Online Appendices to "Confidence Cycles and Liquidity Hoarding"*

Volha Audzei
Czech National Bank

Appendix D. A Simple Model of Credit and the Interbank Market

Here I describe the main mechanism of the model in a simplified setting. For clarity of exposition, I present all the proofs in Appendix E.

There are two time periods and two types of investment opportunities for banks: a storage asset pays R^{res} and a risky asset R^k in the next period. Decisions are made in period 1 and payoffs are realized in period 2. At $t = 1$ banks attract deposits d from a household. I simplify the problem by assuming that deposits are distributed equally among all banks and set $d = 1$. Banks pay R to depositors in the next period. The time subscripts are dropped. There is a continuum of banks normalized to 1 and indexed by i, each with different expectations about the risky asset return, $E^i\hat{R}^k$. In the general equilibrium context, the risky asset is credit to the real sector, so in the simple model I sometimes refer to the risky asset position as credit.

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Banks can participate in the interbank market. The interbank market rate, R^{ib}, is determined endogenously by clearing the market. For simplicity, beliefs are distributed uniformly among banks with mean m and variance σ^2.[1] I think of each banker as a statistician making her best forecast conditional on the available information. Each bank's individual estimate, $E^i\hat{R}^k$, is then assumed to be distributed uniformly with the same variance σ^2. That is, bankers have their own predictions of the risky asset return, $E^i\hat{R}^k$, with variance σ^2, and these predictions are distributed uniformly among banks with mean $\bar{E}\hat{R}^k = m$ and the same variance σ^2.

The beliefs distribution is the only difference in banks' optimization problem, given the amount of net wealth and deposits. Thus, I can rewrite the interbank market clearing condition (12) in the text as follows (a detailed derivation is given in Appendix E):

$$E^m\hat{R}^k - E^l\hat{R}^k = \lambda_b\left(\sigma\sqrt{3} + m - R^{ib}\right) = \lambda_b\left(\bar{R} - R^{ib}\right). \quad \text{(D.1)}$$

The solution to the model is then given by the market clearing condition, (D.1), the definition of repayment probability, (8), and the expressions for marginal investors in terms of the interbank market rate:

$$E^l\hat{R}^k = \frac{R}{\lambda_b + 1} + \frac{\lambda_b R^{ib}}{\lambda_b + 1} - \frac{\sqrt{3}\sigma(R^{ib} - 2R^{res})}{R^{ib}}.$$

$$E^m\hat{R}^k = \frac{R^{ib}(\sqrt{3}\lambda_b R^{ib} - 3\lambda_b\sigma + \sqrt{3}R - 3\sigma)}{(\lambda_b + 1)(\sqrt{3}R^{ib} - 6\sigma)}.$$

Combining these four equations gives an equation for the interbank market rate:[2]

$$a * (R^{ib})^3 + b * (R^{ib})^2 + c * (R^{ib}) + d = 0. \quad \text{(D.2)}$$

For the interbank market to function, there must be an interbank rate R^{ib} solving (D.2) that is real and positive. The necessary and sufficient conditions are summarized in the following proposition.

[1] The bounds of the uniform distribution a and b are then $a = m - \sigma\sqrt{3}$ and $b = m + \sigma\sqrt{3}$. In this simplest model, a can be negative.

[2] $a = \sqrt{3}\lambda_b(1 + \lambda_b)$, $b = -\lambda_b\left(\sqrt{3}(\lambda_b + 1)m + 9\lambda_b\sigma + 3\sigma\right)$, $c = 6\sigma\left(\lambda_b\left(\lambda_b m + \sqrt{3}\lambda_b\sigma + m - R^{res}\right) + R - R^{res} - \sqrt{3}\sigma\right)$, and $d = 12\sqrt{3}(\lambda_b + 1)R^{res}\sigma^2$.

PROPOSITION D.1. *The necessary condition for the interbank market to exist is that there is an interbank rate, R_t^{ib}, solving*

$$a * (R^{ib})^3 + b * (R^{ib})^2 + c * (R^{ib}) + d = 0, \tag{D.3}$$

that is real and non-negative.[3] *With $a > 0$, $b < 0$, and $d > 0$, if a positive root exists, it is unique. This positive root exists only if*

$$R^{res} > A(m, \sigma) + \frac{R}{\lambda_b + 1} < 0, \tag{D.4}$$

where $A < 0$.[4] *The sufficient condition for equilibrium with the interbank market is that the marginal lender belief satisfies $E^l \hat{R}^k < p^l\, R^{ib} = Res$, which implies $\sigma\sqrt{3} > (3 + 2\sqrt{2})R\lambda_b/(1 + \lambda_b)$.*

The mathematical proof is given in Appendix E.

PROPOSITION D.2. *Low market beliefs result in less lending.*

The mathematical proof is given in Appendix E.

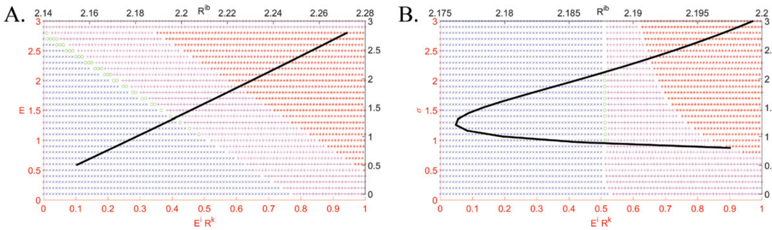
COROLLARY D.1. *With very low diversity of beliefs, there is no lending on the interbank market. With very high diversity of beliefs, lending may occur but will be small.*

Figure D.1 summarizes possible scenarios of interbank market functioning, where panel A shows the impact of mean market beliefs on bank equilibrium allocations and panel B shows the impact of the dispersion of the beliefs and the standard deviation of the bank forecasts. The horizontal axis shows the shares of bankers: hoarders, lenders, investors that do not borrow, and investors that borrow. The black line is the interbank market rate, measured on the upper horizontal axis. When the interbank market rate is not defined, the interbank market collapses: the average beliefs and diversity are too small. If the dispersion is fixed, a decrease in market beliefs about the risky asset return means a shift in the bounds of the beliefs

[3] $a = \sqrt{3}\lambda_b(1 + \lambda_b)$, $b = -\lambda_b\left(\sqrt{3}(\lambda_b + 1)m + 9\lambda_b\sigma + 3\sigma\right)$, $c = 6\sigma\left(\lambda_b\left(\lambda_b m + \sqrt{3}\lambda_b\sigma + m - R^{res}\right) + R - R^{res} - \sqrt{3}\sigma\right)$, and $d = 12\sqrt{3}(\lambda_b + 1)R^{res}\sigma^2$.

