Online Appendix to "Monetary Policy, Commodity Prices, and Misdiagnosis Risk"

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D.1 Commodity Price Shock Inference Using Bayesian Learning

An extension of the model consists in allowing the policymaker to learn from real-time feedback from the economy and the effect of the policy decision. The monetary authority updates its beliefs about the nature of the shocks using Bayesian learning. By updating its initial diagnosis, the monetary authority reduces the inherent procyclicality of misdiagnoses.

The learning setup is as follows. On impact ($t = 1$), the monetary authority acts on its prior beliefs about the source of the shock. Given the economy's reaction, the monetary authority then updates its views in the form of a posterior probability about whether the economy experienced a demand shock:

$$P_1\left(D|\pi_1, y_1\right) = \frac{L\left(D|\pi_1, y_1\right) P_0\left(\mathrm{D}\right)}{L\left(D|\pi_1, y_1\right) P_0\left(\mathrm{D}\right) + L\left(S|\pi_1, y_1\right) P_0\left(\mathrm{S}\right)},$$

where $L(D|\pi_1, y_1)$ and $L(S|\pi_1, y_1)$ are, respectively, the likelihood of demand and supply shocks based on observed output and inflation and the model's implied output and inflation.[1]

[1] The likelihood function would in general incorporate all observable variables in the model. Here, we abstract from those variables that do not enter the monetary policy reaction function.

At time $t = 2$, the posterior distribution $P_1(D|\pi_1, y_1)$ is used as the (proto-)prior for the new posterior. In general, the updating follows the recursive equation:

$$P_t\left(D|\pi_t, y_t\right) = \frac{L\left(D|\pi_t, y_t\right) P_{t-1}\left(D|\pi_{t-1}, y_{t-1}\right)}{\begin{array}{c} L\left(D|\pi_t, y_t\right) P_{t-1}\left(D|\pi_{t-1}, y_{t-1}\right) \\ +\, L\left(S|\pi_t, y_t\right) P_{t-1}\left(S|\pi_{t-1}, y_{t-1}\right) \end{array}},$$

which by recursive substitution can be written as

$$P_t\left(D|\pi_t, y_t\right) = \frac{L\left(D|\pi_t, y_t\right) P_0\left(D\right) \prod_{\tau=1}^{t-1} P_\tau\left(\pi_\tau, y_\tau|D\right)}{\begin{array}{c} L\left(D|\pi_t, y_t\right) P_0\left(D\right) \prod_{\tau=1}^{t-1} P_\tau\left(\pi_\tau, y_\tau|D\right) \\ +\, L\left(S|\pi_t, y_t\right) P_0\left(S\right) \prod_{\tau=1}^{t-1} P_\tau\left(\pi_\tau, y_\tau|S\right) \end{array}}.$$

This produces a sequence of posterior probabilities that the monetary authority attaches to the nature of the shocks as new information becomes available and sets the policy interest accordingly:[2]

