Monetary Policy, Commodity Prices, and Misdiagnosis Risk*

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How should monetary policy respond to commodity prices when the underlying drivers are difficult to diagnose accurately? If monetary authorities misdiagnose commodity price swings as being driven primarily by external supply shocks when they are in fact driven by global demand shocks, the conventional wisdom—to look through the first-round effects of commodity price fluctuations—may no longer be sound policy advice.

To analyze this question, we employ the multi-economy DSGE model of Nakov and Pescatori (2010), which splits the global economy into commodity-exporting and non-commodity-exporting economies. In an otherwise conventional DSGE setup, commodity prices are modeled as changing endogenously with global supply and demand developments, including global monetary policy conditions. This framework allows us to explore the implications of monetary policy decisions when there is a risk of misdiagnosing the drivers of commodity prices.

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We first confirm that monetary authorities deliver better economic performance when they are able to accurately identify the global nature of the shocks, i.e., global supply and demand shocks, driving commodity prices. Moreover, we show that when it is difficult to identify these shocks, monetary authorities can minimize some of the adverse feedbacks from misdiagnoses by targeting core inflation. Finally, we highlight the implications of misdiagnosis risk in the case where the monetary authority misinterprets supply-driven increases in commodity prices as demand driven; the contraction in both output and core inflation is larger than in the case of an accurate diagnosis.

In light of recent empirical studies documenting the significance of global demand in driving commodity prices, these findings call for giving greater prominence to global factors in domestic monetary policymaking and highlight potential gains from focusing on accurate diagnoses of domestic and global sources of shocks.

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1. Introduction

Over the past decade, global commodity prices have experienced wide swings, reaching historically high levels in the run-up to the Great Recession before plummeting as the global economy collapsed. Prices subsequently rebounded with the global economic recovery but then fell again amid significant policy concerns. While challenging, this type of volatility is not a new environment for policymakers. Even though most commodity prices remained broadly stable during the so-called Great Moderation, they were quite volatile in the 1970s amid geopolitical tensions that pushed oil price volatility to then-unprecedented levels.

It was the experience of the 1970s that forged the conventional wisdom about how monetary authorities should respond to commodity price fluctuations. Commodity price fluctuations were seen largely as the result of exogenous supply shocks; in such an environment, the conventional wisdom that emerged was that, when facing such swings, monetary authorities should look through the first-round price effects and only respond to the second-round effects on wage and inflation expectations. In practice, this suggested a monetary policy focus on core inflation.

Views about the drivers of global commodity price swings have been evolving, especially in recent years, as a growing body of statistical evidence points to a new interpretation of commodity price swings. Kilian (2009), for example, finds evidence that oil price fluctuations have been increasingly influenced by demand from commodity-hungry emerging market economies (EMEs). The most recent literature has not challenged this view: Kilian and Baumeister (2016) argue that the oil price decline in 2015 should also be ascribed to a slowdown in global economic activity; and Stuermer (2017) and Fueki et al. (2018) emphasize the role of demand shocks in a historical perspective. In a similar vein, Sussman and Zohar (2016) report that commodity price fluctuations can be taken as a proxy for global demand. In a broader context, Filardo and Lombardi (2014) note the growing prominence of these global demand shifters for EME inflation dynamics. This evidence raises doubts about the relevance of the conventional wisdom.

The prominence of *endogenous* commodity price swings has important implications for monetary policy, given its central role in influencing aggregate demand. The relationship between monetary policy decisions and endogenous commodity prices implies an important two-way link. Monetary policy decisions influence aggregate demand and hence commodity prices. Indeed, Anzuini, Lombardi, and Pagano (2013) and Filardo and Lombardi (2014) report evidence that loose monetary policy has had an effect on commodity prices via the global demand channel.[1] At the same time, commodity price swings influence price stability and hence monetary policy decisions.

Given the global nature of commodity prices, misdiagnosis risk is particularly high in a world of many central banks with purely domestic monetary policy mandates. Individual countries may think that because they are sufficiently small, they can reasonably ignore the effect of their own policy decisions on the rest of the world,

[1]There is evidence that U.S. monetary policy plays a special role. Akram (2009) finds that lower interest rates in the United States boost commodity prices via an exchange rate channel.

and hence on commodity prices. This would be the case if all economies were hit by uncorrelated idiosyncratic shocks. However, global shocks imply that central banks are likely to respond in a correlated way which would endogenously feed back on commodity prices. A failure to internalize the endogenous feedbacks could contribute to economic and financial stability concerns (Caruana, Filardo, and Hofmann 2014; Rajan 2015).

From a modeling perspective, this discussion suggests the importance of developing monetary policy models with endogenously determined commodity prices and monetary authorities that are subject to misdiagnosis risk. To date, the bulk of the theoretical literature has stayed clear of models with endogenous commodity prices (see, e.g., Leduc and Sill 2004, Carlstrom and Fuerst 2006, Montoro 2012, Natal 2012, and Catao and Chang 2015). Moreover, this literature has generally focused on how a monetary authority should respond to exogenous movements in oil prices, e.g., whether it is optimal to target core or headline inflation and whether commodity price movements have far-reaching implications for the tradeoff between stabilizing output and controlling inflation. For example, Blanchard and Galí (2010) have gone so far as to argue that an increase in commodity prices driven by foreign demand can still be treated by a domestic monetary authority as an external supply shock. Such a conclusion is less tenable in models of endogenous commodity prices and correlated monetary policy reaction functions.

Various theoretical papers have addressed the endogeneity of commodity prices in small-scale dynamic stochastic general equilibrium (DSGE) models (e.g., Backus and Crucini 2000; Bodenstein, Erceg, and Guerrieri 2008; and Nakov and Nuño 2013). However, these models have generally ignored monetary policy, focusing instead on oil price determination and the frictions affecting it. Nakov and Pescatori (2010) is an early attempt to characterize monetary policy tradeoffs in a DSGE model in which oil prices are determined endogenously. Another important contribution to this literature is Bodenstein, Guerrieri, and Kilian (2012), who highlight, as we do, the benefits of identifying the nature of the shocks hitting the economy.

Our model extends this class of models by considering the policy challenges facing a monetary authority when it tries to infer the source of commodity price shocks.[2] Namely, there is a risk that a monetary authority may misdiagnose a commodity price swing as being driven by an external supply shock when it is, in fact, driven by an endogenous global demand shock, and vice versa. In our model, the commodity price is endogenously determined in equilibrium by the interplay of global demand and commodity supply from two types of commodity-exporting economies—one competitive and one monopolistic. In this setting, the optimal monetary policy response to commodity price swings depends on the perception of the underlying drivers of the swings. Unable to fully know the nature of the drivers, the monetary authority infers them via signal extraction, thereby opening up the possibility of systematic misdiagnoses.

The modeling exercise delivers several policy-relevant implications. First, it is important to distinguish between global demand and supply shocks when responding to commodity prices. If it is possible to accurately diagnose the source of a shock, our model finds that the best response to demand shocks is to lean against them fully (a result consistent with a standard New Keynesian closed-economy model). The best response to commodity supply shocks (i.e., a decrease in commodity prices) is to look through them.

Second, the conventional wisdom of looking through the first-round effects of commodity price swings is not always optimal. In our model, this result arises because our model breaks the "divine coincidence" between inflation and output gap stabilization (e.g., Blanchard and Galí 2007), which is a standard feature of DSGE models with exogenous commodity prices. The breaking of the divine coincidence comes, in part, from the assumption of a monopolistically competitive commodity exporter, and in part from the imperfect information environment.

Third, misdiagnosis risk matters. In the case where the monetary authority misinterprets supply-driven increases in commodity prices as demand driven, the contraction in both output and core inflation is larger than in the case of an accurate diagnosis. This indicates another reason for the breakdown of the divine coincidence in this model (even if the dominant exporter acts as a price taker). This

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[2] See Filardo and Lombardi (2014) for a discussion of commodity price misdiagnosis risks in the context of Asian EMEs.

result underscores the potential benefits of trying to correctly diagnose the sources of commodity price swings when setting monetary policy. For example, a monetary authority that misdiagnoses global demand shocks as external supply shocks amplifies cyclical fluctuations (including commodity prices) and, as a result, destabilizes the economy.

2. Outline of the Baseline Model

We present a global monetary policy model in which commodity prices are determined endogenously, in the spirit of Nakov and Pescatori (2010, hereafter NP). The global economy is split into commodity-importing countries and commodity-exporting ones.