[4] $A = \frac{-3\left(3\lambda_b^3+7\lambda_b+6\right)\sigma^2 - \lambda_b(\lambda_b+1)^2 m^2 + 4\sqrt{3}\lambda_b(\lambda_b+1)m\sigma}{6\sqrt{3}(\lambda_b+1)^2\sigma}$.

Figure D.1. Banks' Beliefs Distribution



Note: Numerical simulations. From left to right: x area—hoarders; o area—lenders; + area—investors who do not borrow; *—investors who borrow. Solid line—interbank rate (upper horizontal axis). The vertical axis shows the mean of the beliefs distribution (panel A) and the standard deviation of the beliefs distribution (panel B). The lower horizontal axis shows the shares of banks making a particular portfolio choice. The numerical values for the parameters are $R^{res} = R = 1.0101, \lambda_b = 0.1$, for panel A: $\sigma = 0.98$, for panel B: $m = 1$.

distribution: the most pessimistic banker becomes even more pessimistic and the most optimistic banker becomes less optimistic. Borrowers (the "*" area) use interbank loans to invest in the risky asset. Intuitively, when borrowers expect a lower return on the risky asset, they are willing to pay less for the interbank loan. With all bankers feeling less optimistic about the risky asset return, there is a larger share of bankers who do not invest themselves and expect a lower probability of loan repayment: there is more hoarding (the "x" area).

The role of the standard deviation in the model is twofold. First, it measures the dispersion of beliefs across banks. Second, it reflects how each bank is uncertain about its own estimate of the future return. If lenders are almost certain about their expectations of a low return, they will assign a low loan repayment probability and ask for a high interbank market rate. At the same time, borrowers are more convinced about their high return expectations and are willing to pay the high rate. Consequently, with a low standard deviation, there is either no lending or very low lending at a high interbank rate. As the standard deviation increases, so does the uncertainty among borrowers. They are willing to pay a lower interbank rate. When the uncertainty and dispersion are very large, there is still lending, but its volume is negligible (see Figure D.1). If the diversity of beliefs is large, the bounds of the distribution widen and there are some very optimistic borrowers. This pushes the interbank market rate up.

Policy Effects

A Functioning Interbank Market. I now consider the impact of policy actions in the context of the simple model I have developed. First, I assume that the interbank market is functioning, but some "superior agent," which I call the central bank, would like to increase interbank lending and/or stimulate credit to the real economy.

PROPOSITION D.3. *A low policy rate increases lending, lowers the interbank market rate, and increases the supply of credit to the real economy.*

The mathematical proof is given in Appendix E.

Liquidity Provision. In this simple framework without liquidity concerns, the provision of liquidity to banks, whether targeted or untargeted, does not affect the functioning of the interbank market. All bankers would allocate all of their available funds according to the decision rules above. The provision of funds to optimists increases credit to the real economy, given that optimists exist. If the liquidity provision is untargeted and the funds are distributed equally among the banks, the pessimistic banks hoard it, as their main concern is counterparty risk and low returns on risky assets. In this regard, targeting only optimistic banks can increase credit. Again, in the general equilibrium context, the feedback from prices and bank balance sheets reverses the predictions: the untargeted policy results in better general outcomes than the targeted policy.

To sum up, in light of my model, if a central bank wants to increase lending on a market where banks are concerned about counterparty risks, the effects of a policy that does not address those concerns are limited.

Interbank Market Collapse. I now consider the case in which the interbank market collapses due to low market expectations about the returns on the risky asset. Obviously, providing banks with additional funds will not revive interbank lending, but, if provided to optimists, such funds could increase credit to the real economy. The size of this effect is conditional on the share of investors. A reduction of the safe interest rate makes the safe asset relatively less attractive. But when the banks are pessimistic about the returns on the risky asset, the effects of such policy are limited. This is summarized by the next proposition.

PROPOSITION D.4. *The effect of a policy rate reduction is limited by the mean market belief.*

A formal proof is provided in Appendix E. The only tool that might have a potential effect is *a reduction in the policy rate.* However, this has a very limited or zero effect if market beliefs are very low, which also means a very low interbank rate. In this case, even with a low reserve rate, hoarding is still more attractive than interbank lending.

Overall, if banks are concerned about low returns to a risky asset and estimate a low probability of loan repayment, policy actions have a very limited effect. Liquidity provision policies enhance credit through optimistic bankers only, with the rest of the funds ending up in reserves. A low interest rate policy restores the market only if market beliefs are not very pessimistic and stimulates credit to the real economy among banks expecting the risky asset to pay more than the storage asset.

Appendix E. Derivations for the Simple Model

Interbank Market Clearing

The market clearing condition for the interbank market with a uniform beliefs distribution is

$$F_{m,\sigma_v^2}\left(E^m \hat{R}^k\right) - F_{m,\sigma_v^2}\left(E^l \hat{R}^k\right) = \lambda_b \left(1 - F_{m,\sigma_v^2}\left(R^{ib}\right)\right).$$

The cumulative distribution function for the continuous uniform distribution is $\frac{x-a}{b-a}$. Then the market clearing condition is rewritten as

$$\frac{E^m \hat{R}^k - a}{b-a} - \frac{E^l \hat{R}^k - a}{b-a} = \lambda_b \left(1 - \frac{R^{ib} - a}{b-a}\right)$$

$$\Rightarrow E^m \hat{R}^k - E^l \hat{R}^k = \lambda_b \left(b - R^{ib}\right),$$

where b is the upper bound on the beliefs distribution, denoted as $\bar{R}$ in Appendix D.

