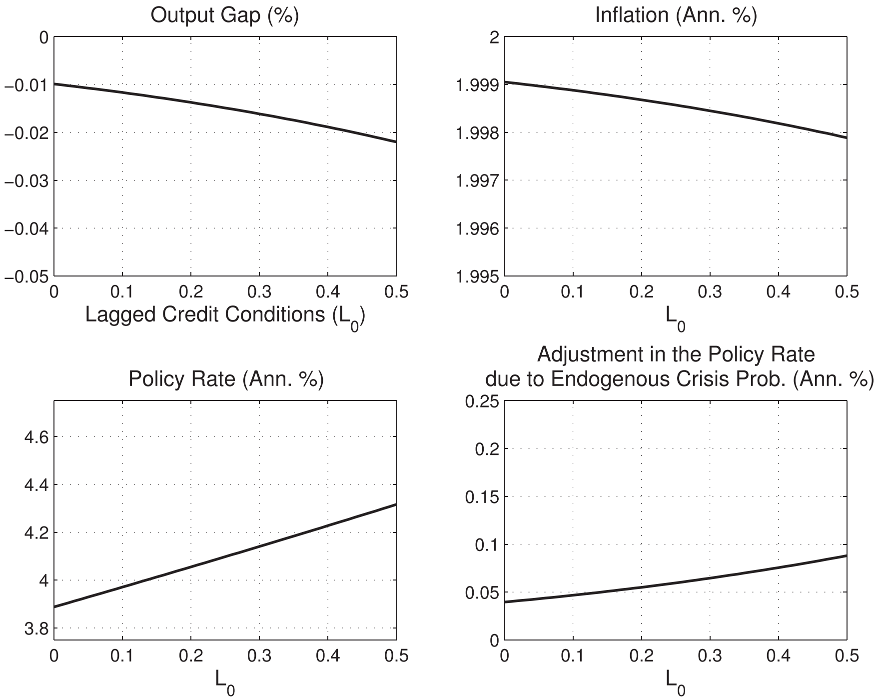
E.2 A Model with Credit–Output Linkages

In our baseline model, financial conditions affect the economy only via their effects on crisis probability. However, credit booms are often associated with output booms. Accordingly, we consider a model in which increases in financial conditions lead to an increase in output. Specifically, we modify the aggregate demand equation to include financial conditions as follows:

$$y_1 = E_1^{ps} y_2 + \sigma(i_1 - E_1^{ps}\pi_2) + \alpha_L(L_1). \tag{E1}$$

Figure E2 shows the optimal policy in this model with credit–output linkage. The optimal policy rate increases with financial conditions. This is because financial conditions act as demand shocks in this version of the model. The central bank can offset the effects of increases in financial conditions on the output gap by increasing the policy rate. Thus, the credit–output linkage gives the central bank another incentive to raise the policy rate during a credit boom, over and above the crisis probability motive described earlier. The central bank would raise the policy rate in response to credit booms by less if the crisis probability was hypothetically constant. This is shown in the bottom-right panel of figure E2, which shows the additional increase in the policy rate due to financial stability concerns. Consistent with our results in the baseline model, financial stability concerns imply a very small additional increase in the optimal policy rate.