The commodity-importing countries are treated as one representative economy. It imports the commodity both for consumption and as an input in production of final goods and services. The firms produce final goods and services in a monopolistically competitive way in the face of nominal rigidities.

The commodity-exporting economies comprise a dominant commodity-exporting economy and a fringe of smaller competitive exporters. The dominant commodity-exporting economy has market power and sets prices above marginal cost. The fringe of small exporting countries operates competitively, taking the global commodity price as given. Consumers in these commodity-exporting countries buy final goods and services from the commodity-importing economy.[3]

The role of the central bank is to set monetary policy à la Taylor (1993). We extend the NP model by considering two types of uncertainty that the central bank faces. The first is real-time data uncertainty. Because inflation and output are typically only observed with a lag, we model the central bank as reacting to *expected* inflation and the *expected* output gap. The central bank can observe and

[3]Note that cross-border financial autarky is assumed, implying that current accounts are balanced in each period. Also, trade is assumed to be carried out in a common global currency, suppressing potential tradeoffs from exchange rate dynamics. These assumptions streamline the analysis and allow us to highlight the key implications of misdiagnosis risk which would be more complex in a richer model.

respond to commodity prices in real time. Relaxing the perfect information assumption opens up one way, within this class of models, for us to explore the consequences of central bank miscues in setting the policy rate. Second, and much less frequently addressed in the literature, is uncertainty about whether the commodity price is driven by demand or supply shocks. We assume that central banks cannot directly observe the source of commodity price shocks, but policymakers use available data to infer them. We explore the inferential challenges and implications that arise from the misdiagnosis of shocks, thereby shedding further light on the practical use of DSGE models.

The rest of this section sketches out the structure of the model. In addition to the two key points mentioned above, we also deviate from the Nakov and Pescatori (2010) setup in three other respects: (i) we introduce the commodity good into the households' utility function, which allows us to consider nontrivial policy responses to differences between headline and core inflation; (ii) we interpret the commodity as a broad basket of commodities rather than focusing narrowly on oil;[4] and (iii) we solve the Nash (instead of the Ramsey) problem for the dominant producer so as to reflect realistic information constraints facing producers.[5]

2.1 Representative Households and Firms in the Commodity-Importing Economy

Imported commodities play two roles in this economy. For households, commodities enter the consumption basket; as mentioned above, we make this dependence explicit to highlight the monetary policy implications of headline and core inflation. For firms in the commodity-importing economy, commodities are used in the production of final goods and services.

[4]This is consistent with the view that commodity markets as a whole remain an important driver of short-run inflation dynamics and often reflect cartel-like behavior for metals (and minerals) and anti-competitive behavior for foodstuffs (see Organisation for Economic Co-operation and Development 2012).

[5]The computer code for the model is available at http://www.macromodelbase.com/.

2.1.1 Representative Households

The representative household has a utility function over consumption (C_t) and labor (L_t),

$$U_{t_0} = E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} exp\left(e_t\right) \left[\ln\left(C_t\right) - \frac{L_t^{1+v}}{1+v}\right], \tag{1}$$

where e_t is a preference shock v and is the inverse of the labor supply elasticity. Consumption is defined as a Cobb-Douglas aggregator of *final* goods and services, $C_{Y,\,t}$, and the commodity, $\mathfrak{M}_{C,t}$:

$$C_t = \left(C_{Y,t}\right)^{1-\gamma} \left(\mathfrak{M}_{C,t}\right)^{\gamma}. \tag{2}$$

$C_{Y,\,t}$ is a Dixit-Stiglitz aggregate of a continuum of differentiated goods and services, $C_{Y,\,t}(z)$, of the form $C_{Y,t} = \left[\int_0^1 C_{Y,t}\left(z\right)^{\frac{\varepsilon-1}{\varepsilon}} dz\right]^{\frac{\varepsilon}{\varepsilon-1}}$.

The household maximizes its intertemporal utility subject to the period budget constraint,

$$C_t = \frac{W_t\, L_t}{P_t} + \frac{B_{t-1}}{P_t} - \frac{1}{R_t}\frac{B_t}{P_t} + \frac{\Gamma_t}{P_t} + \frac{T_t}{P_t}, \tag{3}$$

where W_t is the nominal wage, P_t the price of the consumption good, B_t the end-of-period nominal bond holdings, R_t the riskless nominal gross interest rate, Γ_t the share of the representative household's nominal profits, and T_t the net transfers from the government.

2.1.2 Firms

Monopolistically competitive firms in the commodity-importing economy produce final goods and services using a Cobb-Douglas technology:

$$Y_t\left(z\right) = A_t L_t\left(z\right)^{1-\alpha} \mathfrak{M}_{Y,t}\left(z\right)^{\alpha}, \tag{4}$$

where $\mathfrak{M}_{Y,t}$ is the commodity input and α denotes the commodity share in production. The firms' cost-minimization problem implies an expression for the real marginal cost, given the real commodity market price, $Q_t \equiv P_{\mathfrak{M},t}/P_t$:

$$MC_t\left(z\right) = \left(\frac{W_t}{P_t}\right)^{1-\alpha} Q_t^{\alpha} \Big/ \left[A_t\left(1-\alpha\right)^{1-\alpha}\alpha^{\alpha}\right].^{6} \tag{5}$$

These first-order conditions imply the demand equations for the commodity:

$$\mathfrak{M}_{Y,t}\left(z\right) = \alpha\frac{MC_t}{Q_t}Y_t\left(z\right). \tag{6}$$

Under Calvo pricing, and following Benigno and Woodford (2005), the implied nonlinear Phillips curve for final goods and services—i.e., core inflation ($\Pi_{Y,t}$)—is

$$\theta\Pi_{Y,t} = 1 - (1-\theta)\left(\frac{N_t}{D_t}\right)^{1-\varepsilon}, \tag{7}$$

where N_t/D_t is the equilibrium relative price, with $D_t = Y_t/C_t + \theta\beta E_t\left[\left(\Pi_{Y,t+1}\right)^{\varepsilon-1}\mathrm{D}_{t-1}\right]$ and $N_t = \mu Y_t MC_t/C_t + \theta\beta E_t\left[\left(\Pi_{Y,t+1}\right)^{\varepsilon}\mathrm{N}_{t-1}\right]$.

2.1.3 Aggregation

Aggregating firms' demand for labor and the commodity yields a set of equations with which to solve the model. Even though labor and commodity demand differ across firms due to Calvo staggered pricing, the aggregate-demand equations resemble those of the individual firm's demand equations except for an adjustment capturing the effect of price dispersion. Higher inflation increases the dispersion of prices, and this increased price dispersion boosts labor and commodity demands for a given level of output.

[6]Note that all firms face the same real marginal costs given the constant-returns-to-scale technology and competitive factor markets. In this model, the real commodity price is proportional to the inverse of the importing economy's terms of trade.

2.2 Representative Households and Firms in the Commodity-Exporting Economies

The commodity market is modeled as comprising a dominant commodity exporter and a fringe group of competitive commodity exporters. The dominant exporter produces monopolistically, and the fringe produces competitively. For completeness, the households in these economies face conventional decision problems.

2.2.1 Dominant Commodity Exporter

In each period, the dominant exporter chooses its supply to maximize profits, taking global demand as given and internalizing the response of the competitive fringe. The dominant exporting economy produces the commodity according to the technology:

$$\mathfrak{M}_t = Z_t I_t^{*,D}, \tag{8}$$

where Z_t is an exogenous productivity shifter and $I_t^{*,D}$ is the intermediate input (bought from the commodity-importing economy). Productivity evolves exogenously according to

$$\ln Z_t = (1-\rho_z)\ln \bar{Z} + \rho_z \ln Z_{t-1} + \varepsilon_t^z, \tag{9}$$

where $\varepsilon_t^z \sim i.i.d.N\left(0, \sigma_z^2\right)$. Shocks to Z_t can then be interpreted as *global commodity supply shocks.*

The household utility function depends only on consumption of final goods and services:

$$U_{t_0}^{*,D} = E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \ln\left(C_t^{*,D}\right) \tag{10}$$

subject to the period budget constraint, $P_{Y,t}C_t^{*,D} = \Gamma_t^{*,D}$, which equates consumption expenditure to dividends from commodity production, $\Gamma_t^{*,D}$.