PROPOSITION D.1. *The necessary condition for the interbank market to exist is that there is an interbank rate, R_t^{ib}, solving*

$$a * (R^{ib})^3 + b * (R^{ib})^2 + c * (R^{ib}) + d = 0, \tag{E.1}$$

that is real and non-negative.[5] *With $a > 0$, $b < 0$, and $d > 0$, if a positive root exists, it is unique. This positive root exists only if*

$$R^{res} > A(m, \sigma) + \frac{R}{\lambda_b + 1} < 0, \tag{E.2}$$

where $A < 0$.[6] *The sufficient condition for equilibrium with the interbank market is that the marginal lender belief satisfies $E^l \hat{R}^k < p^l\, R^{ib} = Res$, which implies $\sigma\sqrt{3} > (3 + 2\sqrt{2})R\lambda_b/(1 + \lambda_b)$.*

Proof. Denote as $\Delta = b^2 - 3ac$ the discriminant of the cubic Equation (E.1), and $rib^{(1)}$, $rib^{(2)}$, and $rib^{(3)}$ its three roots, where the first one is always real and the other two may be real. With $a > 0$, $b < 0$, and $d > 0$, the first root is always negative. The last two roots are real and distinct from the first only if $\Delta > 0$. The condition for $\Delta > 0$ is (E.2). If the parameters are such that $\Delta > 0$, the second root is always negative and the third is always positive. Therefore, for a positive real solution to exist, the necessary condition is (E.2).

However, given the interbank market rate, for the interbank market to exist there must be a marginal lender with the belief $E^l R^k$ determined from $p^l\, R^{ib} = R^{res}$, and this belief should be smaller than $p^l\, R^{ib}$. I analyze the definition of the marginal lender and then show under which conditions $E^l R^k < R^{res}$:[7]

[5] $a = \sqrt{3}\lambda_b(1 + \lambda_b)$, $b = -\lambda_b\left(\sqrt{3}(\lambda_b + 1)m + 9\lambda_b\sigma + 3\sigma\right)$, $c = 6\sigma\left(\lambda_b\left(\lambda_b m + \sqrt{3}\lambda_b\sigma + m - R^{res}\right) + R - R^{res} - \sqrt{3}\sigma\right)$, and $d = 12\sqrt{3}(\lambda_b + 1)R^{res}\sigma^2$.

[6] $A = \frac{-3\left(3\lambda_b^3+7\lambda_b+6\right)\sigma^2-\lambda_b(\lambda_b+1)^2m^2+4\sqrt{3}\lambda_b(\lambda_b+1)m\sigma}{6\sqrt{3}(\lambda_b+1)^2\sigma}$.

[7] For the purposes of exposition here and in the following proofs, I substitute a steady-state assumption on $R^{res} = R$.

$$E^l\hat{R}^k = \frac{R^{res}2\sigma\sqrt{3}}{R^{ib}} - \sigma\sqrt{3} + \frac{R}{(1+\lambda_b)} + \frac{\lambda_b}{(1+\lambda_b)}R^{ib},$$

$$\frac{R^{res}2\sigma\sqrt{3}}{R^{ib}} - \sigma\sqrt{3} + \frac{R}{(1+\lambda_b)} + \frac{\lambda_b}{(1+\lambda_b)}R^{ib} < R^{res},$$

$$\Rightarrow \tag{E.3}$$

$$\frac{R^{res}2\sigma\sqrt{3}}{R^{ib}} - \sigma\sqrt{3} - \frac{R\lambda_b}{(1+\lambda_b)} + \frac{\lambda_b}{(1+\lambda_b)}R^{ib} < 0,$$

$$\Rightarrow \tag{E.4}$$

$$p^l 2\sigma\sqrt{3} - \left(\sigma\sqrt{3} + \frac{R\lambda_b}{(1+\lambda_b)}\right) + \frac{\lambda_b}{(1+\lambda_b)}R^{ib} < 0, \tag{E.5}$$

$$R^{res}2\sigma\sqrt{3} - R^{ib}\left(\sigma\sqrt{3} + \frac{R\lambda_b}{(1+\lambda_b)}\right) + \frac{\lambda_b}{(1+\lambda_b)}(R^{ib})^2 < 0, \tag{E.6}$$

where I have used $R^{res} = p^l R^{ib}$. The last inequality could be rewritten as

$$(R^{ib} - x'_1)(R^{ib} - x'_2) < 0, \tag{E.7}$$

$$x'_1 = \frac{\left(\sigma\sqrt{3}(1+\lambda_b) + R\lambda_b\right) - (1+\lambda_b)\sqrt{\Delta_1}}{2\lambda_b}, \tag{E.8}$$

$$x'_2 = \frac{\left(\sigma\sqrt{3}(1+\lambda_b) + R\lambda_b\right) + (1+\lambda_b)\sqrt{\Delta_1}}{2\lambda_b}, \tag{E.9}$$

$$\Delta_1 = \left(\sigma\sqrt{3} + \frac{R\lambda_b}{(1+\lambda_b)}\right)^2 - 8\frac{R^{res}\sigma\sqrt{3}\lambda_b}{(1+\lambda_b)}. \tag{E.10}$$

Because the LHS in (E.6) is parabolic, the inequality is satisfied when

$$x'_1 < R^{ib} < x'_2, \tag{E.11}$$

given that the discriminant, Δ_1, is positive. That is,

$$\left(\sigma\sqrt{3}+\frac{R\lambda_b}{(1+\lambda_b)}\right)^2 > 8\frac{R^{res}\sigma\sqrt{3}\lambda_b}{(1+\lambda_b)},$$

$$(\sigma\sqrt{3})^2-6\sigma\sqrt{3}\frac{R\lambda_b}{(1+\lambda_b)}+\frac{R\lambda_b}{(1+\lambda_b)}^2 > 0,$$

which is satisfied for $\sigma\sqrt{3} < (3-2\sqrt{2})\frac{R\lambda_b}{(1+\lambda_b)}$ or for $\sigma\sqrt{3} > (3+2\sqrt{2})\frac{R\lambda_b}{(1+\lambda_b)}$. The first case would contradict $\sigma\sqrt{3} > \frac{R^{ib}}{2} > R^{res}$. Then, the condition on σ is $\sigma\sqrt{3} > (3+2\sqrt{2})\frac{R\lambda_b}{(1+\lambda_b)}$. □