Figure E2. A Model with Credit–Output Linkage



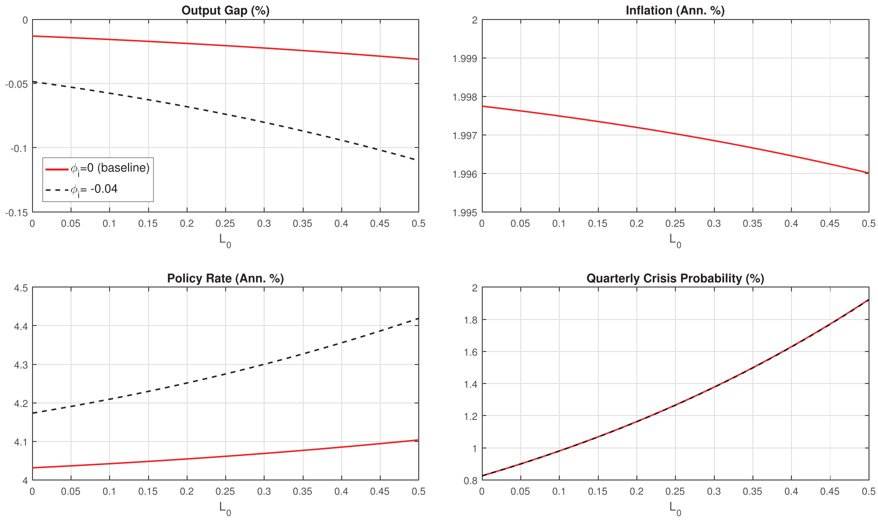
Data Source: Authors' calculations.
Note: This figure shows the optimal policy as a function of the initial level of the credit condition variable, L_0.

E.3 A Model with a Direct Effect of Interest Rates on Credit

Throughout the paper, we assume that the coefficient on the interest rate in the leverage equation is zero, motivated by the fact that the estimated coefficient is not statistically significant. In this section, we consider how optimal policy changes if this parameter is non-zero and there is a direct channel through which the interest rate affects credit conditions.

Figure E3 shows optimal policy when $\phi_i = -0.40$. This value is one standard deviation below the point estimate of -0.26 reported in appendix B. Not surprisingly, if there is a direct channel from the interest rate to credit conditions, the interest rate adjustment

Figure E3. Leverage and Optimal Policy: With and Without a Direct Channel from Interest Rates to Leverage



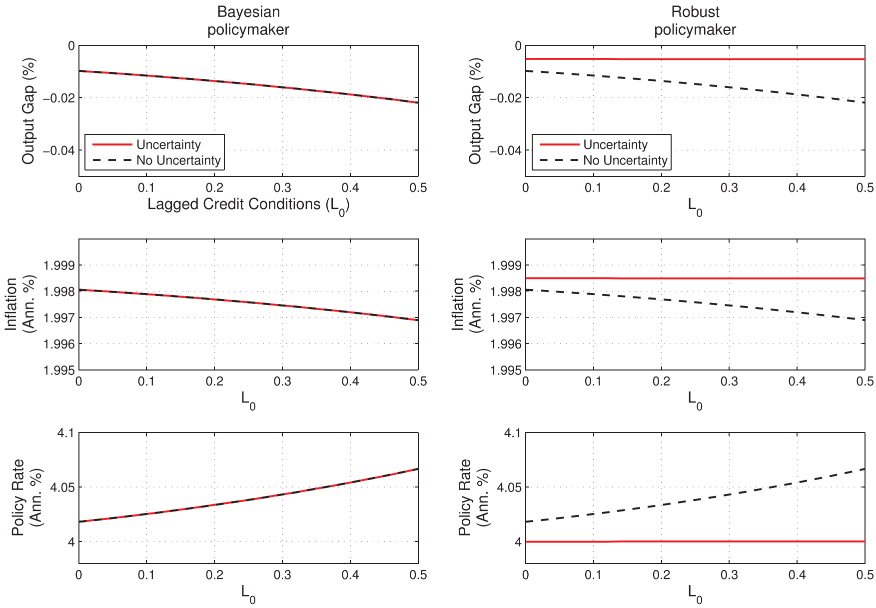
Data Source: Authors' calculations.
Note: This figure shows the optimal policy as a function of the initial level of the credit condition variable, L_0.

is more effective in reducing the crisis probability, and the optimal interest rate adjustment is larger. In this calibration, the optimal interest rate adjustment is 22 basis points larger with $\phi_i = -0.4$ than with $\phi_i = 0$ (compare 25 basis points versus 3 basis points in the baseline).