2.2.2 Fringe of Competitive Commodity Exporters

The fringe exporters comprise a continuum of atomistic firms indexed by $j \in [0, \Omega_t]$. Each produces a quantity $X_t(j)$ of the commodity according to technology of the form

$$X_t(j) = \xi(j) Z_t I_t^{*,F}(j), \text{ subject to capacity constraint}$$
$$X_t(j) \in \left[0, \bar{X}\right], \tag{11}$$

where $[\xi(j) Z_t]^{-1}$ is the marginal cost of economy j defined by an idiosyncratic shock and global supply shock. Input $I_t^{*,F}(j)$ is an intermediate input used in commodity production and is bought from the commodity-importing economy. Each firm maximizes profits, taking as given the international real price of the commodity, Q_t,

$$\max Q_t X_t(j) - \frac{\frac{P_{Y,t}}{P_t} X_t(j)}{\xi(j) Z_t} s.t.\, X_t(j) \in \left[0, \bar{X}\right]. \tag{12}$$

The resulting supply by the fringe of competitive exporters is $X_t = \Omega_t Z_t Q_t$.

Like the setup for the dominant exporting economy, the household utility function in the fringe economies depends only on the aggregate consumption of final goods and services:

$$U_{t_0}^{*,F} = E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \ln\left(C_t^{*,F}\right), \tag{13}$$

subject to the following period budget constraint, $P_{Y,t} C_t^{*,F} = \Gamma_t^{*,F}$, which equates consumption expenditures to dividends earned by the fringe commodity exporters, $\Gamma_t^{*,F}$.

2.2.3 Aggregate Production of the Commodity

In the case of a perfectly competitive fringe market of commodity exporters, the equilibrium commodity price is equal to marginal costs:

$$Q_t^{PC} = Z_t^{-1}, \tag{14}$$

and the quantity produced is given by the global demand at that price. Having market power, the dominant commodity exporter determines its supply by internalizing the optimal response of the fringe and the global demand function for the commodity. The dominant producer's objective function is

$$\max_{\{\mathfrak{M}_t\}_{t_0}^{\infty}} E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \ln \left(Q_t^{1/1-\gamma} \mathfrak{M}_t - \mathfrak{M}_t / Z_t \right) \tag{15}$$

given commodity demand and supply from fringe suppliers, $\mathfrak{M}_t = \mathfrak{M}_{C,t} + \mathfrak{M}_{Y,t} - X_t$.[7]

The first-order condition of this problem determines the commodity price (in real terms):

$$Q_t = \Psi_t Z_t^{-1}, \tag{16}$$

where $\Psi_t \equiv 1/(1-\eta_t)$ is the commodity market markup and $\eta_t = \mathfrak{M}_t / (\mathfrak{M}_t + 2X_t)$ is the elasticity of substitution of the global demand for the commodity (in absolute value). Accordingly, the commodity price is a markup over marginal cost, the latter being influenced by commodity supply shocks, firm-specific supply shocks, and shifts in g_t.[8]

2.2.4 Market Clearing

Under the assumption that all relevant markets clear, there is a well-defined aggregate demand for final goods and services equal to aggregate supply,

$$Y_t = C_{Y,t} + C_t^{*,D} + I_t^{*,D} + +C_t^{*,F} + +I_t^{*,F} + g_t, \tag{17}$$

[7]Note that the dominant exporter takes as given the macroeconomic variables of the fringe producers, e.g., $(C, MC, Y, \Omega, \Delta)$, and internalizes the fringe's reaction, but not the feedbacks of its actions on the macroeconomic performance of the commodity-importing economy. Nakov and Pescatori (2010) analyze this case in which the dominant exporter completely internalizes its actions on the importing country. Technically, they solve the Ramsey problem of the dominant exporter, which may be useful in capturing longer-run supply behavior in the data. Our Nash solution is more restrictive but may be seen as more reasonable when considering a central bank's short-run tradeoffs in a world with incomplete information. Further details on the dominant exporter's problem are provided in appendix B.

[8]In addition, the markup $\Psi_t \equiv 1/(1-\eta_t)$ is an increasing function of the dominant commodity exporter's market share relative to that of the competitive fringe of commodity exporters. The limiting case is when $\mathfrak{M}_t \to 0$ corresponds to perfect competition while $X_t \to 0$ is the case of a single monopolist.

which includes final goods consumption in the commodity-importing economy and the aggregate consumption and intermediate goods demanded by the dominant and the competitive fringe of the commodity-exporting countries, respectively (superscript D denotes the dominant commodity-exporting economy and F the competitive fringe).[9] Finally, g_t[10] is an aggregate demand shock, which captures a positive shift in the demand for the final good produced by the commodity-importing country. Note that the monetary authority cannot fully offset the effect of this shock in the same way as it could with a markup shock.

2.3 Characterizing Monetary Policy

Monetary policy in the commodity-importing country is modeled as a linear Taylor-type rule informed by deviations from model-consistent benchmarks for output, inflation, and the interest rate. This section defines the benchmarks and analyzes the implications of alternative policy rules.

2.3.1 Optimal Benchmarks

Benchmark output gaps can be derived by substituting the equations for labor demand, labor supply, aggregate demand, and commodity demand for production into the aggregate production function. The log-level of output in terms of marginal costs, the dispersion of prices, productivity, and the real commodity price is[11]

$$y_t = \frac{1}{1-\alpha} a_t + \frac{\alpha}{1-\alpha}(mc_t - q_t) + \frac{1}{1+v}\Upsilon\left(mc_t + \frac{\gamma}{1-\gamma} q_t\right). \tag{18}$$

[9]The structural parameters in the model are calibrated to be in line with those found in the literature (see table A.1 in appendix A). Fiscal policy is modeled as a linear rule: $T_t = \tau P_{Y,t} Y_t$.

[10]g_t is modeled as an AR(1) process with zero mean, i.e., $g_t = \rho_g g_{t-1} + \varepsilon_t^g$.

[11]The level of output is $Y_t = \left(\frac{A_t}{\Delta_t}\right)^{1/(1-\alpha)} \left[\frac{(1-\alpha)MC_t\Delta_t}{(Q_t)^{-\gamma/(1-\gamma)} - \alpha MC_t \Delta_t}\right]^{1/(1+v)} \left[\alpha \frac{MC_t\Delta_t}{Q_t}\right]^{\alpha/(1-\alpha)}$ and $\Upsilon \equiv \left[1 - \frac{\alpha}{\mu} Q^{\gamma/(1-\gamma)}\right]^{-1} \geq 1$. A full derivation is provided in appendix C.

The *level of natural output*, y_t^n, is defined as the level of output consistent with a flexible-price equilibrium. In this case, the marginal cost is constant, $MC_t = \mu^{-1}$, and there is an absence of price dispersion, $\Delta_t = 1$. In log-linear terms, the level of natural output, y_t^n, is of the form

$$y_t^n = \frac{\alpha}{1-\alpha} a_t - \left(\frac{\alpha}{1-\alpha} - \frac{1}{1+v} \frac{\gamma}{1-\gamma} \Upsilon \right) q_t. \tag{19}$$

As shown in equation (19), commodity price fluctuations have two opposing effects on the level of natural output. From the side of production (i.e., the first term in the parentheses), an increase in the commodity price has a qualitative effect similar to that of a negative productivity shock; it reduces the level of natural output. From the side of consumption (i.e., the second term in the parentheses), an increase in the commodity price increases the level of natural output. The latter term reflects an increase in labor due to a negative income effect from a higher commodity price.

The *natural output gap*, $\hat{y}_t^n = \left[\frac{\alpha}{1-\alpha} - \frac{1}{1-v} \Upsilon \right] mc_t$, measures the difference between the actual and the natural level of output. This implies that responding to the natural output gap is equivalent to responding to real marginal costs, up to a scale factor.