PROPOSITION D.2. *Low market beliefs result in less lending.*

Proof. In the simple model, lending is given by $E^mR^k - E^lR^k$. Taking the derivative with respect to the average market belief, m:

$$\begin{aligned}\frac{\partial\left(E^mR^k-E^lR^k\right)}{\partial m} &= \frac{\partial E^mR^k}{\partial R^{ib}}\frac{\partial R^{ib}}{\partial m}-\frac{\partial E^lR^k}{\partial R^{ib}}\frac{\partial R^{ib}}{\partial m}\\ &= \frac{\partial R^{ib}}{\partial m}\left(\frac{\partial E^mR^k}{\partial R^{ib}}-\frac{\partial E^lR^k}{\partial R^{ib}}\right)\\ &= \frac{\left(\frac{\partial E^mR^k}{\partial R^{ib}}-\frac{\partial E^lR^k}{\partial R^{ib}}\right)}{\left(1+\frac{1}{\lambda}\left(\frac{\partial E^mR^k}{\partial R^{ib}}-\frac{\partial E^lR^k}{\partial R^{ib}}\right)\right)},\end{aligned}$$

where the last equality is derived from the interbank market clearing condition

$$R^{ib}=m+\sqrt{3}\sigma-\frac{1}{\lambda}\left(E^mR^k-E^lR^k\right),$$

with the derivative with respect to the average market belief being

$$\begin{aligned}\frac{\partial R^{ib}}{\partial m} &= 1-\frac{1}{\lambda}\left(\frac{\partial E^mR^k}{\partial R^{ib}}\frac{\partial R^{ib}}{\partial m}-\frac{\partial E^lR^k}{\partial R^{ib}}\frac{\partial R^{ib}}{\partial m},\right)\\ &= \frac{\partial R^{ib}}{\partial m}=\frac{1}{\left(1+\frac{1}{\lambda}\left(\frac{\partial E^mR^k}{\partial R^{ib}}-\frac{\partial E^lR^k}{\partial R^{ib}}\right)\right)}.\end{aligned}$$

With the marginal lender and the marginal investor defined, respectively, as $E^m R^k = R^{ib} p^m$ and $R^{res} = R^{ib} p^l$, and $p^i = \frac{1}{2} - \frac{\left(R+\lambda_b R^{ib}\right)}{2\sigma\sqrt{3}(1+\lambda_b)} + \frac{E^i \hat{R}^k}{2\sigma\sqrt{3}}$, I get

$$\frac{\partial E^l R^k}{\partial R^{ib}} = \frac{\lambda_b}{(1+\lambda_b)} - \frac{2\sigma\sqrt{3}R^{res}}{\left(R^{ib}\right)^2}, \tag{E.12}$$

$$\frac{\partial E^m R^k}{\partial R^{ib}} = \frac{\partial p^m}{\partial R^{ib}} R^{ib} + p^m,$$

$$\Rightarrow \frac{\partial E^m R^k}{\partial R^{ib}} = \frac{2\sigma\sqrt{3}p^m - \frac{\lambda_b}{(1+\lambda_b)} R^{ib}}{2\sigma\sqrt{3} - R^{ib}}. \tag{E.13}$$

$\frac{\partial E^m R^k}{\partial R^{ib}} - \frac{\partial E^l R^k}{\partial R^{ib}}$ can then be rewritten as

$$\frac{2\sigma\sqrt{3}p^m - \frac{\lambda_b}{(1+\lambda_b)} R^{ib}}{2\sigma\sqrt{3} - R^{ib}} - \frac{\lambda_b}{(1+\lambda_b)} + \frac{2\sigma\sqrt{3}R^{res}}{\left(R^{ib}\right)^2} =$$
$$\frac{2\sigma\sqrt{3}R^{ib}(p^m - p^l) + 2\sigma\sqrt{3}(2\sigma\sqrt{3}p^l - \frac{\lambda_b}{(1+\lambda_b)} R^{ib})}{(2\sigma\sqrt{3} - R^{ib})R^{ib}} > 0.$$

Because $p^m > p^l$ and $(1+\lambda_b)E^l R^k > \lambda_b R^{ib}$, and the model implies that $R > E^l R^k > \frac{\lambda_b R^{ib}}{1+\lambda_b}$, then $p^l = \frac{R}{R^i b} > \frac{\lambda_b}{1+\lambda_b}$. Moreover, $2\sigma\sqrt{3} > R^{ib}$. Then it follows that $2\sigma > 0\sqrt{3}p^l - \frac{\lambda_b}{(1+\lambda_b)} R^{ib} > 0$. □

Proposition D.3. *A low policy rate increases lending, lowers the interbank market rate, and increases the supply of credit to the real economy.*

Proof.

$$\frac{\partial R^{ib}}{\partial R^{res}} = -\frac{1}{\lambda}\left(\frac{\partial E^m R^k}{\partial Rib}\frac{\partial R^{ib}}{\partial R^{res}} - \frac{\partial E^l R^k}{\partial R^{res}}\right).$$

Note that the sign of the derivative of interbank lending $\left(\frac{\partial E^m R^k}{\partial Rib}\frac{\partial R^{ib}}{\partial R^{res}} - \frac{\partial E^l R^k}{\partial R^{res}}\right)$ is the opposite of $\frac{\partial R^{ib}}{\partial R^{res}}$. Then

$$\frac{\partial E^m R^k}{\partial R^{ib}} = \frac{2\sigma\sqrt{3}p^m - \frac{\lambda_b}{(1+\lambda_b)} R^{ib}}{2\sigma\sqrt{3} - R^{ib}}$$

and

$$E^l R^k = \frac{R}{1+\lambda_b} + \frac{\lambda_b R^{ib}}{1+\lambda_b} - \sqrt{3}\sigma + \frac{2\sqrt{3}\sigma R^{res}}{R^{ib}}$$

$$\frac{\partial E^l R^k}{\partial R^{res}} = \frac{\lambda_b}{1+\lambda_b}\frac{\partial R^{ib}}{\partial R^{res}} + \frac{2\sqrt{3}\sigma}{R^{ib}} - \frac{2\sqrt{3}\sigma R^{res}}{\left(R^{ib}\right)^2}\frac{\partial R^{ib}}{\partial R^{res}}.$$