E.3.1 Uncertainty

How does the uncertainty regarding this parameter affect optimal policy? In considering the effects of uncertainty in the context of the Bayesian policymaker, we consider a situation where ϕ_i can take three values, $[-0.28, 0, 0.28]$ with equal probabilities. The high and low values are motivated as plus and minus two times the standard error of the estimate shown earlier, respectively. For the robust policymaker, we consider a setup where the hypothetical agents can choose ϕ_i from $[-0.28, 0.28]$.

Figure E4. Optimal Policy Under Uncertainty: Uncertain Elasticity of Credit Conditions to Output



Data Source: Authors' calculations.
Note: This figure shows the optimal policy as a function of the initial level of the credit condition variable, L_0, with and without uncertainty, for a Bayesian (left) and robust (right) policymaker.

As shown in the left column of figure E4, the presence of uncertainty leaves the optimal interest rate adjustment essentially unchanged. A mean-preserving spread in ϕ_i leads to a mean-preserving spread in the credit condition, L_1. The non-linearity of our logit crisis probability function implies that, for any given choice of i_t, the crisis probability is higher with uncertainty than without uncertainty. However, uncertainty does not change the slope of the crisis probability with respect to the policy rate in a quantitatively meaningful way. The elasticity is slightly smaller, and thus the optimal policy rate is slightly lower, with uncertainty than without uncertainty, but the difference is negligible.

Moving on to the robust policymaker, as shown in figure E4, the robust policymaker leaves the interest rate unchanged from the long-run equilibrium level of 4 percent. The hypothetical evil agent responds to a deviation of the interest rate from the long-run equilibrium rate of 4 percent by choosing the largest possible ϕ_i to maximize the crisis probability. When the range of ϕ_i from which the hypothetical evil agent can choose is small, the anticipation that the largest ϕ_i will be chosen later by the hypothetical evil agent makes it undesirable for the central bank to raise the policy rate, lowering the optimal policy rate. When the range of ϕ_i is sufficiently large, as in the case with our calibration, it becomes optimal to choose zero interest rate. In this case, lowering the interest rate further reduces welfare for the central bank because the hypothetical evil agent will respond to a negative deviation by choosing the smallest possible ϕ_i, which makes it undesirable for the central bank to lower the policy rate. Thus, the robust policymaker chooses to leave the interest rate at the long-run equilibrium rate of 4 percent.

E.4 Alternative Objective Functions

Figure E5 shows the outcome of optimal policy when λ is lower than our baseline value. In the figure, we use $\lambda = 0.003$, a value in line with a microfounded value if the objective function of the central bank is seen as the second-order approximation to the household's welfare.

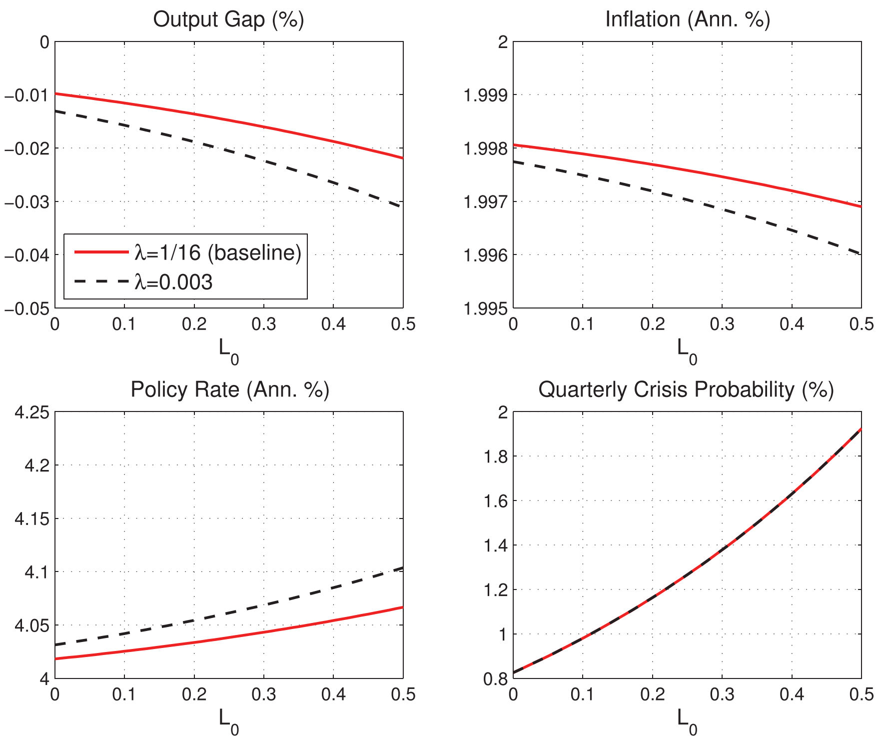
A smaller λ means the weight on the inflation stabilization objective is larger relative to the weight on the output stabilization objective. In our particular calibration, a smaller λ increases the benefit of policy tightening due to a lower crisis probability in the future, relative to the cost of policy tightening due to worse economic activities today. As a result, the optimal interest rate adjustment is larger when λ is smaller. In our calibration, the optimal interest rate is larger by 1–5 basis points with a smaller λ.