Similarly, the *log-level of efficient output*, y_t^e, is defined with respect to the efficient allocation, i.e., flexible prices and no monopolistic distortions in the commodity market or in the final goods market (which implies that $Q_t^e = Z_t^{-1}$ and $\mu^e = 1$):[12]

$$y_t^e = \frac{\alpha}{1-\alpha} a_t - \left(\frac{\alpha}{1-\alpha} - \frac{1}{1+v} \frac{\gamma}{1-\gamma} \Upsilon^e \right) z_t. \tag{20}$$

A key difference is that commodity markup shocks do not affect the level of efficient output: such output is instead affected only by fluctuations associated with supply shocks in the commodity market. As a consequence, a demand-driven increase in the commodity price

[12]With $\Upsilon^e \equiv \left[1 - \alpha Z^{-\gamma/(1-\gamma)} \right]^{-1}$. The relationship between Υ^e and Υ depends on the extent of monopolistic distortions. Υ^e and Υ are equal only if both markets are perfectly competitive or if the commodity is not used for production (that is, $\alpha = 0$).

would leave the benchmark efficient output gap unchanged. However, a negative commodity supply shock would decrease both the natural and efficient output levels, albeit by different amounts.[13] The efficient output gap, $\hat{y}_t^e$, which is defined as the difference between actual output and the efficient level of output, is of the form[14]

$$\hat{y}_t^e = \hat{y}_t^n - \left(\frac{\alpha}{1-\alpha} - \frac{1}{1+v} \frac{\gamma}{1-\gamma} \Upsilon \right) \hat{\Psi}_t - \frac{1}{1+v} \frac{\gamma}{1-\gamma} (\Upsilon - \Upsilon^e) Z_t, \tag{21}$$

where this is the welfare-relevant output gap and is equal to the natural output gap plus a term that depends on the commodity price markup and the commodity supply shock.

2.3.2 Breaking Down the Divine Coincidence

Both core inflation and headline inflation are determined by the natural output gap, expected inflation, and commodity price changes. Expressed in log-linear terms, the equations for core inflation and headline inflation are, respectively,

$$\pi_{Y,t} = \kappa_y \hat{y}_t^n + E_t \pi_{Y,t+1} \text{ and} \tag{22}$$

$$\pi_t = \pi_{Y,t} + \frac{\gamma}{1-\gamma} \Delta q_t. \tag{23}$$

Equation (22) describes the determinants of core inflation, and equation (23) describes headline inflation written in the form of a Phillips curve for aggregate final goods with $\hat{y}_t^n$ the natural output gap. Stabilization of the natural output gap is equivalent to stabilization of core inflation. And, in that case, headline inflation would vary proportionally with changes in real commodity prices. Equation (22) can be written in terms of the efficient output gap:

$$\pi_{Y,t} = \kappa_y \hat{y}_t^e + E_t \pi_{Y,t+1} + u_t, \tag{24}$$

[13]The level of efficient output contracts less than the level of natural output in response to a negative commodity supply shock because the commodity price markup partially offsets the effects of supply shocks on the commodity price.

[14]$\hat{\Psi}_t$ and z_t are Ψ_t and Z_t expressed in log-deviations from the steady state; $\Upsilon^e \equiv \left[1 - \alpha Z^{-\gamma/(1-\gamma)}\right]^{-1}$.

where u_t is an endogenous cost-push shock, which is a function of both $\hat{\Psi}_t$ and z_t. In this model, the divine coincidence featured in models with exogenous commodity prices is broken. It is no longer possible to simultaneously stabilize core inflation and the welfare-relevant output gap. The tradeoff arises from the effect of commodity price fluctuations on the level of efficient output. An increase in commodity price markups generates a positive cost-push shock, which puts upward pressure on core inflation but lowers the efficient output gap.

3. Monetary Policy Responses to Commodity Prices

With the model outlined in section 2 (and calibrated as reported in appendix A), we now explore the performance of alternative monetary policy rules assuming different types of imperfect information. This allows us to focus on both the possibility that central banks may misdiagnose the nature of the shocks hitting an economy and the associated implications. The imperfect information settings also highlight the absence of the divine-coincidence property: monetary policy cannot perfectly stabilize inflation and the output gap unless the central bank is able to accurately identify the nature of underlying shocks.[15]

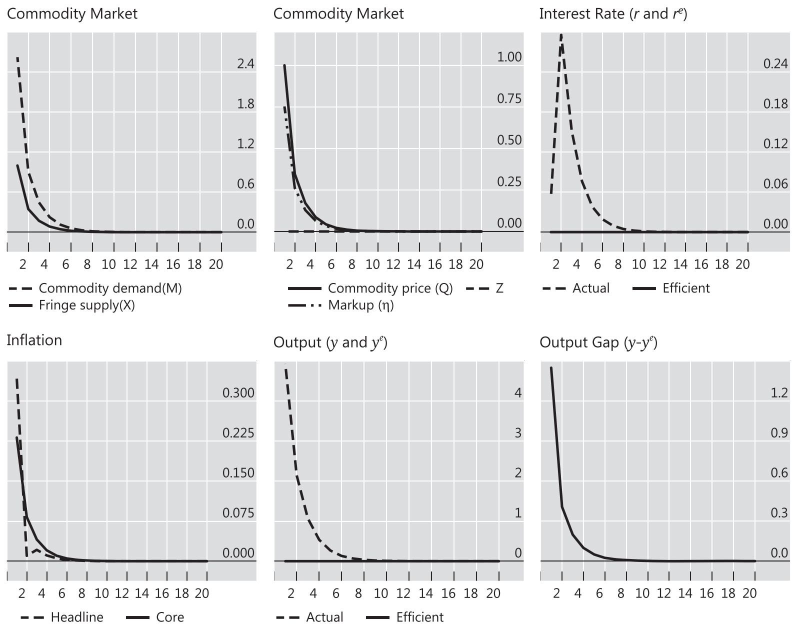
3.1 Monetary Policy Responses under Conventional Data Uncertainty

The baseline policy rule assumes that the monetary authority responds to expectations of core inflation and the efficient output gap. It is a log-linear Taylor-type rule:

$$r_t = E_{t|t-1}\left[r_t^e + \varphi_{core}\pi_{Y,t} + \varphi_y\left(y_t - y_t^e\right)\right] + \varphi_{com}\Delta q_t, \qquad (25)$$

[15]Consistent with the Nakov and Pescatori (2010) model, we focus on the implications of uncertainty for monetary policy tradeoffs only in the commodity-importing economy by assuming nominal rigidities are absent in the commodity-exporting countries.

Figure 1. Responses to a Positive Aggregate-Demand Shock (ε_t^g)



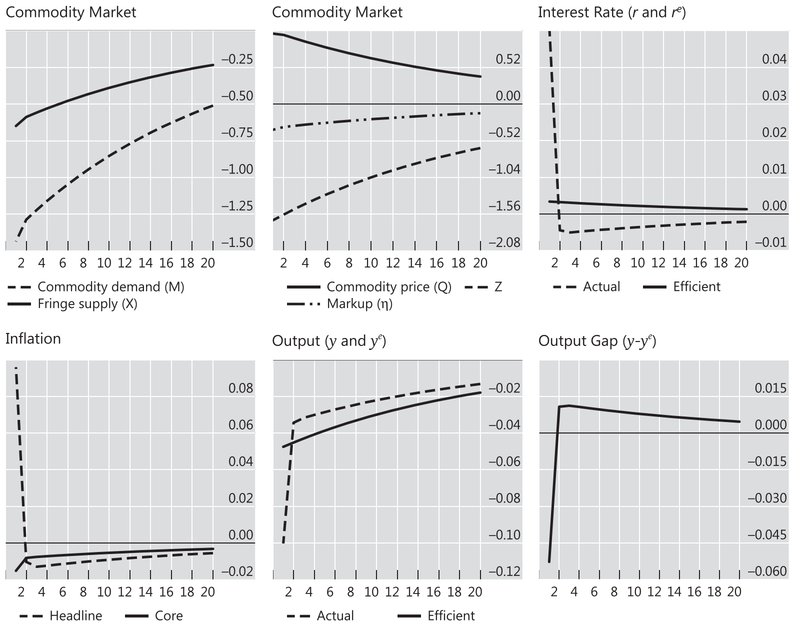
Notes: Impulse responses to an aggregate-demand shock using policy rule $r_t = E_{t|t-1}(r_t^e + \varphi_{core}\pi_{Y,t} + \varphi_y \hat{y}_t^e) + \varphi_{com}\Delta q_t$. The shock is calibrated so as to generate a 1 percent increase in commodity prices.

which includes a benchmark interest rate (r_t^e), core inflation $(\pi_{Y,t})$, and output gap $\hat{y}_t^e = (y_t - y_t^e)$. We assume that the monetary authority responds to *expected* inflation and output gap, whereas the change in the commodity price (Δq_t) is available in real time.

The performance of policy rule (25) is graphically assessed using impulse responses to demand and supply shocks.[16] Figure 1 plots the responses to a positive aggregate demand shock, defined as an exogenous shift in aggregate demand through g_t in equation (17). Figure 2 plots the response to a negative supply shock, defined as

[16]The coefficients of the policy rule are set to $\varphi_{core} = 1.5$, $\varphi_y = 0.5$, and $\varphi_{com} = 0.05$.