Then

$$\frac{\partial R^{ib}}{\partial R^{res}} = \frac{\frac{2\sqrt{3}\sigma}{R^{ib}}}{\lambda_b + \frac{2\sigma\sqrt{3}p^m - \frac{\lambda_b}{(1+\lambda_b)}R^{ib}}{2\sigma\sqrt{3}-R^{ib}} - \frac{\lambda_b}{1+\lambda_b} + \frac{2\sqrt{3}\sigma R^{res}}{(R^{ib})^2}} > 0.$$

With $\frac{2\sqrt{3}\sigma}{R^{ib}} > 0$, the size of the derivative is determined by the denominator. $\lambda_b > \frac{\lambda_b}{1+\lambda_b}$ and $2\sigma\sqrt{3}p^m > \frac{\lambda_b R^{ib}}{1+\lambda_b}$, as $2\sigma\sqrt{3}p^m > 2\sigma\sqrt{3}p^l > \frac{\lambda_b R^{ib}}{1+\lambda_b}$. See the proof for Proposition D.2. □

PROPOSITION D.4. *The effect of a policy rate reduction is limited by the mean market belief.*

Proof. Suppose that $E^l R^k > R^{res}$. Then, for the policy rate reduction to restore lending, the change should be such that $E^l R^k < R^{res}$:

$$\frac{\partial E^l R^k}{\partial R^{res}} = \frac{\lambda_b}{1+\lambda_b}\frac{\partial R^{ib}}{\partial R^{res}} + \frac{1}{1+\lambda_b} + \frac{2\sqrt{3}\sigma}{R^{ib}} - \frac{2\sqrt{3}\sigma R^{res}}{\left(R^{ib}\right)^2}\frac{\partial R^{ib}}{\partial R^{res}}$$

$$= \frac{2\sqrt{3}\sigma}{R^{ib}}\left(\frac{\lambda_b + \frac{2\sigma\sqrt{3}p^m - \frac{\lambda_b}{(1+\lambda_b)}R^{ib}}{2\sigma\sqrt{3}-R^{ib}}}{\lambda_b + \frac{2\sigma\sqrt{3}p^m - \frac{\lambda_b}{(1+\lambda_b)}R^{ib}}{2\sigma\sqrt{3}-R^{ib}} - \frac{\lambda_b}{1+\lambda_b} + \frac{2\sqrt{3}\sigma R^{res}}{(R^{ib})^2}}\right) + \frac{1}{1+\lambda_b}.$$

The numerator of the first term is larger than the denominator, as

$$\frac{2\sqrt{3}\sigma}{R^{ib}}\left(\frac{\lambda_b + \frac{2\sigma\sqrt{3}p^m - \frac{\lambda_b}{(1+\lambda_b)}R^{ib}}{2\sigma\sqrt{3}-R^{ib}}}{\lambda_b + \frac{2\sigma\sqrt{3}p^m - \frac{\lambda_b}{(1+\lambda_b)}R^{ib}}{2\sigma\sqrt{3}-R^{ib}} - \frac{\lambda_b}{1+\lambda_b} + \frac{2\sqrt{3}\sigma R^{res}}{(R^{ib})^2}}\right) > 1,$$

$$\lambda_b\left(2\sqrt{3}\sigma - R^{ib}\right) + 2\sigma\sqrt{3}p^m - \frac{2\sqrt{3}\sigma R^{res}}{R^{ib}} > 0,$$

with $\frac{R^{res}}{R^{ib}} = p^l$:

$$\lambda_b \left(2\sqrt{3}\sigma - R^{ib}\right) + 2\sigma\sqrt{3}\left(p^m - p^l\right) > 0.$$

This is true, as $2\sqrt{3}\sigma - R^{ib} > 0$ (the results of Propositions D.1 and D.2) and $p^m - p^l > 0$. That is, the derivative $\frac{\partial E^l R^k}{\partial R^{res}} > 1$.

Now consider how the difference $E^l R^k - R^{res}$ changes with respect to R^{res} :

$$\frac{\partial E^l R^k}{\partial R^{res}} - 1 > 0.$$

That is, the function is increasing in R^{res} and is increasing faster than R^{res}. A downward shift in the reserves reduces both the right- and left-hand sides of the inequality $E^l R^k > R^{res}$, with $E^l R^k$ declining faster than R^{res}. □

Appendix F. The Agency Problem

My agency problem differs from that of Gertler and Karadi (2011) in several respects. First, in my model, banks have the possibility to put their funds in reserves. Second, banks are heterogeneous, with a share of them investing in a risky asset. Last but not least, some banks participate in the interbank market, transferring some funds from pessimistic to optimistic banks.

Each bank maximizes the terminal worth, discounted by the stochastic discount factor $\beta^j \Omega_{t,t+j}$ arising from the household problem. The value is

$$\begin{aligned} V_t &= \max E_t \sum_{j=0}^{\infty} (1-\theta)\,\theta^j \beta^{j+1} \Omega_{t,t+1+j}\left(N_{t+1+j}\right) \\ &= \max E_t \sum_{j=0}^{\infty} (1-\theta)\,\theta^j \beta^{j+1} \Omega_{t,t+1+j}\left\{\left(R^k_{t+1+j} - R_{t+j}\right) Q_{t+j} S_{t+j}\right. \\ &\qquad \left. + \left(R^{res}_{t+j} - R_{t+j}\right) Res_t + R_{t+j} N_{t+j}\right\}. \quad \text{(F.1)} \end{aligned}$$

Equation (F.1) resembles the terminal worth equation in Gertler and Karadi (2011), the only difference being that I apply it on the average level. Note that the budget constraint of the aggregated bank is $Q_t S_t + Res_t = N_t + B_t$. Also, only those banks with the lowest return expectations hoard funds in reserves (the others either invest themselves or lend funds to be invested by others): $Res_t = s_t^h (N_t + B_t) = s_t^h (Q_t S_t + Res_t)$, with s_t^h being the share of hoarders. And $Q_t S_t = \left(1 - s_t^h\right)(Q_t S_t + Res_t)$. The terminal worth is

$$E_t \sum_{j=0}^{\infty} (1-\theta)\,\theta^j \beta \Omega_{t,t+1+j} \{ \left(R_{t+1+j}^k - R_{t+j}\right)\left(1 - s_{t+j}^h\right)(Q_t S_{t+j} + Res_{t+j})$$

$$+ \left(R_{t+j}^{res} - R_{t+j}\right) s_{t+j}^h (Q_t S_{t+j} + Res_{t+j}) + R_{t+j} N_{t+j} \}$$

$$= E_t \sum_{j=0}^{\infty} (1-\theta)\,\theta^j \beta^{j+1} \Omega_{t,t+1+j} \{ \left(\left(1 - s_{t+j}^h\right)\right.$$

$$\left. \times R_{t+1+j}^k + s_{t+j}^h R_{t+j}^{res} - R_{t+j}\right)(Q_t S_{t+j} + Res_{t+j}) + R_{t+j} N_{t+j} \}.$$