Since λ is the parameter for the central bank's preference, we will not examine the effects of uncertainty regarding this parameter.

E.5 Comparison with the Taylor Rule

In this section, we contrast the allocations and the interest rate adjustment under optimal policy with those under a standard Taylor rule. The blue vertical lines in figure E6 show the outcomes that

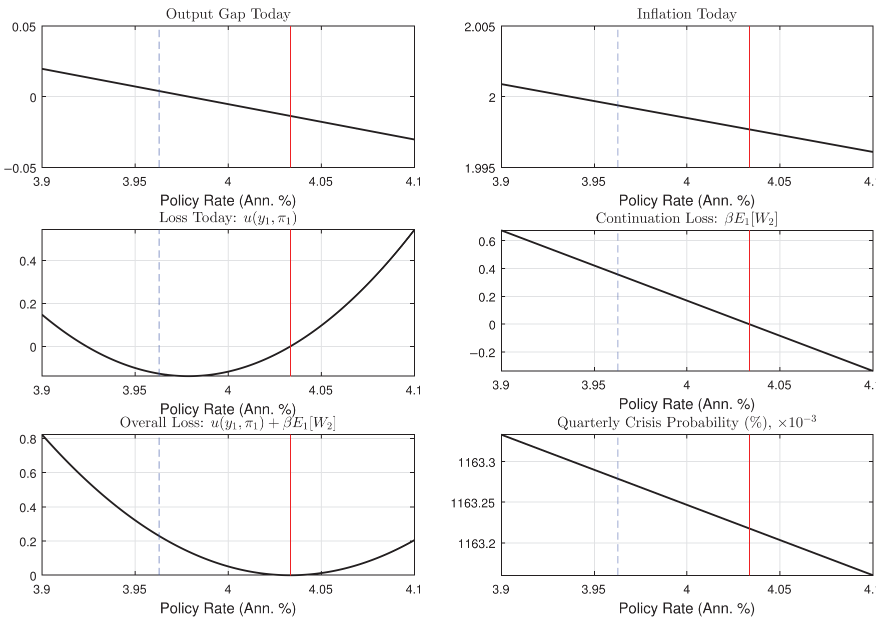
Figure E5. Leverage and Optimal Policy: Alternative Weights on Output Stabilization



Data Source: Authors' calculations.
Note: This figure shows the optimal policy as a function of the initial level of the credit condition variable, L_0.

would prevail under a Taylor rule with the inflation coefficient of 2 and the output gap coefficient of 0.25. The policy rate will be slightly below the long-run equilibrium rate of 4 percent, as the small possibility of the crisis lowers today's inflation and output gap via expectations, leading the policy rate to adjust downward. While the deviation of the interest rate prescribed by the standard Taylor rule from the optimal rate is very small, this exercise demonstrates the sub-optimality of the Taylor rule that ignores the financial stability conditions.