Figure 2. Responses to a Negative Supply Shock (ε_t^z)



Notes: Impulse responses to a (negative) supply shock. Other details are in figure 1.

an exogenous shift in Z_t in equation (9). Both shocks are calibrated to produce, on impact, a 1 percent increase in the commodity price.

Following a demand shock (figure 1), the commodity price rises (top-center panel). The demand shock also drives up output (but not the level of the efficient output) and the output gap. Both core and headline inflation initially rise. With the efficient interest rate unchanged, the policy rule calls for an increase in the policy rate (top-right panel). For roughly six quarters, the interest rate and output gap remain elevated. Note that the intuitively plausible shapes of the response underscore the reasonableness of our calibration for policy analysis.

A negative supply shock (figure 2) drives up the commodity price, but the policy rate response as well as the macroeconomic ones are modest. This constellation of responses confirms that the

Table 1. Expected Welfare Loss for Alternative Efficient Policy Rules, Percent of Steady-State Consumption ($\times$ E – 4)

	Model				
	(1)	**(2)**	**(3)**	**(4)**	**(5)**
Welfare Loss →	**–3.80**	**–6.05**	**–5.91**	**–4.98**	**–5.27**
Policy Rule Specifications for Each Model					
φ_{core}	2.0	—	1.5	1.5	2.0
φ_{head}	—	2.0	—	—	—
φ_{y}	—	—	0.5	0.5	1.0
φ_{com}	—	—	—	0.05	0.1

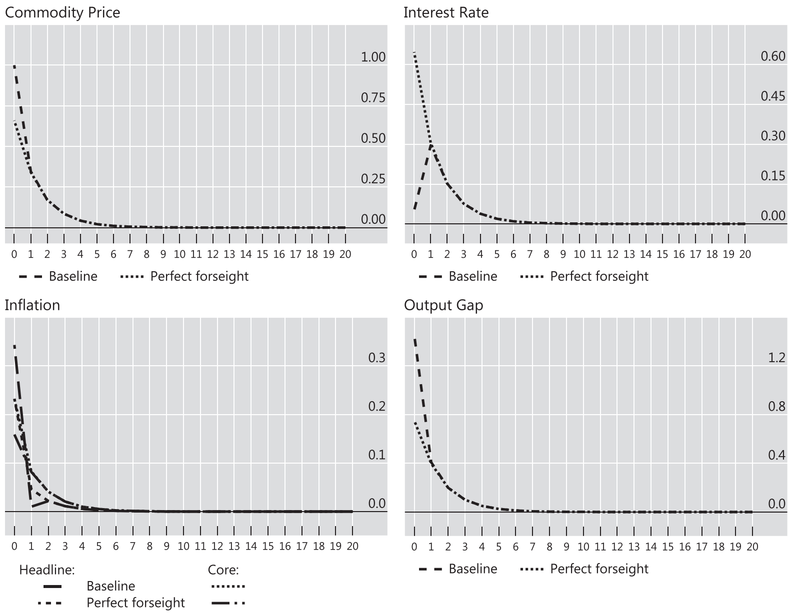
Notes: The unconditional expected welfare, EW_t, is assessed using a second-order solution of the model, where $W_t = U(C_t, L_t, e_t) + \beta E_t W_t$. The welfare cost in terms of steady-state consumption is equivalent to $\{exp((1-\beta)(EW_t - W))\} \times 100$. The Taylor-type monetary policy rule is specified as $r_t = E_{t|t-1}[r_t^e + \varphi_{core}\pi_{Y,t} + \varphi_{head}\pi_t + \varphi_y(y_t - y_t^e)] + \varphi_{com}\Delta q_t$.

baseline model (with its imperfect information assumption) delivers the conventional wisdom that monetary authorities should essentially look through the first-round effects of supply shocks.

Table 1 provides an alternative metric of model performance based on the model's welfare criterion under different policy rules. The first column corresponds to a rule with a response only to expected core inflation; it achieves the lowest (expected welfare) loss. Columns 2–5 correspond to alternative policy rules. The rule in column 2 considers a response only to expected headline inflation; this produces the worst outcome, indicating the superiority of targeting core versus headline inflation in our model. Column 3 introduces an additional response to the expected output gap, and columns 4 and 5 add a direct response to the commodity price change.[17]

[17] One drawback of evaluating expected welfare losses with simple linear rules is that the results may be sensitive to the variances of the shocks. We have conducted a battery of tests to confirm that our findings are robust to a range of reasonable variances.

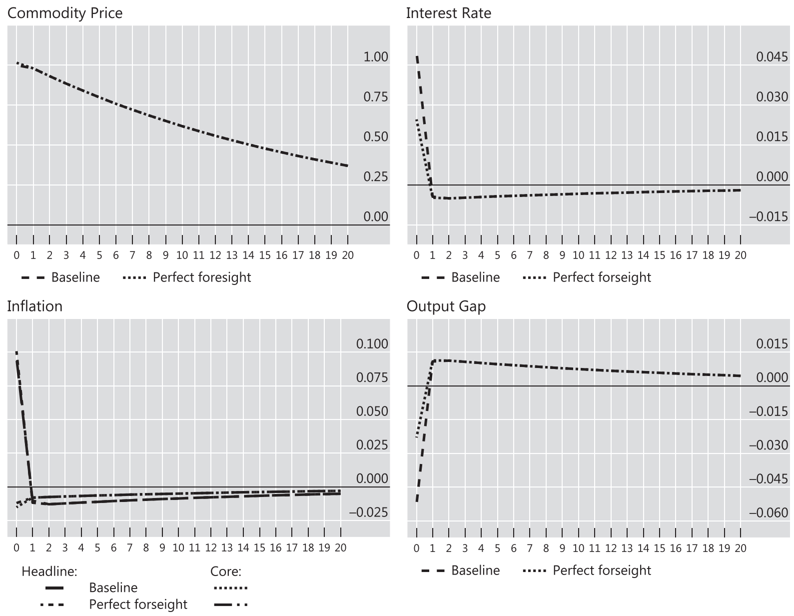
Figure 3. Response to a Positive Aggregate-Demand Shock under Full Information



Notes: Impulse responses to an aggregate-demand shock using the baseline policy rule of figure 1 and the full-information rule $r_t = r_t^e + \varphi_{core}\pi_{Y,t} + \varphi_y\,(y_t - y_t^e) + \varphi_{com}\Delta q_t$. The shock is calibrated so as to generate a 1 percent increase in commodity prices under the baseline policy rule.

While results suggest that responding to commodity prices can improve performance over responding to headline inflation, they leave open questions such as whether monetary authorities can achieve even better performance by inferring the nature of the shocks driving commodity price developments. Data uncertainty indeed induces substantial changes in the reaction to shocks. To illustrate this, we compare the baseline results with those under full information, i.e., when the monetary authority is able to observe with certainty macrovariables at each point in time. Figures 3 and 4 compare the reaction of the system to demand and shocks with the baseline. The overall picture is that first-round effects generated by uncertainty are already sizable. Looking at the aggregate-demand shock (figure 3) highlights that under full information the monetary

Figure 4. Response to a Negative Supply Shock under Full Information



Notes: Impulse responses to a supply shock using the baseline policy rule of figure 1 and the full-information rule $r_t = r_t^e + \varphi_{core}\pi_{Y,t} + \varphi_y\left(y_t - y_t^e\right) + \varphi_{com}\Delta q_t$. The shock is calibrated so as to generate a 1 percent increase in commodity prices under the baseline policy rule.

authority increases the interest rate more aggressively compared with the baseline. This is no surprise, given that the monetary authority is fully aware of the effects of the shock on output. By doing so, the monetary authority not only dampens cyclical fluctuations but also moderates the increase in commodity prices and hence in inflation. Also, under a supply shock (figure 4) full information helps the monetary authority stabilize output, although in this case differences are less striking.

3.2 Monetary Policy Responses under Misdiagnosis Risk

In addition to uncertainty about current output and inflation, when setting policy rates monetary authorities do not have precise knowledge of the underlying nature of the shocks affecting commodity prices. This section considers the challenges of inferring the nature of the shocks and implications arising from possible misdiagnoses, i.e., the risk that a monetary authority may misinterpret the source of the shock driving changes in commodity prices.