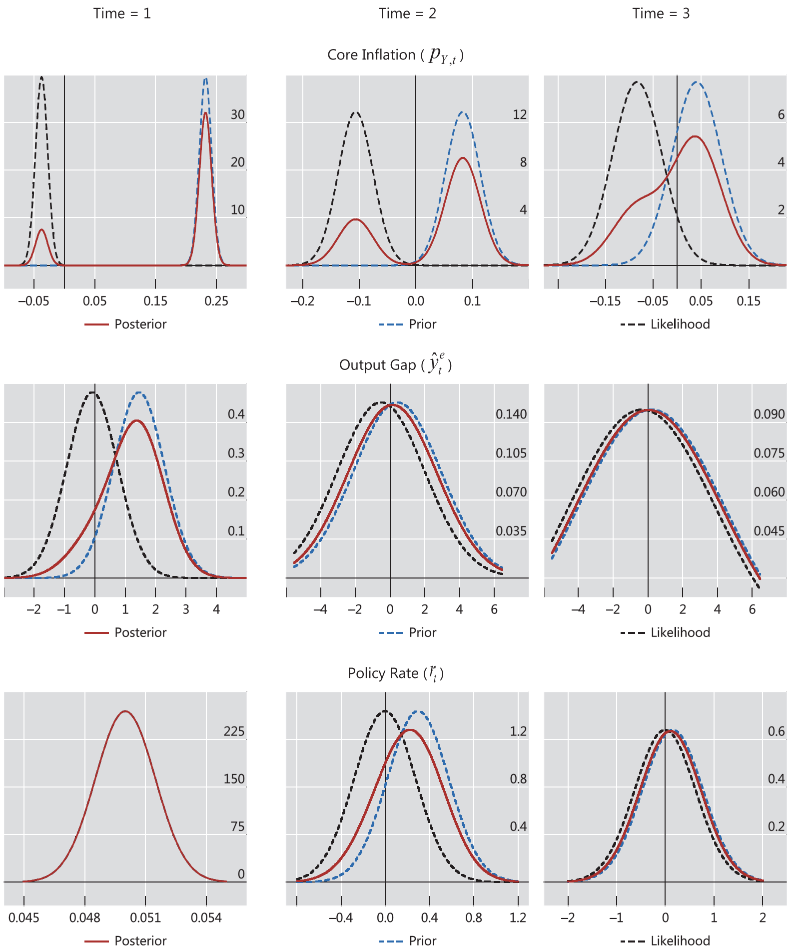
$$\begin{aligned} r_t &= E_{t|t-1}^{bayes}\left[r_t^e + \varphi_{core}\pi_{Y,t} + \varphi_y \hat{y}_t^e\right] + \varphi_{com}\Delta q_t \\ &= E_{t|t-1}\left[r_t^e + \varphi_{core}\pi_{Y,t} + \varphi_y \hat{y}_t^e\right] + \varphi_{com}\Delta q_t + e_t^{bayes}, \end{aligned}$$

with e_t^{bayes} the new misperception error. We first consider the case of a supply shock misdiagnosed as a demand shock (misdiagnosis case A in figure A.1). We start by setting the monetary authority's prior probability of a demand shock at $P_0(D) = 0.99$ and of a supply shock $P_0(S) = 0.01$.

In $t = 1$, the monetary authority observes output and inflation. Given that the authority believes the originating shock came from the demand side, the mass of the prior probability distribution is centered around the outcomes that would have occurred if the originating shock had been a demand shock (the dashed blue distribution which is calibrated to the $t = 1$ baseline responses). The likelihood reflects the information about the "true" supply shock and the endogenous monetary policy "shock" induced by the initial misdiagnosis; the likelihood is centered around the initial response

[2]Note that past "mistakes" during the learning process will influence the state of the economy. Our impulse responses propagate these mistakes over time and hence differ fundamentally from the impulse responses earlier in the paper.

Figure A.1. Misperception Case A with Bayesian Learning



Notes: Case A: A supply shock misdiagnosed to be a demand shock. Probability distributions for core inflation, the output gap, and the policy rate, as perceived by the monetary authority at three different points in time after the initial shock has occurred. "Prior" corresponds to the distribution under the demand shock initially assumed by the monetary authority (D); "Likelihood" is the probability distribution of observed variables that actually occur from a combination of the initial supply shock and the monetary policy error induced by misperception (S); and "Posterior" is the weighted average of the two, where the weights are given by the posterior probabilities assigned to D and S.

of the misperception A case. The resulting posterior distribution reflects the differences between the prior and the likelihood.

For core inflation at $t = 1$, the likelihood and the prior are very different, leading to a bimodel posterior distribution. The likelihood and prior distribution of the output gap (second row) is less precise and does not result in a bimodal posterior distribution.

At $t = 2$ and $t = 3$, the (proto-)priors and the likelihoods are closer to each other, as the monetary authority learns about the initial misdiagnosis. The posterior distributions become more unimodal. Note, however, that some of the convergence is due to strong equilibrium forces even in the case of policy "mistakes" that were seen in the earlier impulse responses.