Figure E6. A Key Tradeoff Faced by the Central Bank (with a Taylor Rule)



Data Source: Authors' calculations.
Note: In this figure, L_0 is set to 0.2, which is roughly the average value of this variable in the United States over the past five decades. In the bottom-left panel, the red solid vertical line shows the optimal policy rate—the policy rate that minimizes the overall loss—while the blue dashed line shows the rate implied by a simple Taylor rule. The welfare losses are expressed as the one-time consumption transfer at time 1 that would make the household as well off as the household in a hypothetical economy with efficient levels of consumption and labor supply, expressed as a percentage of the steady-state consumption, as described in Nakata and Schmidt (2014). Welfare losses are normalized to be zero at the optimum.

Appendix F. The Zero Lower Bound Constraint

F.1 The Policy Tradeoff

In our baseline model, we asked the question of "how should the central bank respond to a credit boom" when the economy is in the non-crisis state today (at time $t = 1$). In this section, we modify the model in order to ask the same question, but when the economy is in a recession and the policy rate is at the zero lower bound (ZLB).

The aggregate demand equation is modified so that there is a negative demand shock, Ω_1, at time $t = 1$.

$$y_1 = E_1^{ps} y_2 + \sigma(i_1 - E_1^{ps} \pi_2 - i^*) - \Omega_1, \tag{F1}$$

where the variable Ω_1 is set so that the optimal shadow policy rate is minus 50 basis points at $L_0 = 0.2$ ($\Omega_1 = 0.0113$), that is, the policymaker is constrained by the zero lower bound.

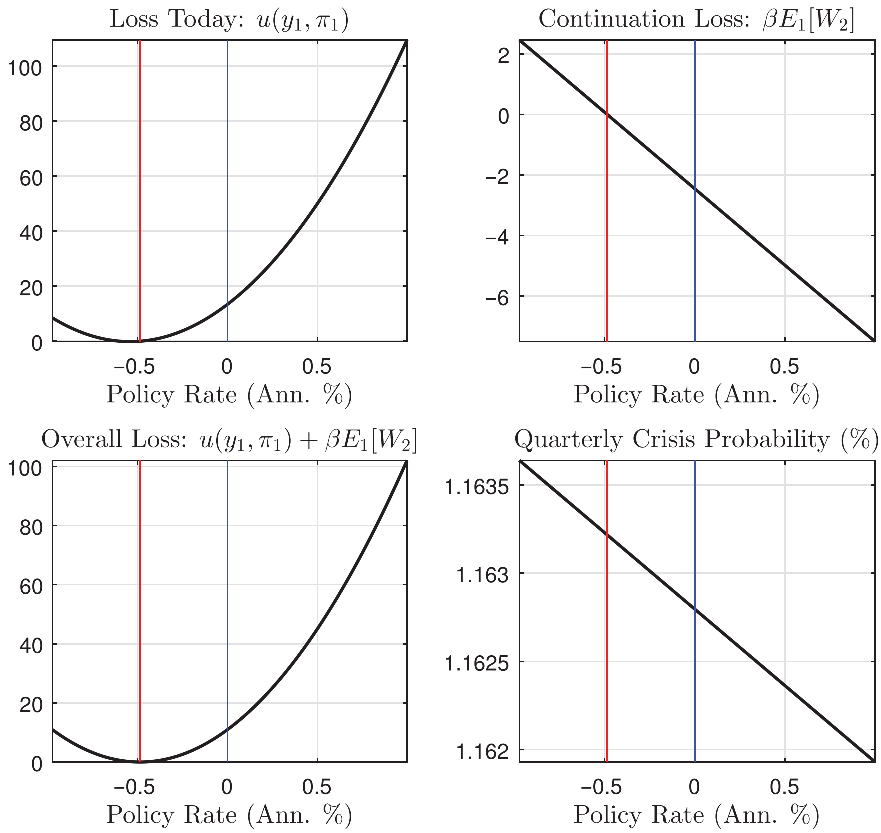
As shown in figure F1, the tradeoff facing the central bank is the same as described in the previous section. In addition, since the optimal shadow policy rate is negative, the constrained optimal policy for the nominal short-term interest rate is zero for $0 \leq L_0 \leq 0.5$ (figure F2). As shown in figure F3, the optimal actual policy rate can be positive for a sufficiently large L_0 under alternative parameterization. In our model, this happens when the severity of the crisis is comparable to that of the Great Depression.