We cast the problem of shock identification in a classical signal-extraction problem.[18] Starting with the assumption that the monetary authority does not observe supply and demand shocks (ε_t^z and ε_t^g), it can infer them from available observations. Consider a simple linear model for commodity prices:

$$q_t = -z_t + \psi_t = H_t' \xi_t, \tag{26}$$

where z_t and ψ_t are the supply and the demand shock in the central banks' signal-extraction model, $H_t = [-1 \quad 1]$, and $\xi_t = [z_t \quad \psi_t]'$. The unconditional variance of ξ_t is

$$P \equiv var\left(\xi_t\right) = \left[\begin{array}{cc} \sigma_z^2 & \sigma_{z\psi} \\ \sigma_{z\psi} & \sigma_\psi^2 \end{array}\right]. \tag{27}$$

Given this informational structure, the monetary authority infers the sources of commodity price fluctuations by solving a signal-extraction problem using a Kalman filter, i.e.,

$$E_t^{ma}\left[z_t \quad \psi_t\right]' = M q_t, \tag{28}$$

where $M = PH\left[H_t' PH\right]^{-1}$ is a weighted average of the variances and covariances of z_t and ψ_t; M is calculated as

$$M = \frac{x}{x^2 - 2\rho x + 1}\left[\begin{array}{c} \rho - x \\ \frac{1}{x} - \rho \end{array}\right], \tag{29}$$

where $\rho = corr\left(z_t, \psi_t\right)$ and $x = \sigma_\psi / \sigma_z$.[19]

Three cases of equation (28) shed light on the tradeoffs facing the monetary authority. In the first case (type A), when $x \to 0$, the volatility of the commodity supply shock is high relative to that of

[18] An online appendix, available at http://www.ijcb.org, also describes the Bayesian learning approach to the problem.

[19] See chapter 13 of Hamilton (1994) for the derivation.

the commodity market markup. In this case, the monetary authority attributes nearly all of the fluctuations to demand shocks. That is,

$$if\ x \to 0, \qquad E_t^{ma}\left[z_t \quad \psi_t\right]' \to \left[0 \quad q_t\right]'.$$

In the second case (type B), the fluctuations are attributed to supply shocks. That is,

$$if\ x \to \infty, \qquad E_t^{ma}\left[z_t \quad \psi_t\right]' \to \left[-q_t \quad 0\right]'.$$

In the last case (type C), the monetary authority attributes the commodity price fluctuations partially to each component of the commodity price, taking into account the relative volatility and correlation in equation (29).

Armed with these expectations, we can rewrite the monetary authority's policy rule as

$$\begin{aligned} r_t &= E_{t|t-1}^{ma}\left[r_t^e + \varphi_{core}\pi_{Y,t} + \varphi_y \hat{y}_t^e\right] + \varphi_{com}\Delta q_t \\ &= E_{t|t-1}\left[r_t^e + \varphi_{core}\pi_{Y,t} + \varphi_y \hat{y}_t^e\right] + \varphi_{com}\Delta q_t + e_t, \end{aligned} \tag{30}$$

where

$$\begin{aligned} e_t = &\left[E_{t|t-1}^{ma}\left(r_t^e\right) - E_{t|t-1}\left(r_t^e\right)\right] + \varphi_{core}\left[E_{t|t-1}^{ma}\left(\pi_{Y,t}\right) - E_{t|t-1}\left(\pi_{Y,t}\right)\right] \\ &+ \varphi_y\left[E_{t|t-1}^{ma}\left(\hat{y}_t^e\right) - E_{t|t-1}\left(\hat{y}_t^e\right)\right]. \end{aligned}$$

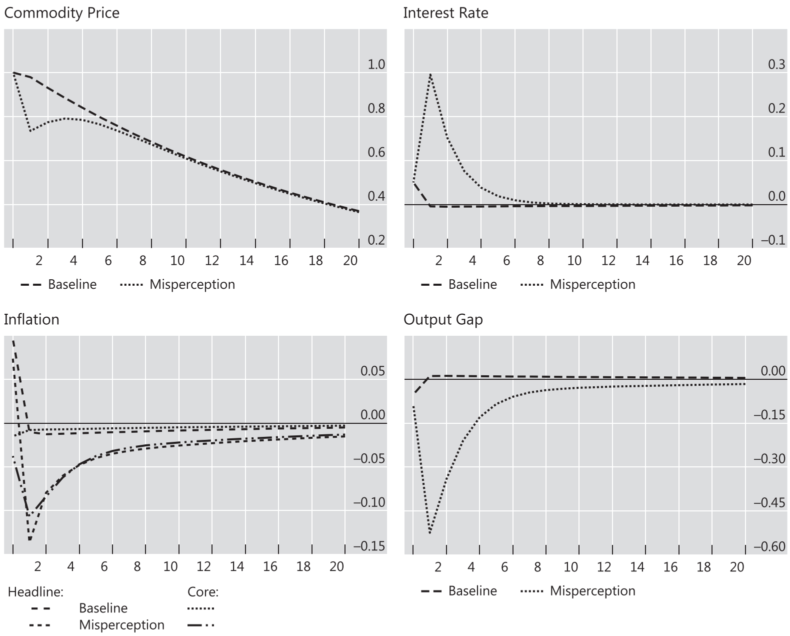
The e_t corresponds to a misdiagnosis error, which is *endogenous,* and $E_{t|t-1}^{ma}$ denotes the expectations under the incorrect diagnosis on the source of the shock. Note that when the monetary authority imputes the change in commodity prices to the wrong type of shock, its estimates of endogenous variables will be incorrect, leading to persistent errors in interest rate setting.[20]

To investigate the implications of the endogenous error, we replace equation (25) in our baseline model with equation (30). The resulting impulse responses highlight the findings.

Figure 5 shows the impulse responses to a commodity supply shock in the misdiagnosis case A. Even though the commodity price

[20]If the monetary authority correctly identifies the source of the shock, the error is zero.

Figure 5. Responses to a Negative Supply Shock under Misdiagnosis Type A

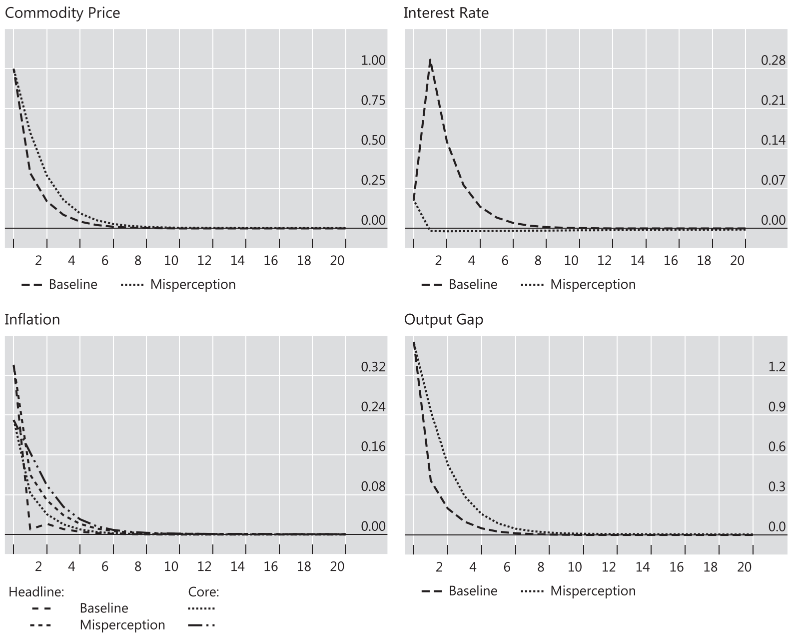


Notes: Impulse responses to a negative commodity supply shock when the monetary authority attributes all the fluctuation in the commodity price to an aggregate-demand shock (misdiagnosis type A). Other details are in figure 1.

is driven by a supply shock, the monetary authority misdiagnoses it as a traditional demand-driven commodity shock. If the monetary authority fails to recognize that an increase in the commodity price is driven by (external) supply conditions, the consequence is overly tight monetary policy accompanied by a sizable drop in both output and inflation. The commodity price responds in a more muted way than in the baseline case because of tighter monetary policy. Core and headline inflation both fall because of the slowing economy.