I then have to restrict banks from borrowing from the household. Otherwise, for a non-negative $\beta^j \Omega_{t,t+j}\left(R_{t+1+j}^k - R_{t+j}\right)$ a bank prefers to borrow indefinitely from the household. The limited government guarantee gives rise to an agency problem, as only a fraction $(1-\lambda)$ can be recovered, regardless of whether the funds were lost due to an unfortunate investment or diverted by a fraudulent banker. For a depositor willing to participate, the following constraint must be met:

$$V_t \geq \lambda\,(Q_t S_t + Res_t)\,,$$

where V_t is the value the banker would lose and $\lambda\,(Q_t S_t + Res_t)$ is the gain from diverting. That is, the continuation value should be larger than the gain from diverting. I rewrite (F.1) as

$$V_t = v_t\,(Q_t S_t + Res_t) + \eta_t N_t,$$

where

$$v_t = E_t \left\{ (1-\theta)\,\beta \Omega_{t,t+1} \left(\left(1 - s_t^h\right) R_{t+1}^k + s_t^h R_t^{res} - R_t\right) \right.$$
$$\left. + \beta \Omega_{t,t+1} \theta \chi_{t,t+1} v_{t+1} \right\},$$
$$\eta_t = E_t \left\{ (1-\theta) + \beta \Omega_{t,t+1} \theta z_{t,t+1} \eta_{t+1} \right\},$$

$$\chi_{t,t+1} = \frac{Q_{t+1}S_{t+1} + Res_{t+1}}{Q_t S_t + Res_t},$$
$$z_{t,t+1} = \frac{N_{t+1}}{N_t},$$

and the expression for the financial accelerator as

$$Q_t S_t + Res_t = \frac{\eta_t}{\lambda - v_t} N_t = \varphi_t N_t,$$

where φ_t is the leverage ratio, limiting the amount of assets an intermediary can acquire as a proportion of a bank's net worth.

To determine the leverage ratio, the household needs to form expectations about future risky asset returns. I assume that the household has a belief equal to the mean market belief.

Appendix G. The Rest of the Model Equations

Household. There is a representative risk-averse household in the economy which has utility from consumption and disutility from labor. The household owns all the firms and all the banks. The household solves the following problem subject to a budget constraint:

$$\max_{C_t, L_t, D_t} E_t \sum_{i=0}^{\infty} \beta^i \left[\ln(C_{t+i} - hC_{t+i-1}) - \frac{\chi}{1+\varphi} L_{t+i-1}^{1+\phi} \right], \quad \text{(G.1)}$$

$$\text{s.t. } C_t + B_t = W_t L_t + R_{t-1} B_{t-1} + \Pi_t + T_t, \quad \text{(G.2)}$$

where C, L, B, and T stand for consumption, labor supply, deposits in banks, and tax, respectively. W and R are the real wage and the real gross return on bank deposits. Π_t is net transfers from financial and non-financial firms to the household. β, ϕ, $\chi > 0$, and $\beta < 1$.

Bank deposits are guaranteed by the government, which, in the case of bank insolvency, repays the deposits and interest to the household, but only a fraction $(1 - \lambda)$ thereof.

The first-order conditions state that the marginal disutility of labor is equal to the marginal utility of consumption and that the nominal return on bank deposits should, at the margin, compensate the consumer for postponing consumption to the next period:

$$[C_t]: \ \rho_t = (C_t - hC_{t-1})^{-1} - \beta h E_t (C_{t+1} - hC_t)^{-1}, \tag{G.3}$$

$$[L_t]: \ \rho_t W_t - \chi L_t^{\phi} = 0, \tag{G.4}$$

$$[B_t]: \ E_t \beta \Omega_{t,t+1} R_{t+1} = 1, \tag{G.5}$$

where $\Omega_{t,t+1} \equiv \frac{\rho_{t+1}}{\rho_t}$ is a stochastic discount factor and ρ_t is the marginal utility of consumption.

Intermediate Goods Producers. Intermediate goods producers combine labor and capital using the Cobb-Douglas production function:

$$Y_t = A_t (U_t K_{t-1})^{\alpha} L_t^{1-\alpha}, \tag{G.6}$$

where K_{t-1} stands for capital, L_t stands for labor, and A_t is total factor productivity. U_t is the utilization rate of capital.

In each period, an intermediate goods producer chooses labor demand and demand for capital to maximize its current and next-period profits. The profit consists of the revenues from production and the resale value of the depreciated capital net of payments on claims S_t and labor costs. The price of a unit of the intermediate good is $P_{m,t}$ and the cost of buying new capital is Q_t. The producer then chooses the utilization rate and labor demand as

$$[L_t]: \ (1-\alpha)\frac{P_{m,t}Y_t}{L_t} = W_t, \tag{G.7}$$

$$[U_t]: \ (\alpha)\frac{P_{m,t}Y_t}{U_t} = \delta'(U_t) K_{t-1}. \tag{G.8}$$

As firms make zero profit, they distribute the return on capital to holders of their claims as in (2) in the text. The wage is then determined by the marginal product of labor. The price of the intermediate good equals the marginal costs.

Capital-Producing Firms. Capital-producing firms maximize the following utility:

$$\max_{In_t} E_t \sum_{k=t}^{\infty} \beta^{T-k} \Omega_{t,k} \left((Q_k - 1) I_{nk} - f\left(\frac{In_k + I_{ss}}{In_{k-1} + I_{ss}}\right)(In_k + Iss)\right), \tag{G.9}$$

where In is net investment, defined as $In_t \equiv I_t - \delta(U_t)\xi_t K_{t-1}$, where $\delta(U_t)\xi_t K_{t-1}$ is the quantity of renovated capital. Iss is steady-state investment and Q_t is the price of capital. Function f is an investment adjustment cost function satisfying the following properties: $f(1) = f'(1) = 0$ and $f''(1) > 0$.