F.2 The Zero Lower Bound and Parameter Uncertainty

Uncertainty regarding the effectiveness of interest rate policy in influencing the crisis probability affects both types of policymakers—the Bayesian and the robust policymakers—already facing a large contractionary shock in the same way as it affects the two types of policymakers in normal times. As shown in the left column of figure F4, the unconstrained optimal policy rate is higher in the presence of uncertainty than in the absence of it under the Bayesian policymaker. As shown in the left column of figure F5, the unconstrained optimal policy rate is higher in the presence of uncertainty than in the absence of it under the robust policymaker. For both types of policymakers, the unconstrained optimal policy rates remain below zero, and as a result, the actual optimal policy rate remains at zero.

Uncertainty regarding the severity of the crisis also affects the two types of policymakers facing a large contractionary shock in the same way as it affects them in normal times. As shown in the middle column of figures F4 and F5, the unconstrained optimal policy rate is higher in the presence of uncertainty than in the absence of it.

Figure F1. Optimal Policy Tradeoff and the Zero Lower Bound Constraint

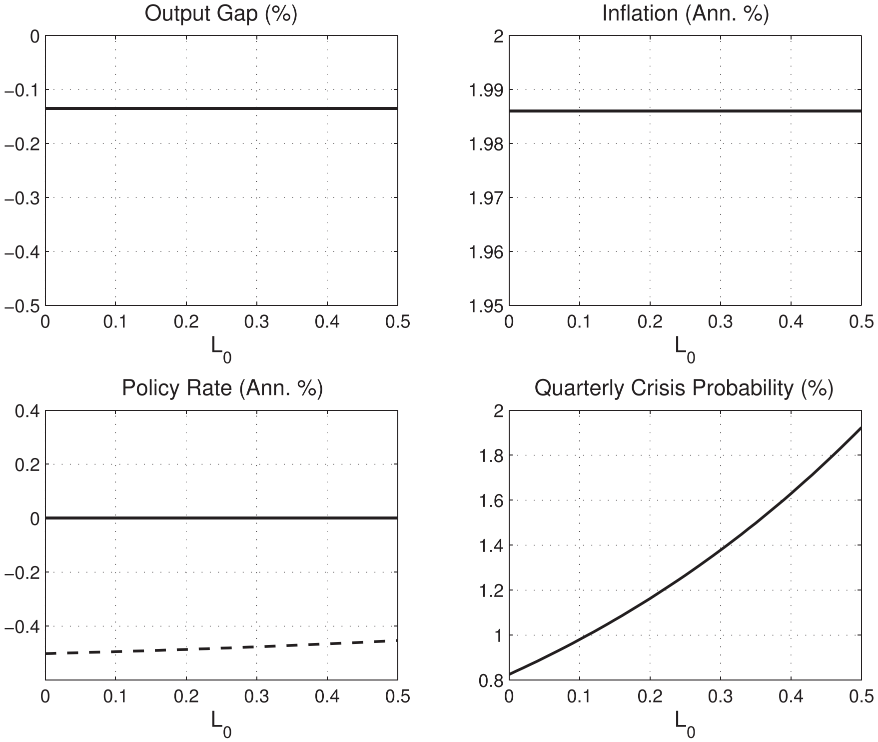


Data Source: Authors' calculations.
Note: In this figure, L_0 is set to 0.2, which is roughly the average value of this variable in the United States over the past five decades. In the bottom-left panel, the red vertical line shows the optimal shadow policy rate—the negative policy rate that would minimize the overall loss. The blue vertical line shows the constrained optimal policy rate at the zero lower bound.