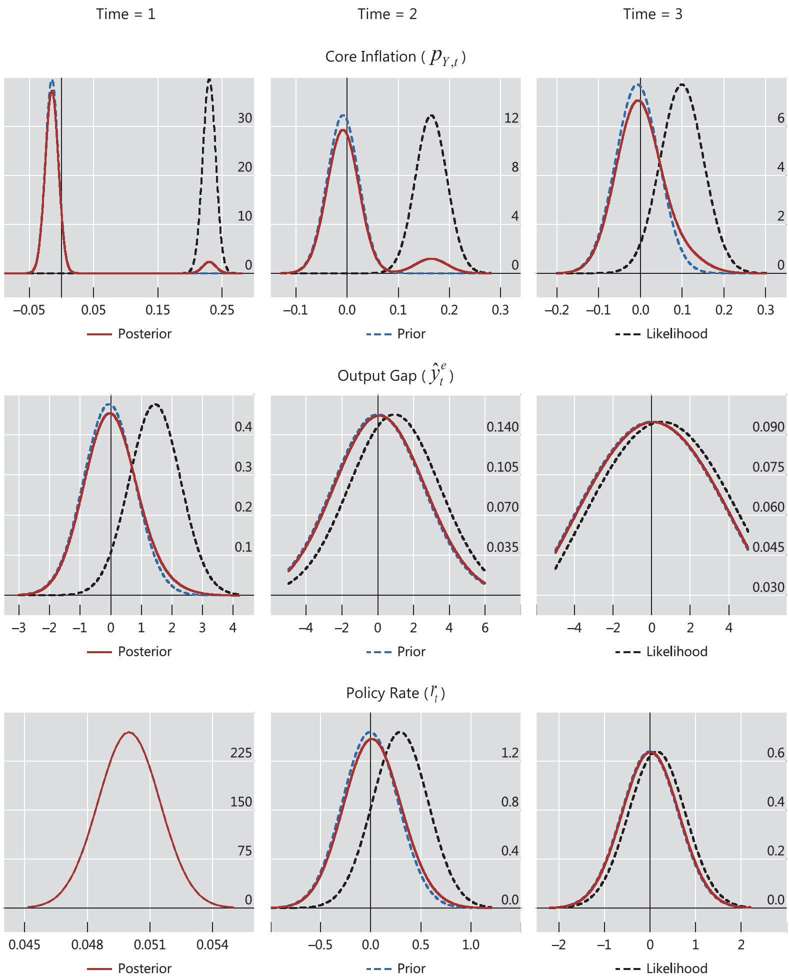
Results for misdiagnosis case B in figure A.2 indicate that the initial monetary policy response is excessively loose. As the monetary authority learns about the mistake and eventually corrects its stance, the initial procyclicality of output and inflation dies down.

This Bayesian exercise suggests that learning is a function of the shock distributions hitting the economy. Our model structure is fairly simple and generates rapid convergence via learning. However, it may be very difficult in real time for a monetary authority to converge so quickly after an initial misdiagnosis. Adding realistic shocks with fat-tailed distributions for all the variables (especially measurement error) would slow the rate of convergence and increase the persistence of the procyclical policy response. As well, parameter uncertainty (such as the extent of commodity price pass-through to inflation and the slope of the Phillips curve, which have been highlighted in recent empirical research) also complicates the inferential problem.[3]

On the other hand, if the commodity price shock is very large relative to the other shocks hitting the economy, the learning in principle should be faster, as the economy's reaction to the true shock will show through more prominently in the posterior distributions. Recent papers trying to parse the nature of oil shocks mentioned in

[3]Addressing parameter uncertainty along the lines of, e.g., Wieland (2000), Rodina (2012), and Cogley, Matthes, and Sbordone (2015) is left for future research. Additional uncertainty of this type is likely to slow the rate of learning about the nature of the commodity price shocks and make the misdiagnosis errors even more persistent.

Figure A.2. Misperception Case B with Bayesian Learning



the Introduction demonstrate just how difficult this can be empirically. And for run-of-the-mill shocks of the type calibrated in this paper, the ability of a monetary authority to learn from past mistakes during the "fog of war" suggests that lingering (endogenous) policy errors associated with misdiagnoses may be a regular feature of the monetary policy environment. Such results have far-reaching implications for all empirical macrofinancial research efforts that assume well-defined exogenous errors in the data.

References

Cogley, T., C. Matthes, and A. Sbordone. 2015. "Optimized Taylor Rules for Disinflation when Agents Are Learning." *Journal of Monetary Economics* 72 (May): 131–47.

Rodina, F. 2012. "The Role of Model Uncertainty and Learning in the US Postwar Policy Response to Oil Prices." *Journal of Economic Dynamics and Control* 36 (7): 1009–41.

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