The first-order conditions for investment give the price of capital, Q_t:

$$
\begin{aligned}
[I_t] : Q_t = 1 + f(\cdot) + \frac{In_k + I_{ss}}{In_{k-1} + I_{ss}} f'(\cdot) \\
- E_t \beta \Omega_{t,t+1} \left(\frac{In_k + I_{ss}}{In_{k-1} + I_{ss}} \right)^2 f'(\cdot).
\end{aligned} \tag{G.10}
$$

Final Goods Producers (Retailers). Final goods producers combine output from intermediate goods producers using the production function:

$$
Y_t = \left[\int_0^1 Y_{ft}^{\frac{\varepsilon-1}{\varepsilon}} df \right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{G.11}
$$

where Y_{ft} is the composite goods output from retailer f and ε is the elasticity of substitution. I follow the Calvo-pricing convention and each period allow only a fraction γ of firms to optimize their prices. Firms are monopolistic competitors and maximize their profit:

$$
\max_{P_t^*} \sum_{i=0}^{\infty} \gamma^i \beta^i \Omega_{t,t+1} \left[\frac{P_t^*}{P_{t+i}} \prod_{k=1}^{i} (1 + \pi_{t+k-1})^{\gamma_p} - P_{m,t+i} \right] Y_{ft+i}, \tag{G.12}
$$

subject to demand from households:

$$
Y_{ft} = \left(\frac{P_{ft}^*}{P_t} \right)^{-\varepsilon} Y_t, \tag{G.13}
$$

where P_t^* is the optimal price set in period t, γ is the fraction of firms which cannot reset their prices but only index to inflation, and $\pi_t = \frac{P_t}{P_{t-1}} - 1$ is the one-period inflation rate.

The problem results in the first-order condition:

$$\sum_{i=0}^{\infty} \gamma^i \beta^i \Omega_{t,t+i} \left[\frac{P_t^*}{P_{t+i}} \prod_{k=1}^{i} (1+\pi_{t+k-1})^{\gamma_p} - \mu P_{m,t+i} \right] Y_{ft+i} = 0, \tag{G.14}$$

where $\mu \equiv \frac{1}{1-\frac{1}{\varepsilon}}$ is a monopolistic markup.

The resulting equation for the price dynamics takes the form

$$P_t = \left[\int_0^1 P_{ft}^{\frac{1}{1-\varepsilon}} df \right]^{1-\varepsilon}, \tag{G.15}$$

$$P_t = \left[(1-\gamma) \left(P_t^*\right)^{1-\varepsilon} + \gamma \left\{ (1+\pi_{t+k-1})^{\gamma_p} P_{t-1} \right\}^{1-e} \right]^{\frac{1}{1-\varepsilon}}. \tag{G.16}$$

The Government and the Central Bank. The government collects lump-sum taxes from households, makes lump-sum transfers to households, T_t, and accepts reserves (the safe asset), Res_t. It also bears some of the costs of conducting policy, Po_t. The government's budget constraint is satisfied when the following holds:

$$G_t + Po_t = T_t + Res_t - R_t^{res} Res_{t-1}. \tag{G.17}$$

The resources in the economy are then distributed between consumption, investment, and government expenditure on policy:

$$Y_t = C_t + I_t + f\left(\frac{In_t + I_{ss}}{In_{t-1} + I_{ss}} \right) (In_t + Iss) + G_t + Po_t. \tag{G.18}$$

The central bank conducts monetary policy according to the simple rule:

$$i_t = (1-\rho_i)\left(i + \kappa_\pi \pi_t + \kappa_y (\log Y_t - \log Y_t^*) \right) + \rho_i i_{t-1} + \epsilon_t, \tag{G.19}$$

where Y^* is flexible output, ε_t is an exogenous monetary policy shock, and i is the steady-state nominal rate. ρ_i is a smoothing parameter lying between zero and one. The real and nominal interest rates are linked via the Fisher equation: $1 + i_t = R_t E_t (1 + \pi_{t+1})$.

Appendix H. Calibrated Parameters from Gertler and Karadi (2011)

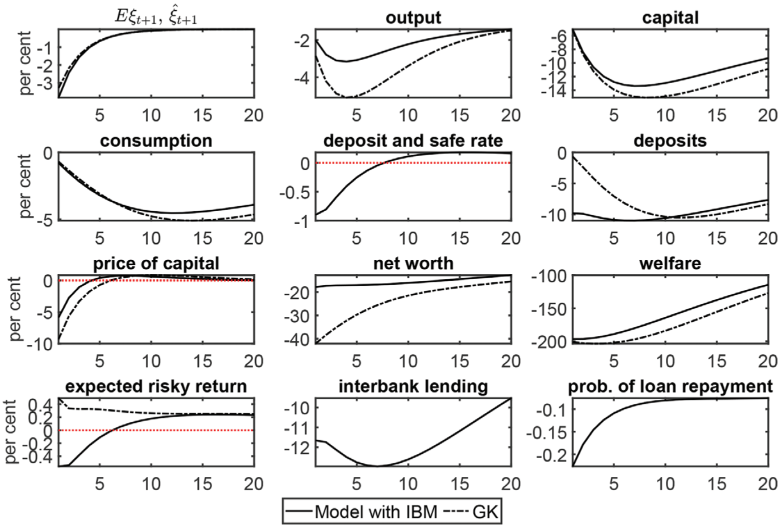
Table H.1. Calibrated Parameters from Gertler and Karadi (2011)

β	0.99	Household's Discount Rate
h	0.815	Habit Parameter
χ	3.409	Relative Utility Weight of Labor
ϕ	0.276	Inverse Frisch Elasticity of Labor Supply
λ	0.381	Fraction of Capital to be Diverted
θ	0.972	Survival Rate of Bankers
α	0.33	Capital Share
U	1	Steady-State Capital Utilization Rate
δ (U)	0.025	Steady-State Depreciation Rate
ζ	7.2	Elasticity of Marginal Depreciation with Respect to Utilization Rate
η_i	1.728	Inverse Elasticity of Net Investment to Price of Capital
ε	4.167	Elasticity of Substitution
γ	0.779	Probability of Keeping Prices Fixed
γ_p	0.241	Measure of Price Indexation
κ_π	1.5	Inflation Coefficient of Taylor Rule
κ_y	0.125	Output Gap Coefficient of Taylor Rule
ρ_i	0.8	Smoothing Parameter of Taylor Rule
$\frac{G}{Y}$	0.2	Steady-State Proportion of Government Expenditure
τ	0.001	Cost of Government Policy
κ	10	Reaction Parameter for Government Policy

Note: Parameter ω is a function of the steady-state values of K, N, and returns and is calculated to be 0.0059.