Since the unconstrained optimal policy rate remains below zero, the actual optimal policy rate remains zero.

The Bayesian policymaker facing a large negative demand shock reduces the policy rate by less under uncertainty regarding the effect of policy on today's inflation and output, as shown in the right

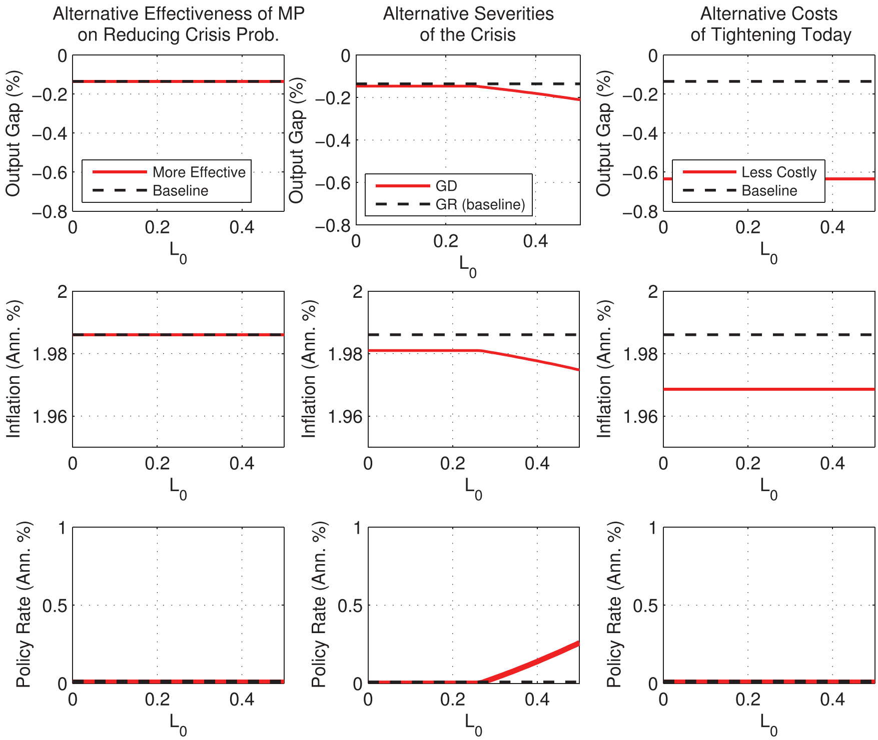
Figure F2. Optimal Policy at the ZLB



Data Source: Authors' calculations.
Note: In the bottom panels, the solid and dashed black lines are, respectively, for the actual and shadow optimal policy rates.

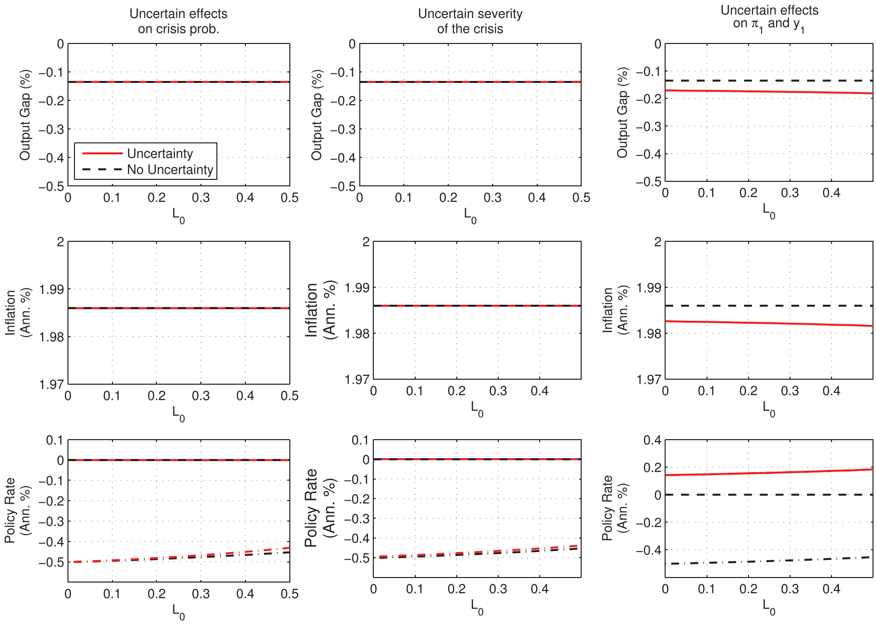
columns of figure F4. This is a manifestation of Brainard's attenuation principle: the Bayesian policymaker responds to the negative demand shock by reducing the policy rate by less under uncertainty. In our calibration, the optimal policy rate becomes positive. Uncertainty regarding the severity of the crisis affects the robust policymaker facing a large negative demand shock in the same way as in normal times. As shown in the right column of figure F4, the unconstrained optimal policy rate is slightly lower in the presence of uncertainty than in the absence of it. The unconstrained optimal policy rate is below zero, and the actual optimal policy rate is zero.