Figure 6 displays the impulse responses to a conventional aggregate demand shock in the case of misdiagnosis type B, i.e., when the rise in the commodity's price is mistakenly attributed to a negative commodity supply shock. In this case, the easier monetary policy associated with the looking-through-the-supply-shock strategy

Figure 6. Responses to a Positive Aggregate-Demand Shock under Misdiagnosis Type B

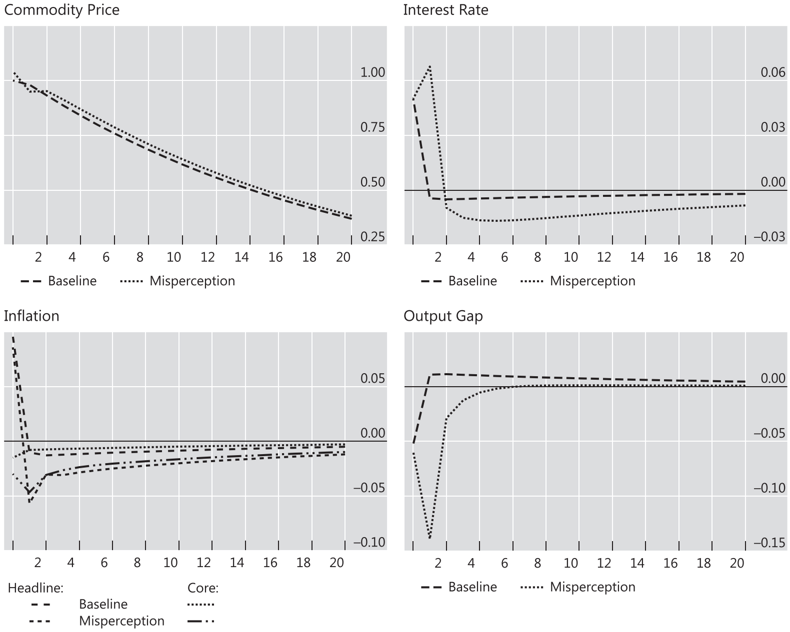


Notes: Impulse responses to an aggregate-demand shock when the monetary authority attributes all the fluctuation in the commodity price to a supply shock (misdiagnosis type B). Other details are in figure 1.

results in higher output and inflation. This type of policy misdiagnosis induces a procyclical increase in the commodity price.

Figures 7 and 8 report the results for the case of misdiagnosis type C, in which the monetary authority implements the optimally weighted response to the commodity price rise based on the historical commodity demand and supply shocks. The standard deviations of the shocks are calibrated to the empirical estimates by Filardo and Lombardi (2014), which yields a ratio of about 1.5 (x in equation (29)). Consistent with misdiagnosis cases A and B, the monetary authority responds excessively to supply shocks and insufficiently to demand shocks: on net, monetary policy appears excessively procyclical in the model.

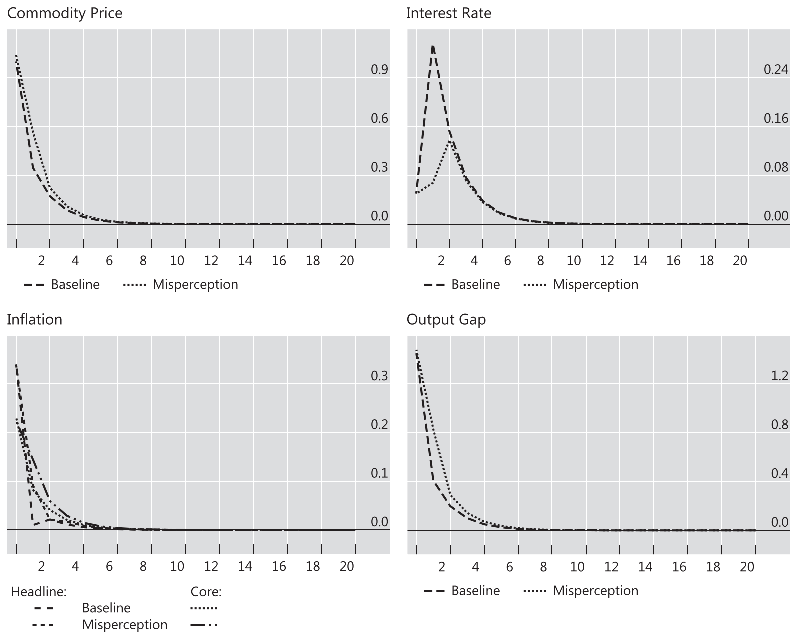
Figure 7. Responses to a Negative Supply Shock under Misdiagnosis Type C



Notes: Impulse responses to a negative commodity supply shock when the monetary authority attributes the fluctuation in the commodity price proportionally to aggregate-demand shock and commodity supply shock, with weights given by the ratio of their standard deviations (misdiagnosis type C). Other details are in figure 1.

The relative importance of these misdiagnosis risks can also be assessed by examining the welfare losses under different policy rules. The left-hand and right-hand panels of figure 9 plot, respectively, the welfare loss of the misdiagnosis types A and B cases (relative to the case of correct identification). In the case where the monetary authority misinterprets a rise in the commodity price as supply driven, the contraction in both output and core inflation is larger than in the full-information case. And, in the case where commodity price fluctuations are driven by global demand, the monetary authority amplifies cyclical fluctuations and, as a result, destabilizes the economy. Hence, it is no surprise that the expected welfare losses are always greater than zero under misdiagnoses.

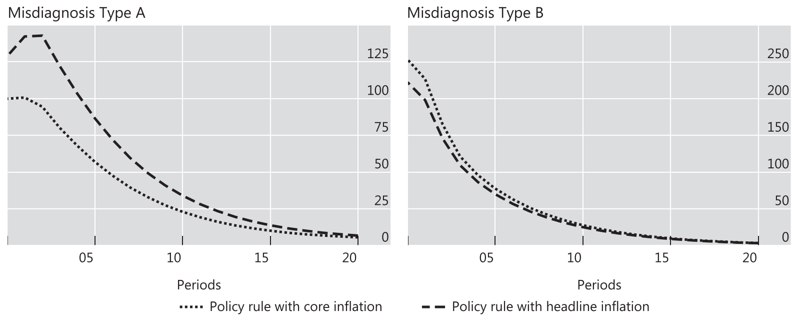
Figure 8. Responses to a Positive Aggregate-Demand Shock under Misdiagnosis Type C



Notes: Impulse responses to an aggregate-demand shock when the monetary authority attributes the fluctuation in the commodity price proportionally to aggregate-demand shock and commodity supply shock, with weights given by the ratio of their standard deviations (misdiagnosis type C), assuming the efficient benchmark policy rule and a 0.5 autoregression coefficient for the shock processes.

With respect to the performance of core and headline policy rules under these cases of misdiagnosis, the results still lend support to policy rules with core inflation rather than headline inflation. In misdiagnosis case A (supply shock treated as if it was demand), the core inflation rule implies a much lower expected loss than the headline one. This is not surprising given the more muted policy response associated with the headline inflation rule. In the misperception B case, the difference in the expected loss between the two rules is small, with a slight edge to the headline rule. If misdiagnosis risk cases A and B were both 50-50, the expected loss criteria

Figure 9. Ratio of the Welfare Losses under Misdiagnosis



Notes: Welfare is computed using a second-order solution of the model, where $W_t = U(C_t, L_t, e_t) + \beta E_t W_t$ and the welfare cost in terms of steady-state consumption is equivalent to $\{exp\left[(1-\beta)(EW_t - W) - 1\right]\} \times 100$.

would support the core inflation rule in this imperfect information environment.

Overall, the signal-extraction results reinforce the earlier findings on the importance of correctly identifying the underlying nature of commodity price shocks. On the modeling side, the misdiagnosis risk and its inherent procyclicality lead to a breakdown of the divine coincidence found in full-information models à la Blanchard and Galí (2007). On the policy side, the case for a core inflation rule is strengthened.

4. Conclusions

In this paper we documented that (i) monetary authorities deliver better economic performance when they are able to accurately identify the global nature of the shocks, i.e., global supply and demand shocks, driving commodity prices; and (ii) when it is difficult to identify these shocks, monetary authorities can minimize some of the adverse feedbacks from misdiagnosis by targeting core inflation. Global shocks also imply that central banks may find themselves responding in a correlated way. If so, it is important to account for this possibility. This reinforces arguments for greater prominence to be given to global factors in domestic monetary policymaking.

One important aspect of monetary policy misdiagnosis risk that deserves further research includes the important issue of parameter uncertainty, especially with respect to the slope of the Phillips curve. Indeed, recent empirical evidence suggests that the slope has flattened, and at least has become more uncertain, across many economies. In such a situation, the main call of this paper for more attention to be given to the nature of the shocks hitting the economy may take on even greater prominence. This may require going beyond the methods advocated in this paper, e.g., exploiting big data with cross-country coverage.