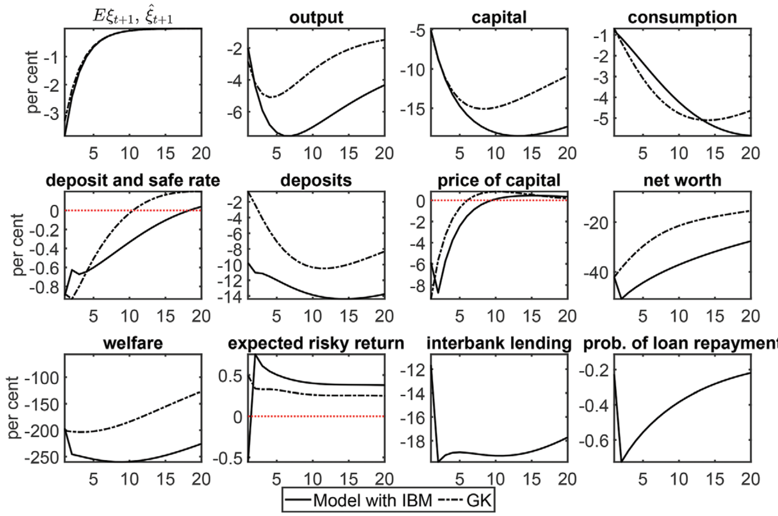
Appendix I. Extended Impulse Responses

Figure I.1. Crisis Simulations: Comparison with the Baseline



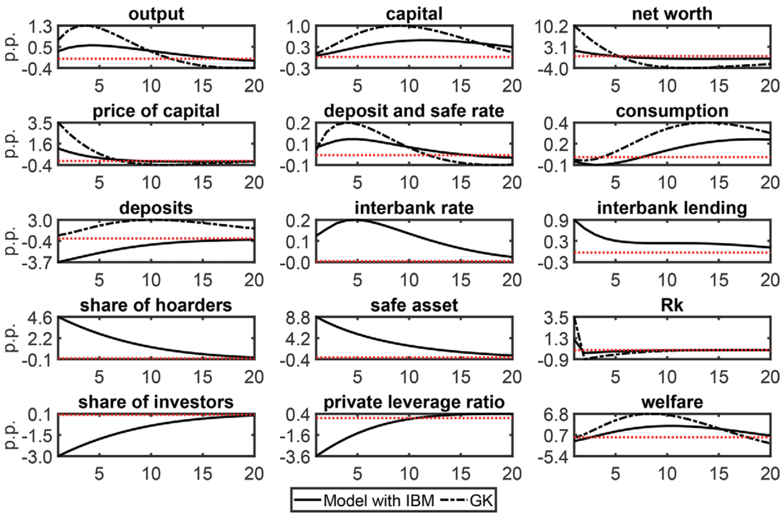
Note: The responses are plotted for a 5 percent transitory shock to capital quality ξ_t without a policy response from the central bank. The solid lines "Model with IBM" show the responses of my model, and the dashed and dotted lines "GK" show the responses of the baseline model of Gertler and Karadi (2011). Welfare is calculated as in Gertler and Karadi (2011) as a second-order approximation of household utility.

Figure I.2. Crisis Simulations: Comparable Net Worth



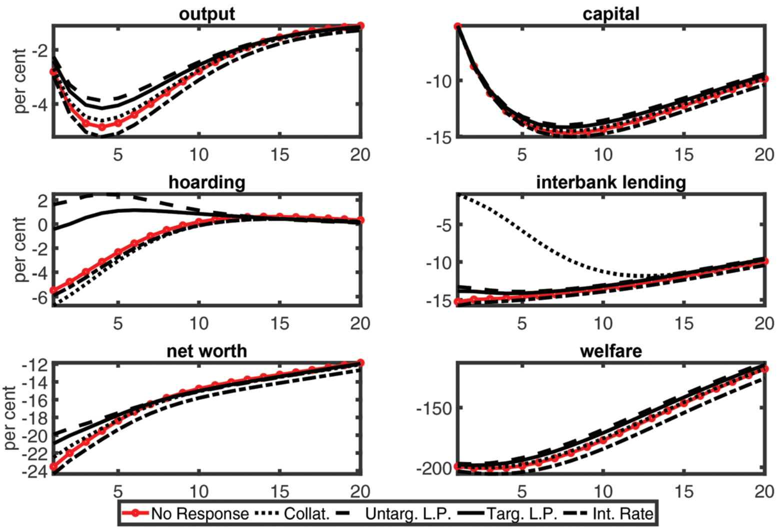
Note: The responses are plotted for a 5 percent transitory shock to capital quality ξ_t without a policy response from the central bank. The solid lines "Model with IBM" show the responses of my model, and the dashed and dotted lines "GK" show the responses of the baseline model of Gertler and Karadi (2011).

Figure I.3. Policy Effects vs. Baseline: Untargeted Liquidity Provision



Note: The responses are plotted for a 5 percent transitory shock to capital quality ξ_t under the policy of untargeted liquidity provision. The solid lines "Model with IBM" show the responses of my model, and the dashed and dotted lines "GK" show the responses of the baseline model of Gertler and Karadi (2011).

Figure I.4. Policy Effects



Note: The responses are plotted for a 5 percent transitory shock to ξ and a 5 percent fall in the average expert opinion, $\bar{E}_t\mu_{t+1}$. The solid line with dots "No Response" shows the model response without the policy response from the central bank, the dotted line "Collat." shows the case where the collateral constraints are relaxed, the dashed line "Untarg. L.P." illustrates the case with untargeted liquidity provision, and the solid line "Targ L.P." shows the case with targeted liquidity provision. The dashed and dotted line "Int. Rate" shows the simulations in which the central bank reduces the policy rate.