Figure F3. Optimal Policy and the ZLB: Alternative Scenarios



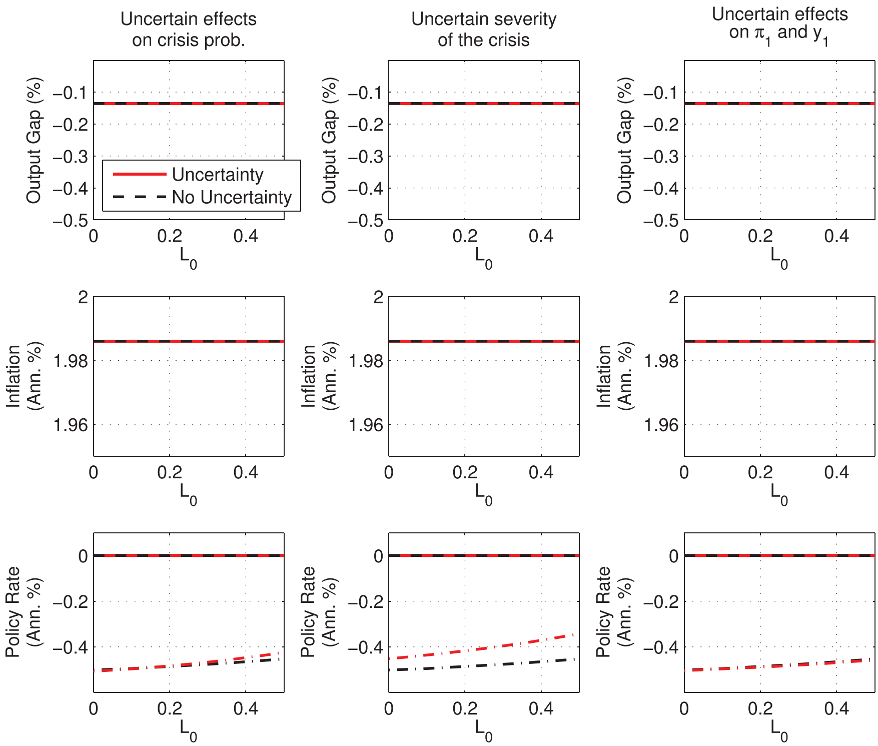
Data Source: Authors' calculations.
Note: This figure shows the optimal policy as a function of the initial level of the credit condition variable, L_0, under alternative calibrations of the model.

Figure F4. Optimal Policy under Uncertainty at the ZLB: Bayesian Policymaker



Data Source: Authors' calculations.
Note: In the bottom panels, the dash-dotted lines correspond to the shadow optimal policy rates.

Figure F5. Optimal Policy under Uncertainty at the ZLB: Robust Policymaker



Data Source: Authors' calculations.
Note: In the bottom panels, the dash-dotted lines correspond to the shadow optimal policy rates.

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