Appendix A. Model Calibration

Table A.1. Baseline Calibration

Structural Parameters	Parameter	Value
Share of Commodity in Consumption Basket	γ	0.10
Share of Commodity in Production Function	α	0.10
Inverse Frisch Labor Supply Elasticity	ν	0.50
Price Elasticity of Substitution	ε	7.66
Quarterly Discount Factor	β	0.99
Price Adjustment Probability	θ	0.75
Size of Competitive Commodity Production Relative to GDP	X/Y	0.10

Notes: For the parameters associated with the commodity market, the choice is a bit more arbitrary due to the less conventional view of the literature. The share of the commodity in the consumption basket is set to 10 percent, which roughly matches the share of primary commodity inputs in the U.S. CPI. For the share of commodities in the production *function,* we also use 10 percent, as in Arseneau and Leduc (2013); given that we focus on commodities rather than oil alone, both values are larger than the 5 percent commonly used in oil-only models. Finally, the size of the competitive commodity production sector relative to GDP is set at 10 percent.

Appendix B. The Dominant Commodity Exporter's Problem

For the dominant commodity exporter, the optimization problem can be written as a series of intratemporal decisions. Under the assumption that the dominant commodity exporter does not fully internalize the actions of the other exporters (i.e., taking as given the macroeconomic variables (C_t, MC_t, Y_t, Δ_t, and Ω_t) of the commodity-importing country), the problem can be written as

$$\max_{\mathfrak{M}_t} \ln \left(Q^{1/(1-\gamma)} \mathfrak{M}_t - \mathfrak{M}_t / Z_t \right)$$
$$\text{s.t. } Q_t = h(\mathfrak{M}_t).$$

The first-order condition of this problem is

$$Q_t = \left[Z_t^{-1} \frac{1}{1 - \zeta^{-1} \eta_t} \right]^{\zeta},$$

where $\eta_t \equiv -\partial \ln Q_t / \partial \ln \mathfrak{M}_t = -h' \left(\mathfrak{M}_t \right) \mathfrak{M}_t) / Q_t$ is the elasticity of commodity demand (in absolute value) and $\zeta = 1 - \gamma$.

The demand for the commodity by the dominant commodity producer takes the form

$$\mathfrak{M}_t = \frac{1}{Q_t} \mathfrak{D}_t - Q_t \mathfrak{E}_t,$$

where $\mathfrak{D}_t \equiv (\gamma C_t + \alpha M C_t Y_t \Delta_t)$ and $\mathfrak{E}_t \equiv \Omega_t Z_t$.

The inverse demand function is

$$Q_t = \frac{1}{2} \frac{\sqrt{\mathfrak{M}^2 + 4 \mathfrak{D}_t \mathfrak{E}_t} - \mathfrak{M}}{\mathfrak{E}_t}.$$

Given this, the elasticity of demand for the commodity is

$$\eta_t = -\frac{f' \left(\mathfrak{M}_t \right) \mathfrak{M}_t}{Q_t} = \frac{\mathfrak{M}_t}{\sqrt{\mathfrak{M}_t^2 + 4 \mathfrak{D}_t \mathfrak{E}_t}} = \frac{\mathfrak{M}_t}{\sqrt{\mathfrak{M}_t^2 + 4 (\mathfrak{M}_t + X_t) X_t}}$$
$$= \frac{\mathfrak{M}_t}{\mathfrak{M}_t + 2 X_t}.$$

Finally, the commodity price markup is

$$\Psi_t = \frac{1}{1-\eta_t} = 1 + \frac{\mathfrak{M}_t}{2X_t}.$$

Appendix C. Output, Inflation, and Interest Rate Benchmarks

C.1 Efficient Output Gap Benchmark

The benchmark output gap can be derived by substituting the equations for labor demand, labor supply, aggregate demand, and commodity demand for production into the aggregate production function. The level of output in terms of marginal costs, the dispersion of prices, productivity, and the real commodity price is

$$Y_t = \left(\frac{A_t}{\Delta_t}\right)^{1/(1-\alpha)} \left[\frac{(1-\alpha)\, MC_t\, \Delta_t}{(Q_t)^{-\gamma/(1-\gamma)} - \alpha MC_t\, \Delta_t}\right]^{1/(1+v)} \times \left(\alpha \frac{MC_t\, \Delta_t}{Q_t}\right)^{\alpha/(1-\alpha)}.$$

The log-linear approximation of the level of output, in deviations from the steady state, is[21]

$$y_t = \frac{1}{1-\alpha} a_t + \frac{\alpha}{1-\alpha}(mc_t - q_t) + \frac{1}{1+v}\Upsilon\left(mc_t + \frac{\gamma}{1-\gamma} q_t\right),$$

where $\Upsilon \equiv \left[1 - \frac{\alpha}{\mu} Q^{\gamma/(1-\gamma)}\right]^{-1} \geq 1$.

The level of efficient output, y_t^e, is defined with respect to the efficient allocation, i.e., flexible prices and no monopolistic distortions in the commodity market or in the final goods market (which implies that $Q_t^e = Z_t^{-1}$ and $\mu^e = 1$):

$$y_t^e = \frac{1}{1-\alpha} a_t + \left(\frac{\alpha}{1-\alpha} - \frac{1}{1+v}\frac{\gamma}{1-\gamma}\Upsilon^e\right) z_t,$$

[21] Note that a linear approximation of the price dispersion, Δ_t, does not appear in this equation because price dispersion is assumed to have only second-order effects on the dynamics, as shown in Benigno and Woodford (2005).

where $\Upsilon^e = \left[1 - \alpha Z^{-\gamma/(1-\gamma)}\right]^{-1}$. The relationship between Υ^e and Υ depends on the extent of monopolistic distortions. Υ^e and Υ are equal only if both markets are perfectly competitive or if the commodity is not used for production (that is, $\alpha = 0$).[22] A key difference is that commodity shocks affecting the markup do not have an effect on the level of efficient output: such output is affected only by fluctuations associated with supply shocks in the commodity market. As a consequence, a demand-driven increase in the commodity price would leave the benchmark efficient output gap unchanged. However, a negative commodity supply shock would decrease the efficient output level.

The *efficient output gap*, $\hat{y}_t^e$, which is defined as the difference between actual output and the efficient level of output, is of the form

$$\hat{y}_t^e = y_t - y_t^e.$$

C.2 Inflation Benchmarks

Expressed in log-linear terms, the equations for headline inflation and core inflation, respectively, are of the form

$$\pi_t = \pi_{Y,t} + \frac{\gamma}{1-\gamma}\Delta q_t$$
$$\pi_{Y,t} = \kappa_y \hat{y}_t^e + E_t \pi_{Y,t+1} + u_t,$$

where u_t is an endogenous cost-push shock, which is a function of both ψ_t and z_t.

In this model, the divine coincidence featured in models with exogenous commodity prices is broken. It is no longer possible to simultaneously stabilize core inflation and the welfare-relevant output gap. The tradeoff arises from the effect of commodity prices on the level of efficient output. An increase in commodity price markups generates a positive cost-push shock, which puts upward pressure on core inflation but lowers the efficient output gap.

[22] More precisely, $\Upsilon^e > (<)\,\Upsilon$, if $\Psi^{\gamma/(1-\gamma)} < (>)\,\mu$, where Ψ and μ are the markups in steady state of the commodity and final goods markets, respectively.

C.3 Interest Rate Benchmarks

The interest rate benchmarks are derived by substituting the equations for the aggregate resources constraint and the definition of the price level into the IS equation:

$$\frac{1}{R_t} = \beta E_t \left[\frac{1}{\Pi_{Y,t+1}} \left(\frac{1 - \alpha MC_t Q_t^{\gamma/(1-\gamma)} \Delta_t}{1 - \alpha MC_{t+1} Q_{t+1}^{\gamma/(1-\gamma)} \Delta_{t+1}} \right) \times \left(\frac{Y_t}{Y_{t+1}} \right) \exp\left(g_{t+1} - g_t\right) \right].$$

The *efficient interest rate* is defined in the case where commodity and final goods markets are perfectly competitive and in this model can be written as

$$r_t^e = (g_t - E_t g_{t+1}) - \left(y_t^e - E_t y_{t+1}^e\right) - \frac{\gamma}{1-\gamma}\left(\Upsilon^e - 1\right)\left(z_t - E_t z_{t+1}\right).$$

Note that the efficient interest rate does not respond to changes in the markup.

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